DEA-Based Production Planning Changes
in General Situation

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Abstract

In great producing organizations (such as factory with several workshops) and chain stores supplying products (such as chain supermarkets and restaurants) under consideration the central decision-making unit, usually there are several Decision-Making Units (DMUs) participate in total production and supply. The purpose of this research is to present new input and output plans, determined by central Decision-Making Unit, for all DMUs. In this plan, demand changes and input changes can be predicted for the next production season. This new production plan has been presented through optimizing the average or overall production performance of the entire organization, measured by BCC efficiency. By using new plans the efficiency of each DMU will be maximized. It is supposed that all DMUs are able to improve their input usages and output productions. To illustrate this approach, two simple numerical examples and a real data for twenty restaurant chains are used.

Keywords: Production plan, Data Envelopment Analysis (DEA), Decision-Making Unit (DMU), centralized Decision-Making Unit, input, output, and BCC efficiency

1. Introduction

The production in great organizations and production supply in chain stores equipped with a central Decision-Making unit are usually executed with cooperation of more than one DMU. If input and output changes in entire organization are predicted for next season, how the central decision-making unit should present new input and output plan for each DMU for next production season to meet the changes and maximize each DMU efficiency. It should be noted that the presented available DEA models can not

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answer the above question. In this paper a model is presented on the basis of BCC model in DEA to answer the above question. Many researchers have examined the production planning in different viewpoints. Chazel et al. [3] studied the deterministic optimization problem of a profit-maximizing firm which plans its sales production schedule. They assume that the storage cost is convex. This allows them to relate the optimal planning problem to the study of a backward integro-differential equation, from which an explicit construction of the optimal plan can be obtained. Pastor et al. [10] presented a case of production planning in a woodturning company. The goal is to meet the demand at minimum cost while subjected to a series of principal conditions. Charnes et al. [2] developed DEA model for measuring relative efficiency of peer decision-making units (DMUs) which have multiple inputs and outputs. Juan Du et al. [7] considered a centralized decision-making environment in a large organization with set of n homogenous DMUs under a central decision-making unit which acts like a supervisor. They suppose demand changes can be predicted for next production season and presented new input and output plan for each DMU for next season in order to meet those changes and all DMUs will be CCR efficient after planning.

In recent years, DEA has been used for DMUs in different ways. Yu MM et al. [11] provided a multi-activity network data envelopment analysis model to show efficiency and effectiveness in railways operations. Eilat et al. [5] studied R&D project evaluations. Meng et al. [9] showed that it is possible to develop DEA models that utilize hierarchical structures of input-output data so that they are able to handle quite large numbers of inputs and outputs. Botti et al. [1] opened prospects for researches that there is a relationship between the organization form. Li et al. [8] presented models for measuring and benchmarking the performance of nations at six summer Olympic games. Cooper et al. [4] edited the handbook on Data Envelopment Analysis with many DEA application examples.

A summary of idea 1 concerning article [7] is stated in section 2. In section 3 first we presented problems of idea 1 in article [7] then new idea based upon BCC efficiency analysis is presented to determine future production planning two eliminate those problems. In section 4 this new idea is executed by two simple examples and twenty fast food restaurants in china [7]. It is concluded in section 5.

2. Background

Juan Du et al. [7] considered a centralized decision-making environment in a large organization with set of n homogenous DMUs under a central decision-making unit which acts like a supervisor. All n DMUs produce the same set of outputs and cost the same set of inputs. Suppose this set of n DMUs have to be evaluated and the ith input and rth output of $\text{DMU}_j$ $(j = 1, \ldots, n)$ are denoted by $x_{ij}$ $(i = 1, \ldots, m)$ and $y_{rj}$ $(r = 1, \ldots, s)$, respectively.
In relation to the question, "If it is possible to predict demand changes in next production season, how the central decision-making unit can present new input and output plan for all DMUs to maximize their efficiencies?" two ideas are proposed in article[7]. Idea 1 first examined CCR efficiency of all DMUs then model (1.2) which is one of DEA models[2] has been presented:

$$\max \ h_j = \frac{\sum_{r=1}^{s} u_r y_{rij}}{\sum_{i=1}^{m} v_i x_{ij}}$$

$$s.t. \ \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ijk}} \leq 1 \ \ k = 1, ..., n$$ (1.2)

In idea 1[7] replies above mentioned question on the basis of model (1.2) by different models and it is supposed that it is possible to predict demand changes for output r th (r=1,...,s) in next production season and is shown by $D_r$. First Juan Du et al.[7] assumed that all demand changes for all outputs are positive or negative. Second they assumed that some demand changes are positive, some are negative and some are zero.

Then by following two steps they determined a new input and output plan for all DMUs. Step 1: Considering output changes only in category $J_p$ and assuming the same positive proportional change denoted by $\Delta_j^{(p)} \geq 0 \ (j=1, ..., n)$ in all inputs, and outputs in $J_p$. Step 2: Considering output changes in category $J_n$ only and assuming the same negative proportional change denoted by $-1 \leq \Delta_j^{(n)} \leq 0 \ (j=1, ..., n)$. Let $\Delta_j^{(p)}$ and $\Delta_j^{(n)}$ be an optimal (local or global) solution for steps 1 and 2, respectively. Therefore, the new input and output plan for DMU$j$ (j=1, ..., n) is as follows:

$$((1 + \Delta_j)x_{ij}, ..., (1 + \Delta_j)x_{mj}; (1 + \Delta_j)y_{ij}, ..., (1 + \Delta_j)y_{sj})$$

3. The model for positive or negative changes

However, the mentioned models in idea 1[7] have some problems:

1.1.3 In all models of idea 1 in [7], all inputs will change with the same proportion. But the portion of all inputs that produces the rth (r=1, ..., s) output may not be the same.

2.1.3 A DMU may not be able to use all its ability perfectly. It means, in some cases, a DMU
may have the ability to change its outputs more than amounts indicated in idea 1[7]. For example, if \( Y_o = (5,10,10)' \) and suppose DMU \( o \) can increase its outputs up to \((10,13,30)'\), and demand changes be predicted as \( (\bar{D}_1, \bar{D}_2, \bar{D}_3) = (5,3,20) \). So from the constraints
\[
\sum_{j=1}^{n} \Delta_j y_{oj} \leq \bar{D}_r , r = 1,\ldots,n \quad \text{we have} \quad \Delta_o y_{ro} \leq \bar{D}_r , r = 1,2,3 \quad \text{then} \quad \Delta_o \leq 0.3 . \quad \text{If} \quad \Delta_o = 0.3 \quad \text{be selected, then} \quad \text{rth output is use} \quad \bar{D}_r - \Delta_o y_{ro} \quad \text{less than its ability.}
\]

The following models eliminate those problems:
Suppose there is set of n homogenous DMUs which have to be evaluated. The original BCC efficiency for DMU \( j \) \((j=1,\ldots,n)\) is measured by the linear program which obtained from model (1.3) [6] :
\[
\begin{align*}
\text{Max} & \quad \frac{U^T Y_p + u_o}{V^T X_p} \\
\text{s.t} & \quad \frac{U^T Y_j + u_o}{V^T X_j} \leq 1 , \quad j = 1,\ldots,n \quad (1.3) \\
& \quad U \geq 1 \epsilon , \quad V \geq 1 \epsilon \\
& \quad u_o \quad \text{free}
\end{align*}
\]

If the upper changes for input \( i \) \((i=1,\ldots,m)\) and output \( r \) \((r=1,\ldots,s)\) can be predicted in the next production season as \( \bar{D}_r \) and \( \bar{E}_i \), respectively. These changes can be positive or negative corresponding to an increasing or decreasing of output \( r \) and input \( i \). In order to meet these changes, the central decision-making unit will determine the most preferred input-output plans for all DMUs. In new idea, the average or overall production performance in the entire organization will be optimized after planning, which means maximizing the BCC efficiency of average input and output levels of all DMUs, base upon DEA models. Notice that these changes for each input and output can be different. Here researchers have considered two cases. In 1.2.3 they assumed that all changes are positive or negative. In 2.2.3 they assumed that some changes are positive, some are negative and some are zero. Then suitable production planning models were developed for above situations.

Since variable return to scale (VRS) is assumed, \( x_{ij} \) \((i=1,\ldots,m)\) and \( y_{ij} \) \((r=1,\ldots,s)\) will change by the different proportion denoted by \( \Delta_{ij} \geq -1 \) \((i=1,\ldots,m)\) and \( \Delta_{ij} \geq -1 \) \((r=1,\ldots,s)\), respectively.

If \( \bar{D}_r \geq 0 \) then \( \Delta_{oj} \geq 0 \), and if \( \bar{D}_r \leq 0 \) then \(-1 \leq \Delta_{oj} \leq 0 \).

If \( \bar{E}_i \geq 0 \) then \( \Delta_{ij} \geq 0 \), and if \( \bar{E}_i \leq 0 \) then \(-1 \leq \Delta_{ij} \leq 0 \).

Furthermore, \( \sum_{j=1}^{n} \Delta_{oj} y_{oj} \leq \bar{D}_r \) \((r=1,\ldots,s)\) and \( \sum_{j=1}^{n} \Delta_{ij} x_{ij} \leq \bar{E}_i \) \((i=1,\ldots,m)\).
Based upon model (1.3) and above assumptions and the average input and output levels of all DMUs, the model (2.3) will be presented as follows:

\[
\begin{align*}
\text{Max} \quad 0 &= \frac{\sum_{r=1}^{s} u_r \left[ \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}) y_{ij} \right] + u_0}{\sum_{i=1}^{m} v_i \left[ \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}') x_{ij} \right]} \\
\text{s.t} \quad \frac{\sum_{i=1}^{m} v_i \left[ \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}') x_{ij} \right]}{\sum_{i=1}^{m} v_i \left[ \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}) y_{ij} \right] + u_0} &\leq 1 \\
\sum_{i=1}^{m} v_i \left( 1 + \Delta_{ij} \right) y_{ij} + u_0 &\leq 1, \quad j = 1, \ldots, n \\
\sum_{j=1}^{n} \Delta_{ij} y_{ij} &\leq D_r, \quad r = 1, \ldots, s \\
\sum_{j=1}^{n} \Delta_{ij}' x_{ij} &\leq E_i, \quad i = 1, \ldots, m \\
v_i \geq \varepsilon, u_r \geq \varepsilon, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s \\
\begin{cases}
\Delta_{ij} \geq 0 \quad \text{when} \quad D_r \geq 0, \quad j = 1, \ldots, n, \quad r = 1, \ldots, s \\
-1 \leq \Delta_{ij} \leq 0 \quad \text{when} \quad D_r \leq 0, \quad j = 1, \ldots, n, \quad r = 1, \ldots, s \\
\Delta_{ij}' \geq 0 \quad \text{when} \quad E_i \geq 0, \quad j = 1, \ldots, n, \quad i = 1, \ldots, m \\
-1 \leq \Delta_{ij}' \leq 0 \quad \text{when} \quad E_i \leq 0, \quad j = 1, \ldots, n, \quad i = 1, \ldots, m
\end{cases}
\end{align*}
\]

When the variables are changed as \( u_i = t u_0 \), \( w_i = t v_i \), \( \mu_r = t u_r \), where \( t = \left[ \sum_{i=1}^{m} v_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}') x_{ij} \right) \right]^{-1} \), the above model (2.3) can be converted to the model (3.3):
Max $\theta = \sum_{r=1}^{s} \mu_r \left( \sum_{j=1}^{n} (1+\Delta_{gj})y_{gj} \right) + \nu u_1$

s.t $\sum_{r=1}^{s} \mu_r \left( \sum_{j=1}^{n} (1+\Delta_{gj})y_{gj} \right) + \nu u_1 \leq 1$

$\sum_{r=1}^{s} \mu_r (1+\Delta_{gj})y_{gj} + \nu u_1 \leq \sum_{i=1}^{m} w_i (1+\Delta'_{ij})x_{ij} , \ j=1,...,n$ \hspace{1cm} (a)

$\sum_{i=1}^{m} w_i \sum_{j=1}^{n} (1+\Delta'_{ij})x_{ij} = 1$

$\sum_{j=1}^{n} \Delta_{gj} y_{gj} \leq \overline{D}_r \hspace{1cm} , \ r=1,...,s$ \hspace{1cm} (3.3)

$\sum_{j=1}^{n} \Delta'_{ij} x_{ij} \leq \overline{E}_i \hspace{1cm} , \ i=1,...,m$

\[
\begin{cases}
\Delta_{gj} \geq 0 \quad \text{when } \overline{D}_r \geq 0 \hspace{0.5cm} , \ j=1,...,n \hspace{0.5cm} , \ r=1,...,s \\
-1 \leq \Delta_{gj} \leq 0 \quad \text{when } \overline{D}_r \leq 0 \hspace{0.5cm} , \ j=1,...,n \hspace{0.5cm} , \ r=1,...,s \\
\Delta'_{ij} \geq 0 \quad \text{when } \overline{E}_i \geq 0 \hspace{0.5cm} , \ j=1,...,n \hspace{0.5cm} , \ i=1,...,m \\
-1 \leq \Delta'_{ij} \leq 0 \quad \text{when } \overline{E}_i \leq 0 \hspace{0.5cm} , \ j=1,...,n \hspace{0.5cm} , \ i=1,...,m \\
w_i \geq \varepsilon \hspace{1cm} , \ \mu_r \geq \varepsilon \hspace{1cm} , \ i=1,...,m \hspace{0.5cm} , \ r=1,...,s
\end{cases}
\]

Let $\Delta'_{gj}^* \ (i=1,...,m), \ \Delta_{gj}^* \ (r=1,...,s)$ be an optimal (local or global) solution for model (3.3). The new input and output plan for DMU $j \ (j=1,...,n)$ is

$\left( (1+\Delta'_{gj})x_{gj} , i=1,...,m; (1+\Delta_{gj})y_{gj} , r=1,...,s \right)$. 

2.2.3 Models for changes in general situation

In the real world some of these changes may be positive, some of them may be negative and some of them may be zero. So all inputs are divided into three categories, which denoted by $h_p, h_n$ and $h_z$, representing the sets of inputs with positive, negative and zero changes, respectively. Also all outputs are divided to three categories denoted by $J_p, J_n$ and $J_z$ which represent the sets of outputs with positive, negative and zero changes, respectively.

Note we have $h_p \cup h_n \cup h_z = \{1,...,m\}$ and $\overline{E}_{ij} > 0$ for all $i_p \in h_p$, 
$\overline{E}_{ij} < 0$ for all $i_n \in h_n$, and $\overline{E}_{ij} = 0$ for all $i_z \in h_z$ for inputs; Also $J_p \cup J_n \cup J_z = \{1,...,s\}$ and $\overline{D}_{ij} > 0$ for all $r_p \in J_p$, $\overline{D}_{ij} < 0$ for all $r_n \in J_n$, 
$\overline{D}_{ij} = 0$ for all $r_z \in J_z$ for outputs. Then new input-output plan for all DMUs will be determined to meet these changes by two steps as follows:
**Step 1.** Considering input changes in the category \( h_p \) and output changes in the category \( J_p \) which denoted by \( \Delta'_{i,j}^{(p)} \geq 0, (i = 1, ..., m) \) and \( \Delta_{r,j}^{(p)} \geq 0, (r = 1, ..., s) \), respectively. Model (4.3) based upon model (1.3) is as follows:

\[
\text{Max} \quad \theta = \sum_{r_p \in h_p} u_{r_p} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta'_{r,j}^{(p)}) y_{r,j} \right) + \sum_{i_p \in h_p} u_{i_p} \left( \frac{1}{n} \sum_{i=1}^{n} y_{i,j} \right) + \sum_{r_p \in h_p} u_{r_p} \left( \frac{1}{n} \sum_{j=1}^{n} y_{r,j} \right) + u_0
\]

s.t.

\[
\sum_{i_p \in h_p} v_{i_p} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta'_{i,j}^{(p)}) x_{i,j} \right) + \sum_{r_p \in h_p} v_{r_p} \left( \frac{1}{n} \sum_{r=1}^{n} x_{r,j} \right) + \sum_{i_p \in h_p} v_{i_p} \left( \frac{1}{n} \sum_{j=1}^{n} x_{i,j} \right) + v_0 \leq 1
\]

\[
( \sum_{r_p \in h_p} (1 + \Delta'_{i,j}^{(p)}) y_{r,j} + \sum_{i_p \in h_p} u_{i_p} y_{i,j} + \sum_{r_p \in h_p} u_{r_p} y_{r,j} ) + u_0 \leq 1 \quad , \quad j = 1, ... , n
\]

\[
\sum_{i_p \in h_p} \Delta'_{i,j}^{(p)} y_{r,j} \leq D_{r_p} \quad , \quad r_p \in J_p
\]

\[
\sum_{i_p \in h_p} \Delta_{i,j}^{(p)} x_{i,j} \leq E_{i_p} \quad , \quad i_p \in h_p
\]

\[
\Delta_{i,j}^{(p)} \geq 0 \quad , \quad j = 1, ... , n , r_p \in J_p
\]

\[
\Delta'_{i,j}^{(p)} \geq 0 \quad , \quad j = 1, ... , n , i_p \in h_p
\]

\[
u_{i_p} , u_{i_p} , u_{r} , u_{r} \geq \varepsilon \quad , \quad r_p \in J_p , r_n \in J_n , r_z \in J_z
\]

\[
v_{i_p} , v_{i_p} , v_{i} , v_{i} \geq \varepsilon \quad , \quad i_p \in h_p , i_n \in h_n , i_z \in h_z
\]

\[
u_0 \text{ free}
\]

When the variables are changed as \( u_1 = t u_{0} \), \( \mu_{r} = t u_{r} \), \( \mu_{r} = t u_{n} \), \( \mu_{r} = t u_{p} \), and \( t v_{i} = w_{i}, \ t v_{i} = w_{i} \), \( t v_{i} = w_{i} \) where

\[
t = \left[ \sum_{i_p \in h_p} v_{i_p} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{i,j}^{(p)}) x_{i,j} \right) + \sum_{i_p \in h_p} v_{i_p} \left( \frac{1}{n} \sum_{j=1}^{n} x_{i,j} \right) + \sum_{i_p \in h_p} v_{i_p} \left( \frac{1}{n} \sum_{j=1}^{n} x_{i,j} \right) \right]^{-1}
\]

model (4.3) can be converted to model (5.3):
Let \( \Delta_{ij}^{(p)} \) \((j = 1, \ldots, n, i_p \in h_p)\), \( \Delta_{ij}^{(0)} \) \((j = 1, \ldots, n, r_p \in J_p)\) be an optimal (local or global) solution for model (5.3).

The result of inputs and outputs for DMU\(_j\) \((j = 1, \ldots, n)\) after step 1 is

\[
\begin{align*}
\sum_{i_p \in h_p} i_p (1 + \Delta_{ij}^{(p)}) x_{i_p} & \leq \bar{D}_p, & r_p \in J_p, \\
\sum_{i_p \in h_p} i_p (1 + \Delta_{ij}^{(p)}) y_{i_p} & \leq \bar{E}_p, & i_p \in h_p, \\
\Delta_{ij}^{(p)} & \geq 0, & j = 1, \ldots, n, r_p \in J_p, \\
\Delta_{ij}^{(0)} & \geq 0, & j = 1, \ldots, n, \quad i_p \in h_p, \\
\mu_i, \mu_p, \mu_r & \geq \varepsilon, & r_p \in J_p, \quad r_n \in J_n, \quad r_z \in J_z, \\
w_{i_p}, w_{i_p}, w_{i_p} & \geq \varepsilon, & i_p \in h_p, \quad i_n \in h_n, \quad i_z \in h_z, \\
u_i & \text{ free}.
\end{align*}
\]

The plan for negative changes is presented in step 2.

**Step 2.** Considering input changes in the category \( h_n \) and output changes in the category \( J_n \) which denoted by \(-1 \leq \Delta_{ij}^{(n)} \leq 0 \) \((j = 1, \ldots, n, i_n \in h_n)\) and \(-1 \leq \Delta_{ij}^{(0)} \leq 0 \) \((j = 1, \ldots, n, r_n \in J_n)\). respectively. Model (6.3) based upon model (1.3) and the result of step 1 is as follows:
When the variables are changed as

$$u_i = \mu u_{t_i}, \mu r_p = t u_{r_p}, \mu r_n = t u_{r_n}, \mu r_j = t u_{r_j}$$

and

$$w_{t_i} = t v_{t_i}, w_{r_p} = t v_{r_p}, w_{r_n} = t v_{r_n}, w_{r_j} = t v_{r_j}$$

where

$$t = \left[ \sum_{i_{ch_p}} v_{t_i} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) x_{ij} \right) + \sum_{i_{ch_n}} v_{r_p} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}^{(n)}) x_{ij} \right) + \sum_{i_{ch_n}} v_{r_n} \left( \frac{1}{n} \sum_{j=1}^{n} (1 + \Delta_{ij}^{(n)}) x_{ij} \right) \right]^{-1}$$

model (6.3) can be converted to model (7.3):
Max \( \theta = \left( \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) y_{ij} \right) + \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(o)}) y_{ij} \right) \right) + \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} y_{ij} \right) \right) + nu_1 \\
\text{s.t.} \left( \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) y_{ij} \right) \right) + \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(o)}) y_{ij} \right) + \sum_{i \in I} \mu_i y_{ij} + u_1 \leq 1 \\
\left( \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) x_{ij} \right) \right) + \sum_{i \in I} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(o)}) x_{ij} \right) + \sum_{i \in I} w_i x_{ij}, \quad j=1,...,n \quad (a) \\
\sum_{i=1}^{n} \Delta_{ij}^{(o)} y_{ij} \leq D_i , \quad r_n \in J_n \\
\sum_{i=1}^{n} \Delta_{ij}^{(o)} x_{ij} \leq E_i , \quad i_n \in h_n \\
-1 \leq \Delta_{ij}^{(o)} \leq 0 , \quad j=1,...,n , r_n \in J_n \\
-1 \leq \Delta_{ij}^{(o)} \leq 0 , \quad j=1,...,n , i_n \in h_n \\
\mu_i, \mu_j, \mu_n \geq \varepsilon , \quad r_p \in I_p , \quad r_n \in J_n \quad r_z \in J_z \\
w_i, w_j, w_k \geq \varepsilon , \quad i_p \in h_p , \quad i_n \in h_n , i_z \in h_z \\
u_i \text{ free}

Suppose \( \Delta_{ij}^{(o)} (j=1,...,n \quad i_n \in h_n) \) and \( \Delta_{ij}^{(o)} (j=1,...,n \quad r_n \in J_n) \) be an optimal(local or global) solution for model (7.3).The new input and output plan for DMU\( j \) (j=1,...,n) is:

\[ \left( \left(1 + \Delta_{ij}^{(p)} \right) x_{ij}, \left(1 + \Delta_{ij}^{(o)} \right) x_{ij}, i_i \text{ } \right), \quad i_p \in h_p , \quad i_n \in h_n \text{ } \quad i_z \in h_z ; \]

\[ \left(1 + \Delta_{ij}^{(p)} \right) y_{ij}, \left(1 + \Delta_{ij}^{(o)} \right) y_{ij}, r_p \in J_p , \quad r_n \in J_n \quad r_z \in J_z \]

**Theorem 1.3** If the objective function of model (3.3) is 1, all \( a \) constraints of model (3.3) are binding in the optimal solution of model (3.3).

**Proof:** If \( \theta^* = 1 \) in model (3.3) it is concluded:

\[ \sum_{i=1}^{n} \mu_i \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{*}) y_{ij} \right) + nu_1^* = 1 \quad (b) \]

We continue the proof by contradiction: suppose that

\[ \exists k \in \{1,...,n\} \sum_{i=1}^{n} \mu_i \left(1 + \Delta_{ik}^{*}\right) y_{ik} + u_1^* < \sum_{i=1}^{m} w_i \left(1 + \Delta_{ik}^{*}\right) x_{ik} \]
Therefore, 
\[ \sum_{j=1}^{n} \left( \sum_{i=1}^{m} \mu_{ij}^{*} (1 + \Delta_{ij}^{*}) y_{ij} + u_{1}^{*} \right) < \sum_{j=1}^{n} \left( \sum_{i=1}^{m} w_{i}^{*} (1 + \Delta_{ij}^{*}) x_{ij} \right) \quad (c) \]

Note that we have
\[ \sum_{i=1}^{m} w_{i}^{*} \sum_{j=1}^{n} (1 + \Delta_{ij}^{*}) x_{ij} = 1 \quad (d) \]

As you see, these two (c) and (d) are in contradiction with (b). Therefore, all (a) constraints of model (3.3) are binding in the optimal solution of model (3.3).

**Theorem 2.3** If the objective function of model (7.3) is 1, all (a) constraints of model (7.3) are binding in the optimal solution of model (7.3).

**Proof:** If \( \theta^{*} = 1 \) in model (7.3) it is concluded:
\[ \left( \sum_{i_{j} \in h_{p}} \mu_{i_{j}}^{*} \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) y_{ij} \right) + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(s)}) y_{ij} \right) + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} \left( \sum_{j=1}^{n} y_{ij} \right) \right) + nu_{1}^{*} = 1 \quad (b) \]

We continue the proof by contradiction: suppose that
\[ \exists k \in \{1, ..., n\}; \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} (1 + \Delta_{ij}^{(k)}) y_{ij} + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} (1 + \Delta_{ij}^{(s)}) y_{ij} + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} y_{ij} + u_{1}^{*} < \]
\[ \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} (1 + \Delta_{ij}^{(p)}) x_{ij} + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} (1 + \Delta_{ij}^{(s)}) x_{ij} + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} x_{ij} \]

Therefore,
\[ \sum_{j=1}^{n} \left( \sum_{i_{j} \in h_{p}} \mu_{i_{j}}^{*} (1 + \Delta_{ij}^{(p)}) y_{ij} + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} (1 + \Delta_{ij}^{(s)}) y_{ij} + \sum_{r_{j} \in I_{s}} \mu_{r_{j}}^{*} y_{ij} + u_{1}^{*} \right) < \]
\[ \sum_{j=1}^{n} \left( \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} (1 + \Delta_{ij}^{(p)}) x_{ij} + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} (1 + \Delta_{ij}^{(s)}) x_{ij} + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} x_{ij} \right) \quad (c) \]

Note that we have
\[ \left( \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(p)}) x_{ij} \right) + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} \left( \sum_{j=1}^{n} (1 + \Delta_{ij}^{(s)}) x_{ij} \right) + \sum_{i_{j} \in h_{p}} w_{i_{j}}^{*} \left( \sum_{j=1}^{n} x_{ij} \right) \right) = 1 \quad (d) \]

As you see, these two (c) and (d) are in contradiction with (b). Therefore, all (a) constraints of model (7.3) are binding in the optimal solution of model (7.3).

### 4. Application

In this section, our DEA-based production planning which is explained in new idea in this paper is used for a set of 20 fast food restaurants that are located in the city of Hefei, Anhui Province, China[7]. These restaurants belong to the same chain, with a central decision-making team containing several members that supervise all branches’ performance and make their future sales plans.

Man-hour and shop size are used as two inputs. As it was said in [7] Man-hour, here, refers to the labor force used within a certain period. Shop size refers to total rental floor space of the restaurant that can be used for the purpose of serving customers.
The outputs are the sales of meat dish, vegetable dish, soup, noodles and beverage. The data for these restaurants and the original BCC efficiency scores of all restaurants are given in Table 1:

Table 1. Data for restaurant chains and their original BCC efficiency scores

<table>
<thead>
<tr>
<th>DMUj</th>
<th>inputs</th>
<th>outputs</th>
<th>Original BCC Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>man-hour $10^3 h$</td>
<td>floor space $10^2 m^2$</td>
<td>meat dish $10^3$ servings</td>
</tr>
<tr>
<td>1</td>
<td>3.20</td>
<td>2.00</td>
<td>2.24</td>
</tr>
<tr>
<td>2</td>
<td>3.40</td>
<td>2.10</td>
<td>2.12</td>
</tr>
<tr>
<td>3</td>
<td>3.10</td>
<td>1.80</td>
<td>2.08</td>
</tr>
<tr>
<td>4</td>
<td>3.80</td>
<td>2.20</td>
<td>2.45</td>
</tr>
<tr>
<td>5</td>
<td>4.20</td>
<td>2.60</td>
<td>2.80</td>
</tr>
<tr>
<td>6</td>
<td>4.10</td>
<td>2.50</td>
<td>2.65</td>
</tr>
<tr>
<td>7</td>
<td>3.80</td>
<td>2.30</td>
<td>2.60</td>
</tr>
<tr>
<td>8</td>
<td>3.80</td>
<td>2.20</td>
<td>2.50</td>
</tr>
<tr>
<td>9</td>
<td>2.90</td>
<td>1.60</td>
<td>2.10</td>
</tr>
<tr>
<td>10</td>
<td>4.20</td>
<td>2.80</td>
<td>2.90</td>
</tr>
<tr>
<td>11</td>
<td>3.40</td>
<td>2.10</td>
<td>2.60</td>
</tr>
<tr>
<td>12</td>
<td>4.00</td>
<td>2.40</td>
<td>2.78</td>
</tr>
<tr>
<td>13</td>
<td>3.80</td>
<td>2.60</td>
<td>2.84</td>
</tr>
<tr>
<td>14</td>
<td>3.40</td>
<td>1.90</td>
<td>2.33</td>
</tr>
<tr>
<td>15</td>
<td>2.80</td>
<td>1.60</td>
<td>2.00</td>
</tr>
<tr>
<td>16</td>
<td>3.50</td>
<td>2.20</td>
<td>2.40</td>
</tr>
<tr>
<td>17</td>
<td>4.20</td>
<td>2.50</td>
<td>2.68</td>
</tr>
<tr>
<td>18</td>
<td>3.30</td>
<td>1.80</td>
<td>2.05</td>
</tr>
<tr>
<td>19</td>
<td>3.60</td>
<td>1.90</td>
<td>2.00</td>
</tr>
<tr>
<td>20</td>
<td>3.10</td>
<td>1.70</td>
<td>2.05</td>
</tr>
</tbody>
</table>

The demand changes for meat dish, vegetable dish, soup, noodles and beverage are predicted as $D_1 = 3$, $D_2 = 2.4$, $D_3 = -3$, $D_4 = 6$, and $D_5 = 3$ and input changes is only for man-hour and it is predicted as $E_1 = 2$ in the next business month. All the above data are collected for a month, and the central decision-making team attempt to arrange input and output plans for all 20 fast food restaurants under the same chain for the next business month. Through this research overall performance of all restaurants is optimized. The results of steps 1 and 2 of new idea for all 20 restaurants and new BCC efficiency scores after planning with $\varepsilon = 10^{-5}$ are shown in Table 2:
Table 2. The results of steps 1 and 2 in new idea for 20 restaurants

<table>
<thead>
<tr>
<th>DMU_j</th>
<th>( \Delta_{1j}^p )</th>
<th>( \Delta_{2j}^p )</th>
<th>( \Delta_{4j}^p )</th>
<th>( \Delta_{5j}^p )</th>
<th>( \Delta_{1j}^{n(p)} )</th>
<th>( \Delta_{3j}^{n(p)} )</th>
<th>New BCC Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.28971</td>
<td>-0.22065</td>
<td>1.00000</td>
</tr>
<tr>
<td>2</td>
<td>0.17899</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.35506</td>
<td>1.00000</td>
</tr>
<tr>
<td>3</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.22762</td>
<td>1.00000</td>
</tr>
<tr>
<td>4</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.19448</td>
<td>1.00000</td>
</tr>
<tr>
<td>5</td>
<td>0.17853</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.39073</td>
<td>1.00000</td>
</tr>
<tr>
<td>6</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.06463</td>
<td>1.00000</td>
</tr>
<tr>
<td>7</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.08189</td>
<td>1.00000</td>
</tr>
<tr>
<td>8</td>
<td>0.29833</td>
<td>0.00000</td>
<td>0.00000</td>
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<td>-0.67710</td>
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</tr>
<tr>
<td>9</td>
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<td>2.08333</td>
<td>0.29131</td>
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<td>3.40275</td>
<td>0.28844</td>
<td>-0.42965</td>
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</tr>
<tr>
<td>12</td>
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<td>0.13697</td>
<td>0.00000</td>
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<td>0.00000</td>
<td>-0.12004</td>
<td>1.00000</td>
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<tr>
<td>13</td>
<td>0.00603</td>
<td>0.19359</td>
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<td>14</td>
<td>0.12203</td>
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<td>-0.59805</td>
<td>1.00000</td>
</tr>
<tr>
<td>18</td>
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<td>0.16561</td>
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<td>-0.25279</td>
<td>1.00000</td>
</tr>
<tr>
<td>19</td>
<td>0.00000</td>
<td>0.16864</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>1.00000</td>
</tr>
<tr>
<td>20</td>
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<td>0.02104</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.25768</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

Note that, all DMUs with new production plans are BCC efficient.

5. Conclusions

From section 3 and examples of section 4 it is concluded that the new idea stated in section 2.3 presents a new production plan for each DMU and all DMUs will be BCC efficient after using their new plans.

Note that the new idea in section 2.3 is eliminated the problems of idea 1[7] which stated in section 1.2 in this paper.

Furthermore, the new idea stated in this paper can consider some inputs without any change. For example, a shop size in 20 fast food restaurant in section 2.4 probably can not change and we considered it fixed, but all the inputs should change in idea 1 in [7].

Here we have two suggestions for further research. First one is if the data be special (for example integer), and all input and output changes can be predicted for the next
production season, a model should be presented to show new input and output plans for all DMUs, so that the changes can be met and all DMUs will be BCC efficient. Second one is if all input and output changes can be predicted for the next production season, a model should be presented to show new input and output plans for all DMUs, so that the changes can be met and the classification of efficient and not efficient DMU, does not change.

References


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