Applied Mathematical Sciences, Vol. 4, 2010, no. 58, 2865 - 2880

Pulsatile Blood Flow through a Catheterized Artery with an Axially Nonsymmetrical Stenosis

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Abstract

Pulsatile flow of blood through a catheterized artery in presence of an axially nonsymmetrical mild stenosis with a velocity slip at stenotic wall has been investigated in this paper. Blood has been represented by a Newtonian fluid. By employing a perturbation analysis, analytic expressions for the velocity profile, flow rate, wall shear stress and effective viscosity, are derived. The influence of stenosis height, shape, slip velocity and radius of catheter on axial velocity, wall shear stress and effective viscosity are represented graphically and discussed. Graphical results show that wall shear stress and effective viscosity decrease while axial velocity increases with velocity slip at wall.

Mathematics Subject Classification: 76Z05

Keywords: Stenosis, Catheter, Newtonian fluid.

1. Introduction

An abnormal growth, formed due to deposits of atherosclerotic plaques in the lumen of an artery is usually called stenosis (atherosclerosis) and, its subsequent and severe growth on the artery wall results in serious circulatory disorders [6, 2, 25]. These disorders in circulatory systems may be included as, narrowing in body passage leading to the reduction and impediment to blood flow in the constricted artery regions, the blockage of the artery in making the flow irregular and causing an abnormality of the blood flow and, the presence of stenosis at one or more of the major blood vessels, carrying blood to the heart or brain etc., could lead to various arterial diseases e.g., myocardial infarction, angina pectoris, cerebral accident, coronary thrombosis, strokes etc.[7, 27, 8].

Catheterization refers to a procedure in which a long, thin, flexible plastic tube (catheter) is inserted into an artery [4]. A catheter with a tiny balloon at the end is inserted into the artery in balloon angioplasty to treat atherosclerosis. The catheter is carefully guided to the location at which stenosis occurs and balloon is inflated to fracture the fatty deposits and widen the narrowed portion of the artery [29]. The insertion of a catheter in an artery will naturally form an annular region between the walls of the catheter and artery. As a result, this will alter the flow field, like modifying the pressure distribution and increasing the resistance. Thus it is of immense importance to study the flow of blood in a catheterized artery.

In recent years, considerable attention has been paid to study the flow of blood in a catheterized artery. Many researchers have analysed the flow of blood in an artery, in presence of a catheter by modeling the catheter and artery as rigid co-axial cylinders and blood as either a Newtonian or a non-Newtonian fluid. McDonald [5] considered the pulsatile blood flow in a catheterized artery and obtained theoretical estimates for pressure gradient corrections for catheters. Karahalios [14] has studied the effect of catheterization on various flow characteristics in an artery with or without stenosis. Jayaraman and Dash [13] addressed a numerical study of blood flow in catheterized curved artery with constriction. Daripa and Dash [20] have analyzed the numerical study of pulsatile blood flow in an eccentric catheterized artery, using a fast algorithm and in considering blood as to behave like a Newtonian fluid. Dash et al. [26] considered the steady and pulsatile flow of blood in a narrow catheterized artery estimated the increase in frictional resistance in the artery due to catheterization, using a Casson fluid model. Sankar and Hemalatha [9] discussed the steady flow of Herschel–Bulkley fluid through a catheterized artery. Sankar [10] has studied a two-fluid model for the pulsatile flow of blood in a catheterized artery, by considering the core layer as a Casson fluid and the peripheral layer as a Newtonian fluid.

Although, blood exhibits a non-Newtonian character at low shear rates [11], at high shear rates, generally found in larger arteries (diameter nearly above 1mm), blood behaves like a Newtonian fluid [19]. Since, stenosis normally generates and develops in large diameter arteries (in the range of 500 to 2000 μm), where blood shows a Newtonian behaviour, it appears to be reasonable in assuming blood to be homogeneous, isotropic, incompressible, Newtonian continuum, having a constant viscosity and density for flow though stenosed arteries (having respective radii 10.0, 5.0, 4.0 and 1.5mm in aorta, femoral, carotid and coronary arteries [30]).

In most of the aforementioned studies, traditional no-slip boundary condition [18] has been employed. However, a number of studies of suspensions in general and blood flow in particular both theoretical [28, 1, 31, 21, 22] and experimental [17, 12], have suggested the likely presence of slip (a velocity discontinuity) at the flow boundaries (or in their immediate neighbourhood). The apparent (effective) viscosity will be lowered, as a result of wall slip [21]. Recently, Misra and Shit [15], Ponalgusamy [24], Biswas and Chakraborty [3] have developed mathematical models for blood flow through stenosed arterial segment, by taking a velocity slip condition at the constricted wall. Thus, it seems that consideration of a velocity slip at the stenosed vessel wall will be quite rational, in blood flow modeling.

With the above motivations an attempt has been made to study the effects of slip (at the stenotic wall) and the influence of stenosis height and shape, on the flow variables (wall shear stress, velocity profiles and effective viscosity) for pulsatile blood flow through a catheterized vessel with an axially nonsymmetrical mild stenosis.

2. Mathematical Formulations

We consider an axially symmetric, laminar, pulsatile and fully developed flow of blood (assumed to be incompressible) through a catheterized circular tube with an axially asymmetric but radially symmetric mild stenosis as shown in Fig. 1. It is assumed that wall of the tube is rigid and the body fluid blood is represented by a Newtonian fluid.

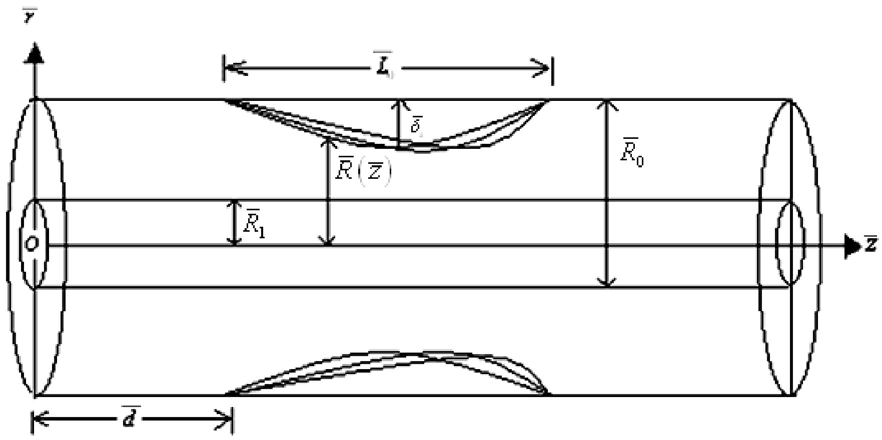

Fig.1.Geometry of an axially nonsymmetrical stenosis with an inserted catheter

The geometry of the stenosis [24] is given by

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - \bar{A}\left[\bar{L}_0^{\,n-1}\left(\bar{z}-\bar{d}\right)-\left(\bar{z}-\bar{d}\right)^n\right]; & \bar{d} \le \bar{z} \le \bar{d}+\bar{L}_0 \\ \bar{R}_0, \text{ otherwise,} & \end{cases} \tag{1}$$

where $\bar{R}\left(\bar{z}\right)$ is the radius of the artery in the stenosed region, $\overline{R_0}$ is the radius of the normal artery, $n\left(\ge 2\right)$ is a parameter (called shape parameter) determining the stenosis shape (the symmetric stenosis occurs when $n=2$), $\bar{L}_0$ is stenosis length and $\bar{d}$ indicates its location. The parameter $\bar{A}$ is given by

$$\bar{A} = \frac{\bar{\delta}_s n^{n/(n-1)}}{\bar{L}_0^n\left(n-1\right)},$$

where $\overline{\delta_s}$ denotes the maximum height of the stenosis located at $\bar{z} = \bar{d} + \bar{L}_0 / n^{n/(n-1)}$, such that $\bar{\delta}_s / \bar{R}_0 << 1$. It has been reported that the radial velocity is negligibly small and can be neglected for a low Reynolds number flow in a tube with mild stenosis [8, 23]. The equations of motion governing the fluid flow are given by

$$\bar{\rho}\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{\tau}\right), \tag{2}$$

$$\frac{\partial \bar{p}}{\partial \bar{r}} = 0. \tag{3}$$

where $\bar{u}$ is the fluid velocity in the axial direction, $\bar{\rho}$ is the density and $\bar{p}$ is the pressure.

The constitutive equation of Newtonian fluid is given by

$$\bar{\tau} = -\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{r}}. \tag{4}$$

where $\bar{\mu}$ is the coefficient of viscosity and τ is the shear stress.
The boundary conditions are given by

$$\bar{u} = \bar{u}_s \text{ at } \bar{r} = \bar{R}\left(\bar{z}\right) \tag{5}$$

$$\bar{u} = 0 \text{ at } \bar{r} = \bar{R}_1 \tag{6}$$

where $\bar{u}_s$ is the slip velocity at the stenotic wall [2] and $R_1\left(<< R_0\right)$ is the radius of the catheter.

Since, the pressure gradient is a function of $\bar{z}$ and $\bar{t}$, we take

$$-\frac{\partial \bar{p}}{\partial \bar{z}}(\bar{z},\bar{t}) = q(\bar{z})\, f(\bar{t}), \tag{7}$$

where $\bar{q}(\bar{z}) = -\frac{\partial \bar{p}}{\partial \bar{z}}(\bar{z},0)$, $f(\bar{t}) = 1 + a\sin\bar{\omega}\bar{t}$, a is the amplitude and $\bar{\omega}$ is the angular frequency of blood flow [6].

Let us introduce the following non-dimensional variables

$$z = \frac{\bar{z}}{\bar{R}_0},\ R(z) = \frac{\bar{R}(\bar{z})}{\bar{R}_0}, R_1 = \frac{\bar{R}_1}{\bar{R}_0}, r = \frac{\bar{r}}{\bar{R}_0},\ t = \bar{t}\,\bar{\omega}\ ,\ L_0 = \frac{\bar{L}_0}{\bar{R}_0}, d = \frac{\bar{d}}{\bar{R}_0}\quad \delta_s = \frac{\bar{\delta}_s}{\bar{R}_0}\ ,$$
$$A = \bar{A}\bar{R}_0^{\,n-1},\ u = \frac{\bar{u}}{\bar{q}_0\bar{R}_0^{\,2}/4\bar{\mu}},\ u_s = \frac{\bar{u}_s}{\bar{q}_0\bar{R}_0^{\,2}/4\bar{\mu}},\ \alpha^2 = \frac{\bar{R}_0^2\bar{\omega}\bar{\rho}}{\bar{\mu}} \tag{8}$$

where α is the pulsatile Reynolds numbers for Newtonian fluid and $\bar{q}_0$ is the negative of the constant pressure gradient in a uniform tube without catheter.
The non-dimensional form of geometry of stenosis is given by

$$R(z) = \begin{cases} 1 - A\left[L_0^{\,n-1}(z-d) - (z-d)^n\right]; & d \le z \le d + L_0 \\ 1,\ \text{otherwise}, & \end{cases} \tag{9}$$

Using non-dimensional variables equations (2) and (4) reduce to

$$\alpha^2\frac{\partial u}{\partial t} = 4q(z)\, f(t) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau), \tag{10}$$

$$\tau = -\frac{1}{2}\frac{\partial u}{\partial r}. \tag{11}$$

With the help of (11), equation (10) becomes

$$\alpha^2\frac{\partial u}{\partial t} = 4q(z)\, f(t) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right). \tag{12}$$

The boundary conditions in the non-dimensional form are given by

$$u = u_s \text{ at } r = R(z), \tag{13}$$

$$u = 0 \text{ at } r = R_1. \tag{14}$$

The non-dimensional volumetric flow rate is given by

$$Q = 4\int_{R_1}^{R(z)} ru(r,z,t)dr\,, \tag{15}$$

where $Q(t) = \dfrac{\overline{Q}(\overline{t})}{\dfrac{\pi\left(\overline{R}_0\right)^4 \overline{q}_0}{8\overline{\mu}}}$; $\overline{Q}(\overline{t}) = 2\pi\int_{\overline{R}_1}^{\overline{R}(\overline{z})} \overline{ru}\left(\overline{r},\overline{z},\overline{t}\right)d\overline{r}$ is the volumetric flow rate.

The effective viscosity μ_e defined as

$$\overline{\mu}_e = \frac{\pi\left(-\dfrac{\partial \overline{p}}{\partial \overline{z}}\right)\left(\overline{R}\left(\overline{z}\right)\right)^4}{\overline{Q}(\overline{t})}, \tag{16}$$

can be expressed in dimensionless form as

$$\mu_e = \frac{\left(R(z)\right)^4}{Q(t)} q(z)\, f(t), \tag{17}$$

where $Q(t)$ is defined in equation (15)

3. Solution

Considering the Womersley parameter to be small, the velocity u can be expressed in the following form

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \ldots\ldots.. \tag{18}$$

Substituting the expression of u from equation (18) in (12), we have

$$\frac{\partial}{\partial r}\left(r\frac{\partial u_0}{\partial r}\right) = -4rq(z)\, f(t), \tag{19}$$

$$\frac{\partial u_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial r}\right). \tag{20}$$

Substituting u from equation (18) into conditions (13) and (14) we get

$$u_0 = u_s, u_1 = 0 \text{ at } r = R(z) \text{ and } u_0 = 0, u_1 = 0,\ r = R_1 \tag{21}$$

To determine u_0 and u_1, we integrate equations (19) and (20) twice with respect to r and use the boundary conditions (21) (for u_1, using the expression obtained for u_0), we have

$$u_0 = \left[1 - \frac{\log\left(\frac{r}{R}\right)}{\log\left(\frac{R_1}{R}\right)}\right] u_s + q(z)\, f(t) \left[\left(R^2 - r^2\right) - \frac{\left(R^2 - R_1^2\right)}{\log\left(\frac{R_1}{R}\right)} \log\left(\frac{r}{R}\right)\right], \tag{22}$$

$$u_1 = \frac{q(z)\, f'(t)}{16} \left[\left(4R^2r^2 - r^4 - 3R^4\right) - \frac{\left(R^2 - R_1^2\right)}{\log\left(\frac{R_1}{R}\right)} \left\{4r^2 \log\left(\frac{r}{R}\right) - 3r^2 + 3R^2\right\}\right.$$
$$\left. - \frac{\log\left(\frac{r}{R}\right)}{\log\left(\frac{R_1}{R}\right)} \left\{4R^2R_1^2 - R_1^4 - 3R^4 - \frac{\left(R^2 - R_1^2\right)}{\log\left(\frac{R_1}{R}\right)} \left(4R_1^2 \log\left(\frac{R_1}{R}\right) + 3R^2 - 3R_1^2\right)\right\}\right] \tag{23}$$

where $R = R(z)$

The expression for velocity u can easily be obtained from equations (18), (22) and (23).

The wall shear stress τ_w (as a result of equations (11) and (18)) becomes

$$\tau_w = -\frac{1}{2}\left(\frac{\partial u_0}{\partial r} + \alpha^2 \frac{\partial u_1}{\partial r}\right)_{r=R(z)} \tag{24}$$

which is determined, by substituting velocity expressions (22) and (23) into the above equation (24), in the form

$$\tau_w = \frac{u_s}{2R\log\left(\frac{R_1}{R}\right)} + q(z)\, f(t) \left\{R + \frac{\left(R^2 - R_1^2\right)}{2R\log\left(\frac{R_1}{R}\right)}\right\} -$$
$$\frac{\alpha^2}{32} q(z)\, f'(t) \left[4R^3 + 2R\frac{\left(R^2 - R_1^2\right)}{\log\left(\frac{R_1}{R}\right)} - \right.$$
$$\left. \frac{1}{R\log\left(\frac{R_1}{R}\right)} \left\{\left(4R^2R_1^2 - R_1^4 - 3R^4\right) - \frac{\left(R^2 - R_1^2\right)}{R\log\left(\frac{R_1}{R}\right)} \left(4R_1^2 \log\left(\frac{R_1}{R}\right) + 3R^2 - 3R_1^2\right)\right\}\right] \tag{25}$$

From equation (15), (22) and (23) the expression for volumetric flow rate is given by

$$Q(t)=\left[2\left(R^2-R_1^2\right)+\frac{2R_1^2\log\left(\frac{R_1}{R}\right)-\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\right]u_s+$$

$$q(z)\,f(t)\left[\left(R^2-R_1^2\right)^2-\frac{\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\left\{\left(R_1^2-2R_1^2\log\left(\frac{R_1}{R}\right)-R^2\right)\right\}\right]+$$

$$\frac{\alpha^2}{48}q(z)\,f^{/}(t)\Big[\left\{18R^4R_1^2+2R_1^6-12R^2R_1^4-8R^6\right\}$$

$$-\frac{\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\left\{6R^4-12R_1^4\log\left(\frac{R_1}{R}\right)+12R_1^4-18R_1^2R^2\right\}+\left\{6R_1^2+\frac{3\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\right\}\mathrm{X}$$

$$\left\{4R^2R_1^2-R_1^4-3R^4-\frac{\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\left\{24R_1^2\log\left(\frac{R_1}{R}\right)+18\left(R^2-R_1^2\right)\right\}\right\}\Big] \tag{26}$$

The effective viscosity μ_e can be found out with the help of equations (17) and (26).

If steady flow is considered, then equation (26) reduces to

$$\left[2\left(R^2-R_1^2\right)+\frac{2R_1^2\log\left(\frac{R_1}{R}\right)-\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\right]u_s+$$
$$q(z)\left[\left(R^2-R_1^2\right)^2-\frac{\left(R^2-R_1^2\right)}{\log\left(\frac{R_1}{R}\right)}\left\{\left(R_1^2-2R_1^2\log\left(\frac{R_1}{R}\right)-R^2\right)\right\}\right]=Q_s \tag{27}$$

where Q_s is the steady state flow rate.

Taking $Q_s=1$ [8], the value of $q(z)$ can easily be found out from equation (27).

In absence of catheter, i.e. when $R_1=0$, the equations (22), (23), (25), (26) reduce to

$$u_0=u_s+q(z)\,f(t)\left(R^2-r^2\right), \tag{28}$$

$$u_1 = \frac{q(z)\,f'(t)}{16}\left(4R^2r^2 - r^4 - 3R^4\right), \tag{29}$$

$$\tau_w = q(z)\,f(t)\,R - \frac{\alpha^2}{8}q(z)\,f'(t)\,R^3, \tag{30}$$

$$Q(t) = \left(R(z)\right)^2\left\{2u_s + q(z)\,f(t)\left(R(z)\right)^2 - \frac{\alpha^2}{6}q(z)\,f'(t)\left(R(z)\right)^4\right\}. \tag{31}$$

The equations (28)-(31) are in good agreement with the result obtained in [3] without body acceleration.

4. Results and Discussions

The present model has been developed to analyse the effects of stenosis height, shape, catheter radius and slip velocity on axial velocity, shear stress and effective viscosity. The value 0.5 is taken for the amplitude a and the pulsatile Reynolds' number α, the range 0-0.2 is taken for the height of the stenosis δ_s. Radius of the catheter is taken in the range 0-0.5 and the value of the stenosis shape parameter is taken from 2 to 6.

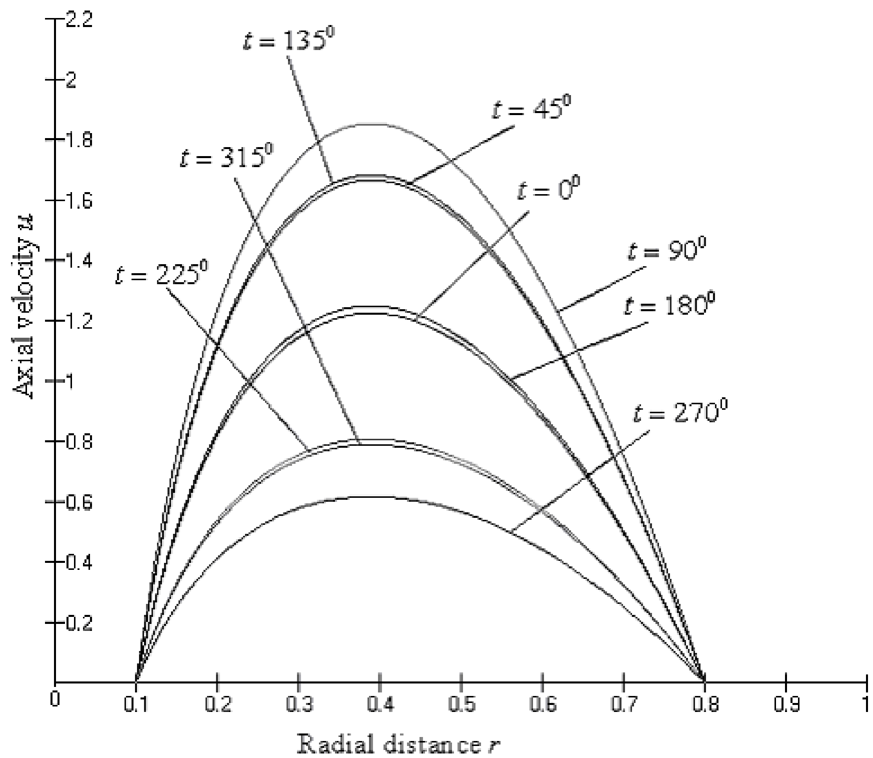

Fig. 2: Variation of axial velocity with radial distance for different values of t

The variation of axial velocity with radial distance for different values of time t and for fixed values of $\delta_s = 0.2, R_1 = 0.1, \alpha = 0.5, z = 8, n = 2$ is presented in Fig. 2. It is found that velocity increases as the time t increases from $t = 0^0$ to $t = 90^0$ and then it deceases from $t = 90^0$ to $t = 270^0$ and then again it increases from $t = 270^0$ to $t = 360^0$.

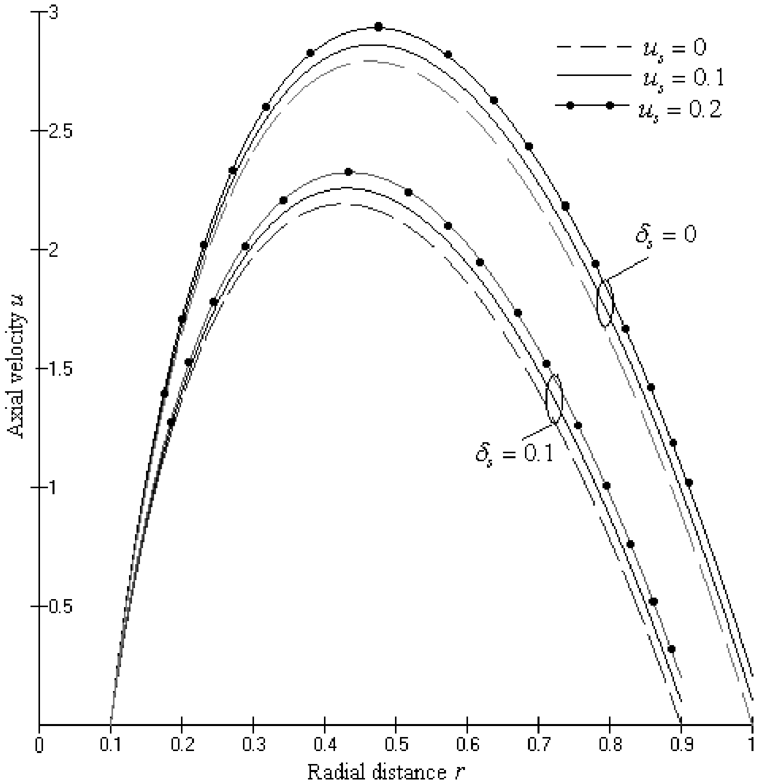

Fig.3: Variation of axial velocity with radial distance for different δ_s and u_s

Fig.3 represents the variation of axial velocity with radial distance for different values of stenosis height δ_s, slip velocity u_s and for fixed values of $R_1 = 0.1, \alpha = 0.5, z = 8, n = 2, t = 45^0$. The magnitude of axial velocity is observed to be more in uniform artery than that in a stenosed tube. Also increase in velocity slip increases the axial velocity in both uniform and stenosed vessels.

Fig. 4 depicts the variation of wall shear stress with axial distance for different values of catheter radius R_1, stenosis shape parameter n, slip velocity u_s and at $\alpha = 0.5, t = 45^0, \delta_s = 0.2$. It is observed that the wall shear stress distribution, in the stenotic region increases with the axial distance in the upstream of the stenosis throat and attains its maximum magnitude at the throat, wherefrom it decreases with the axial distance. The wall shear stress decreases with increasing shape parameter, n in the upstream of the throat but this property reverses in the downstream. The magnitude of τ_w is found to be more in catheterised artery than that in uncatheterized one. However, for any values of n and R_1, employment of velocity slip at wall decreases the wall shear stress.

The variation of wall shear stress with catheter radius R_1 for various magnitudes of u_s and δ_s at $z = 8, n = 2, t = 45^0$ is described in Fig. 5. From figure it is noted that wall shear stress increases with catheter radius in both uniform and stenosed arteries. On the other hand, increase in slip velocity reduces the wall shear stress.

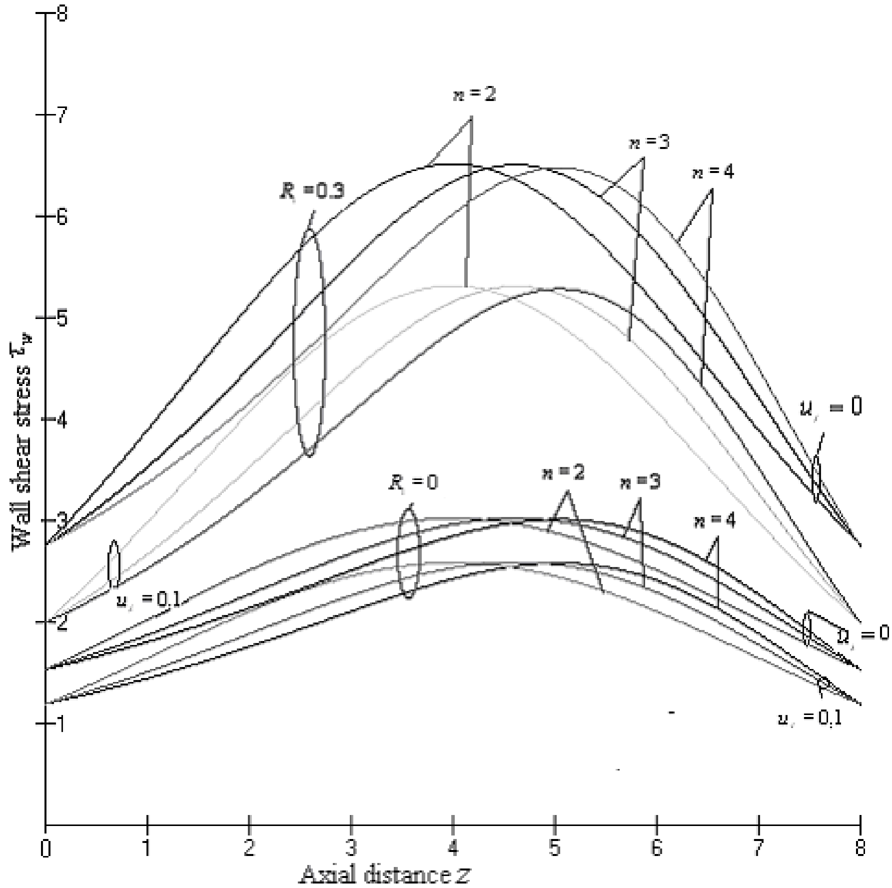

Fig. 4: Variation of wall shear stress with axial distance for different u_s, R_1 **and** n

Fig.6 shows the variation of effective viscosity with the radius of the inserted catheter for different values of shape parameter n and slip velocity u_s at $\delta_s = 0.2, t = 45^0$. It is found that effective viscosity increases with catheter radius significantly but decreases with shape parameter and slip velocity.

Fig.7 describes the variation of effective viscosity with catheter radius for various magnitudes of δ_s and u_s and for fixed values of $n = 2, t = 45^0$. It is noted that μ_e in uniform artery is less in magnitude than that in stenosed tube. Also increase in stenosis height increases the effective viscosity. However, employment of slip velocity reduces the magnitude of μ_e in both uniform and stenosed arteries.

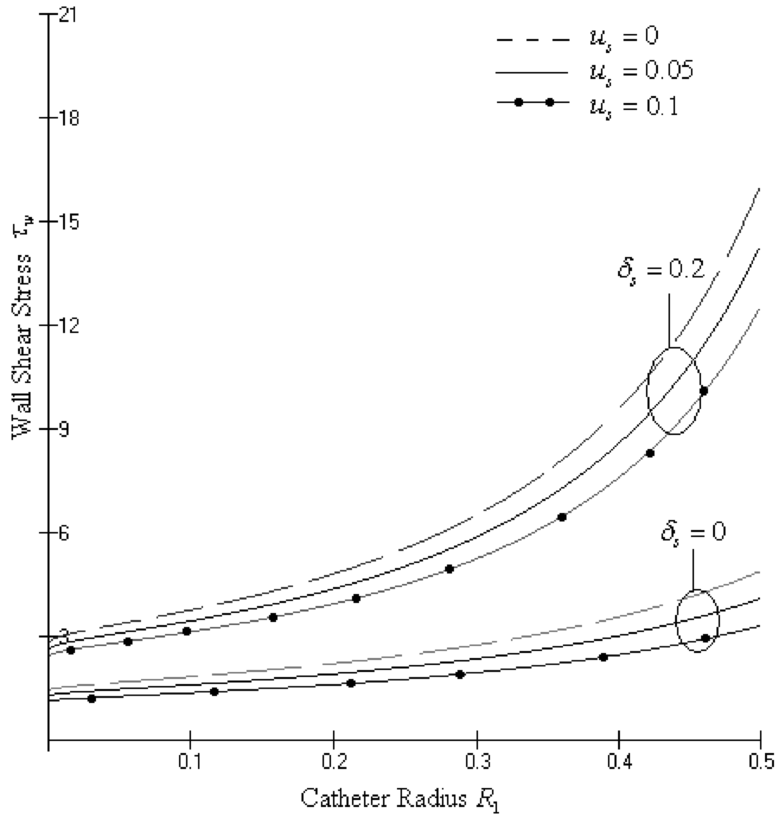

Fig. 6: Variation of wall shear stress with catheter radius for different δ_s and u_s

Concluding Remarks

In this paper an attempt has been made to study the effects of stenosis height, shape and slip velocity on various flow variables. The governing equation of motion has been integrated by employing a perturbation technique considering very small Womersely frequency parameter. It is observed that increase in shape parameter increases the wall shear stress in the upstream of the throat but decreases in the downstream. From the analysis it can be concluded that slip velocity has a significant role to play in reducing wall shear stress and effective viscosity. Since elevation of blood viscosity is considered as a serious risk factor in the cardiovascular, hematological, neoplastic and other disorders [16], the present model may be used as a tool for reducing blood viscosity by using slip velocity at stenotic wall. Also, more interesting models can be studied by considering the severe stenosis and permeability of the vessel wall. These studies will be done in the near future.

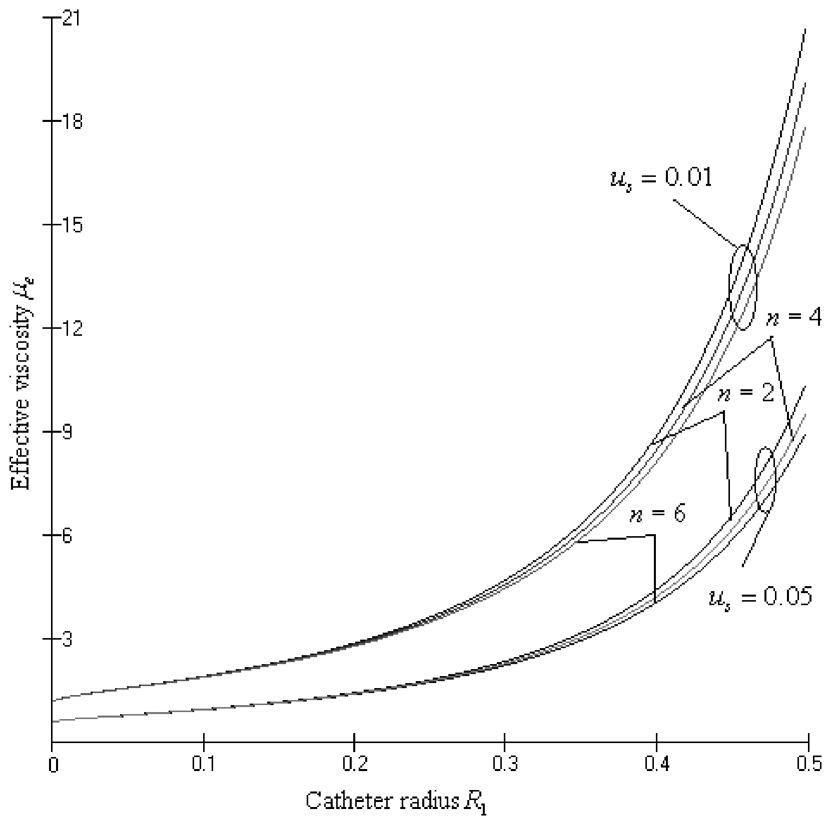

Fig. 7: Variation of effective viscosity with catheter radius for different u_s and n

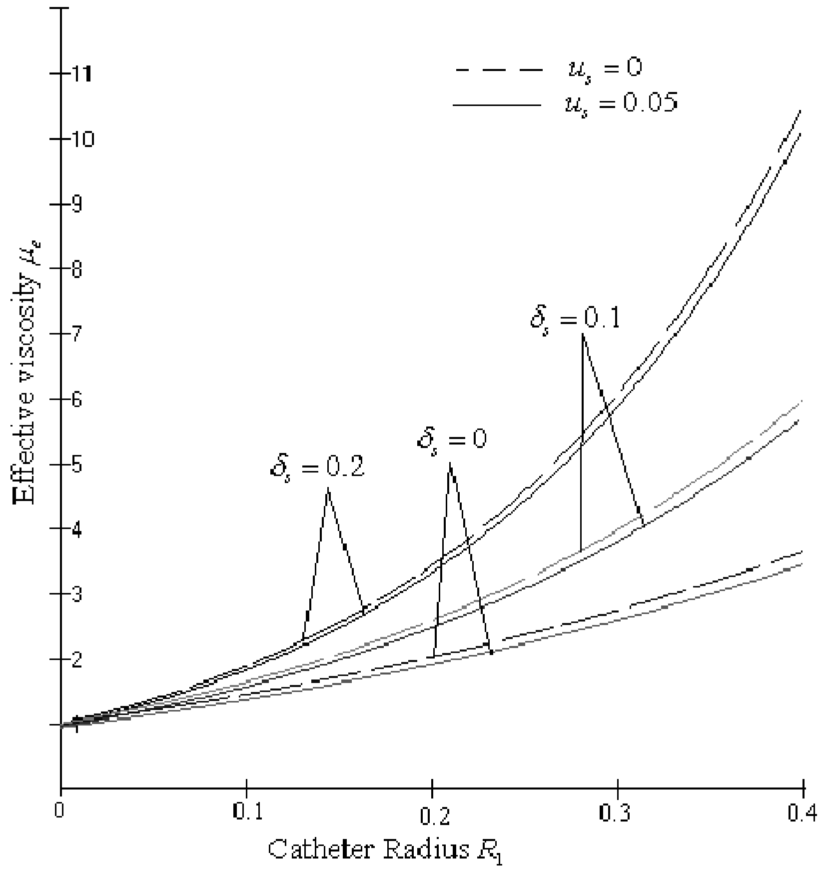

Fig.8: Variation of effective viscosity with catheter radius for different u_s and δ_s

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Received: April, 2010