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Relationship Between Newtonian and MONDian Acceleration

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Abstract

Modified Newtonian Dynamics (MOND) is a theory that suggests a modification in Newtonian dynamics. It is an empirically motivated modification of Newtonian gravity without the need of dark matter. This theory is well fitted by explaining why the velocity of galaxies or stars are in linear and in flat curve. More than 84 galaxies are in a well fitting with MOND, and all fits of these galaxies can be found.

Keywords: Gravitational Theory, MONDian

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1 Introduction

Newtonian gravitational law states the attraction between every two objects attract one another with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This law is compatible with planets in solar system but not with further objects in huge massive systems like galaxies. According to this law the acceleration of a body in orbit is depends on the velocity and size of the orbit. This law accurately explain the relation between velocity and distances for near objects but it is not able to explain for larger velocities of further objects. Therefore it is seems that this law needs a correction for objects that are far away from the center of galaxy. Modified Newtonian Dynamics (MOND) is a theory that suggests a modification of Newton's laws for observed properties of galaxies. This theory created in 1983 by Mordehai Milgrom [1][2] and attempt to explain why the velocities of stars in galaxies were observed to are larger than expected based on Newtonian mechanics. The aim of this paper is to consider the basic idea of MOND with accelerations below $a_0 = 1.2 \times 10^{-10} m/s^2$ the effective gravitational attraction approaches to the usual Newtonian acceleration.

2 Newtonian Modification of Kepler 3^{th} Law

The MOND theory as an empirically motivated modification of Newtonian gravity describe the flat rotation curve without the need of dark matter. Fundamentally this theory created by Mordehai Milgrom in 1983 [1][2]. He introduced a new acceleration a_N, and a constant value a_0 ($= 1.2\times10^{-10} m/s^2$) in his theory. The relationship between real acceleration (or MONDian acceleration) a_M and newtonian acceleration a_N is,

$$a_M = \left(a_0 \frac{F_N}{m}\right)^{1/2} = (a_0.a_N)^{1/2}\,. \tag{1}$$

Here the value of F_N is Newtonian force and a_0 is a new physical constant and having dimensions of accelerating. This value can be experimentally determined from the rotation curve to fit observations. The value of a_0 in terms of Hubble constant and cosmological constant also is,

$$a_0 = 1.2 \times 10^{-10} m/s^2 \approx \frac{c.H_o}{2\pi} \approx \left(\frac{\Lambda}{3}\right)^{1/3}\,. \tag{2}$$

Here c is the speed of light, H is present Hubble constant and Λ is cosmological constant. When the Newtonian acceleration (a_N) is larger than a_0 ($a_N \gg a_0$) then Newtons value of acceleration is valid. When the Newtonian acceleration is very smaller than a_0, ($a_N \ll a_0$) then the gravity is stronger than Newtonian

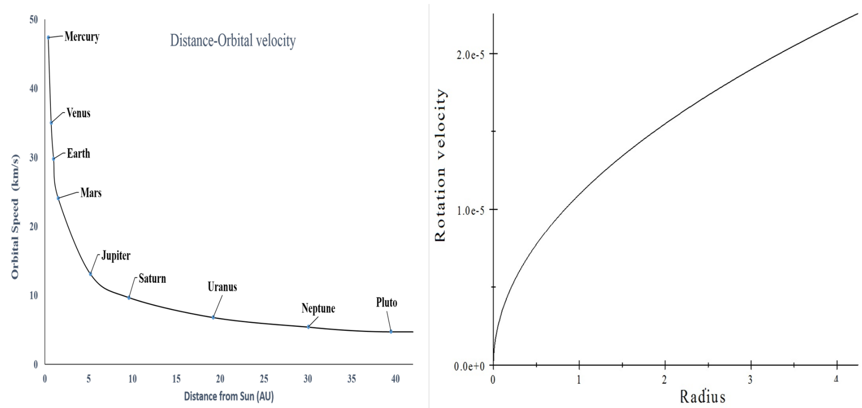

Figure 1: Rotation Curves a) In Newtonian dynamic, b)MONDian.

theory. Therefore MOND is for small acceleration and small acceleration is for very far away from the center of galaxy. The acceleration on the surface of the Earth always is much larger than a_0, therefore always entering Newtonian regime. Therefore,

$$F_N = \frac{dp}{dt} = \frac{d(m.v)}{dt} = F_N = m.a_N. \tag{3}$$

Here m , a_N and F_N are the mass, acceleration and Newtonian force, respectively. By substituting a_N in terms of MONDian acceleration, a_M,

$$a_N = \mu\left(\frac{|\vec{a}_M|}{a_0}\right)\vec{a}_M. \tag{4}$$

into the $F = ma$, a new form will produce which is MOND theory (Milgrom, 1983) [2] as

$$F_M = m.\left[\mu(x)\left(\frac{a_M}{a_0}\right)\right]a_M. \tag{5}$$

The MOND acceleration or MONDified is more accurate acceleration. The value of $\mu(x)$ is known as the interpolating function and in terms of x and a_0 as a new constant. For $a_N \gg a_0$ then $a_M = a_N$ and for $a_N \ll a_0$ then $a_M = \sqrt{a_N a_0}$. In usual form a_N is very larger than a_0 and then,

$$\mu(x)\left(\frac{a_M}{a_0}\right) = 1. \tag{6}$$

and Eq [5] reduced back to classical Newtonian form. Three possible forms of $\mu(x)$ which are acceptable to galaxy data are,

$$\mu(x) = \frac{x}{\sqrt{1+x^2}}. \tag{7}$$

or

$$\mu(x) = \frac{x}{1+x}. \tag{8}$$

and or

$$\mu(x) = 1 - e^{-x}. \tag{9}$$

The function normally used for galaxy fitting, suggested by Milgrom in 1983 and is

$$\mu(x) = \frac{x}{\sqrt{1+x^2}}. \tag{10}$$

Where here μ is a function that interpolates between the Newtonian regime, $\mu(x) = 1$, when $x \gg 1$ and it is in MOND regime, $\mu(x) = x$ when $x \ll 1$ [3]. By equality of Newton's law of gravity and MOND's modification have,

$$G\frac{M}{r^2} = \mu(x)\left(\frac{a_M^2}{a_0}\right) \tag{11}$$

If $a_M \ll a_0$ and therefore $\mu(x)(a_M/a_0) = (a_M/a_0)$, then we have

$$a_M = \left(\frac{GMa_0}{r^2}\right)^{1/2} = \frac{\sqrt{GMa_0}}{r} \tag{12}$$

For a circular motion centrifugal force $F = m(v_0)^2/r$ and with $a = (v_0)^2/r$, then

$$a = \frac{{v_0}^2}{r} = \frac{\sqrt{GMa_0}}{r} \tag{13}$$

and therefore

$${v_0}^2 = \sqrt{GMa_0} \tag{14}$$

For very low acceleration (VLA) a galaxy's rotational velocity approaches to the total mass with G , M and a_0 constant. A given mass, the rotation velocity converges to a constant value and then MOND predict a flat rotation curves as v. For $a_N \ll a_0$ then the gravitation force is

$$F_N = m\left(\frac{a^2}{a_0}\right) \tag{15}$$

In this case the gravitational forces in bound systems mostly is in Newtonian regime and the distance in MOND is,

$$R_M = \sqrt{\frac{GM}{a_0}} \tag{16}$$

Only at large distances from the central mass (e.g. in galaxies), the acceleration declines below a_0 and in this case R is $11.8kpc$ and $M = 10^{11}M_{sun}$. In our solar system, the gravitational acceleration of all planets lies well above a_0.

But for $a = a_0$ then $R = 7700\ AU$ and this is the region for Oort cloud. The rotational curve for $a_N \ll a_0$ must be slightly increasing and can determine the characteristic of acceleration a_0 from the observed circular velocity. At large distance, acceleration is smaller and enters into the MOND form and the circular velocity is

$$v_0 = (GMa_0)^{1/4} \tag{17}$$

The rotation velocity with MOND [1][5], shown in the Fig 2. The relationship

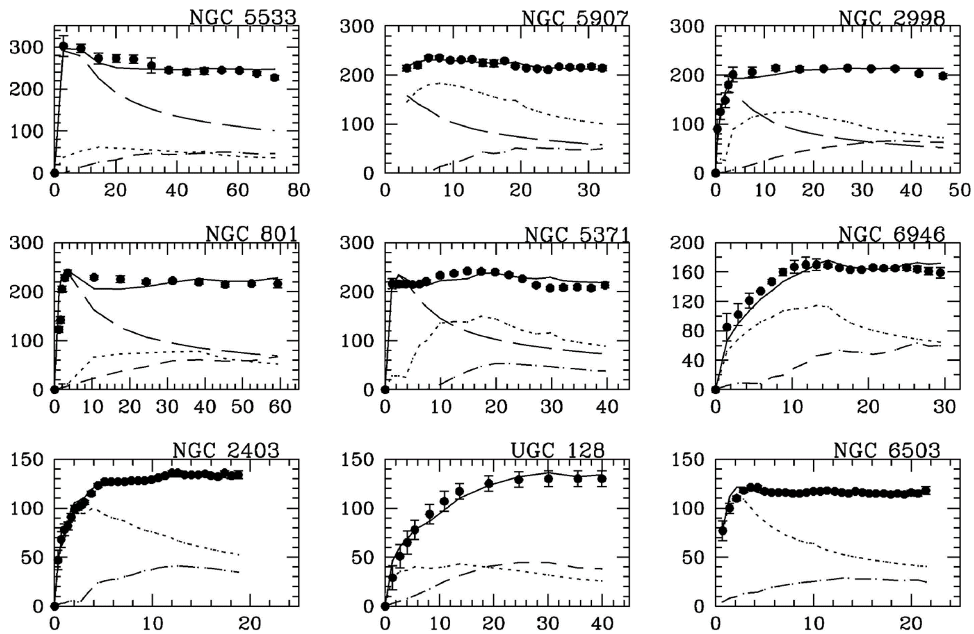

Figure 2: Rotation Curves with MOND [1][5].

between Newtonian and MONDian acceleration Eq [4] is

$$a_N = \left(\frac{a_M}{a_0}\right)\left[1 + \left(\frac{a_M}{a_0}\right)^2\right]^{1/2} a_M \tag{18}$$

and from this equation the value of a_M [4][6] is

$$a_M = \frac{a_N}{\sqrt{2}}\left[1 + \sqrt{1 + \frac{4{a_0}^2}{{a_N}^2}}\right]^{1/2} \tag{19}$$

The Eq [19] can be used for N-body modelling of gravity interaction instead of Newtonian gravitational acceleration. For LSB (Low Surface Brightness Galaxies) and HSB (High Surface Brightness Galaxies)

3 5-Conclusion

More than 84 galaxies are well fitted by MOND theory[7]. The list of galaxies are as: UGC 2885, NGC 5533, NGC 6674, NGC 7331, NGC 5907, NGC 2998, NGC 801, NGC 5371, NGC 5033, NGC 2903, NGC 3521, NGC 2683, NGC 3198, NGC 6946, NGC 2403, NGC 6503, NGC 1003, NGC 247, NGC 7739, NGC 300, NGC 5585, NGC 55, NGC 1560, NGC 3109, UGC 128, UGC 2259, M 33, IC 2574, DDO 170, DDO 168, NGC 3726, NGC 3769, NGC 3877, NGC 3893, NGC 3917, NGC 3949, NGC 3953, NGC 3972, NGC 3992, NGC 4010, NGC 4013, NGC 4051, NGC 4085, NGC 4088, NGC 4100, NGC 4138, NGC 4157, NGC 4183, NGC 4217, NGC 4389, UGC 6399, UGC 6446, UGC 6667, UGC 6818, UGC 6917, UGC 6923, UGC 6930, UGC 6973, UGC 6983, UGC 7089, NGC 1024, NGC 3593, NGC 4698, NGC 5879, IC 724, F563-1 F563-V2 F568-1, F568-3 F568-V1, F571-V1, F574-1, F583-1 F583-4 UGC 1230, UGC 5005, UGC 5999, Carina Fornax, Leo I, Leo II, Sculptor, Sextans Sgr. MOND theory is well supported by galactic observations; however, its extension to large-scale systems and cosmology remains unclear and need further research work.

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