Fuzzy-Chance Constrained Multi-Objective Programming Applications for Inventory Control Model

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Abstract
In this paper we have developed the traditional inventory control model (r,Q) taking into account a multi-item models with two objectives to minimize holding costs and shortage cost and also risk level under constraints including available budgetary, the least service

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level, storage spaces & allowed quantities of shortage. According to proposed model demand is potential and follows the distribution and extra demands encounter lost sales. Space storage is counted as a fuzzy-chance parameter with normal distribution. Budgetary parameters are available and the maximum allowed shortage is a fuzzy triangular number. The methodology that has been used in proposed solution changed model to a crisp multi-objective problem using defuzzification of fuzzy constraints and fuzzy chance-constrained programming methods, and then we solve it using fuzzy arithmetic concepts.

**Keywords:** inventory costs, shortage cost, holding cost, ordering cost, (r,Q) inventory control model, multi objective programming, fuzzy-chance constrained programming, flexible programming, stochastic programming, risk level

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**1. Introduction**

After the publication of the EOQ model by Harris, some complexity and changes in their adjustment caused inefficiency and some developments were in need [21]. The complexity arises in modeling a realistic decision-making inventory situation mainly due to the presence of some non-deterministic information; that is, that they are not able to encode precision and certainty of classical mathematical logic. Actually, a realistic situation is no longer realistic when the imprecise and uncertain information are neglected for the sake of mathematical model building [11]. One concern of studies on the inventory literature is (r,Q) model. This model have been popular in education and practiced at the beginning of infancy of inventory theory. In this regard, various computational procedures have been proposed in the textbooks and scientific studies for determining the control parameter "r" and "Q" [32]. Review (r,Q) model, among the stochastic demand, whenever inventory is the most r, Q would be established [29]. Optimization of an inventory system depends on finding a pair of "r" and "Q" parameters that minimize the long-run average of cost per period [32], but in real-life situations decision-maker not only deals with minimizing costs but also concentrates on the other objectives which are limited. The concept of fuzzy sets theory helps us to solve the inventory management problems. Several researchers are given in next section. After the publication of classical lot-size formula by Harris in 1915, many researchers worked on the EOQ model whose results are currently available in reference books and survey papers [9, 14, 26, and 21]. Other studies like [11, 13, 4, and 2] developed inventory models with variable replenishment production for multi-item inventory problems. According to other studies [5, 6], demand and cost can be practically assumed to be inversely related to each other. Some inventory models have been developed and solved by geometric programming technique drawing on this assumption. Multi-item classical inventory models under resource constraints such as budgetary cost, limited storage area, number of orders, etc. are presented in well-known books [28, 21, 14, 17, and 8]. Ben-daya and Raouf [3] discussed a multi-item inventory model with stochastic demand and Abou-
el-ata and Kotb [17] developed a crisp model inventory model under two restrictions. Initially, fuzzy sets theory presented by Zade in 1965 and Zimmerman used this theory to solve decision making problems [11]. Recently, Mandal et al. [20] investigated a multi-objective fuzzy inventory model with three constraints using fuzzy geometric programming approach. They formulated the model with fuzzy parameters only in fuzzy environment and then solved it using modified geometric programming technique. Lodwick and Bachman [18] used surprise function technique to convert a fuzzy possibility/necessity optimization problem into a deterministic one. Roy and Maiti [27] used geometric programming technique to solve both single item and multi-item fuzzy inventory problems. Hadley and Whitin [14] first extended the classical EOQ inventory model to the stochastic model. An application of fuzzy sets to inventory control models is presented in [31]. In the survey [11] named "Multi-item stochastic and fuzzy stochastic inventory models under two restrictions" by Das et al., the models are formulated under total budgetary and space constraints. Here the goal is to minimize the costs and all fuzzy models represented as stochastic and non-linear ones are solved by gradient technique and chance constrained programming. Yahua [32] presented two procedures which determine optimal values for the control parameters (i.e., reorder-point and order-quantity) when the holding costs are non-quasi-convex. In Chu et al. [19] study, a new form of partial backorder policy (PB2) with two-segment backorder control limits is introduced. In other study Mnanas and Maiti [20] presented a multi-item inventory model with two-storage facilities under stochastic constraints that are solved by Goal Programming Method (GPM) and fuzzy simulation-based single/multi-objective genetic algorithm (FSGA /FSMOGA) otherwise. In another study on stochastic inventory, Ouyang and Chang [24] attempted to apply the fuzzy sets concepts to deal with the uncertain backorders and lost sales. Hariga [15] incorporated a common stochastic continuous review inventory (r,Q) model and developed method for optimizing economic orders. Eynan and Kropp [12] examined a periodic review system under stochastic demand with variable stock out costs by using a Taylor series expansion to approximate part of the cost function. The other examples of developing stochastic inventory models are presented in [23, 25, 30] studies. Nowadays, existence of a mixed environment or the coexistence of imprecision and uncertainty in an inventory is a realistic phenomenon [2]. Therefore, in this paper we developed inventory control model (r,Q) with two objectives as minimizing costs and risk level under limitations such as available budgetary, the performance level, number of shortages, and fuzzy-chanced constrained of storage space. Shortages are assumed to be allowed and likely to face with lost sales. Inventory costs depend on quantities. Available budgetary level is fuzzy factor and storage space is a stochastic parameter with fuzzy mean and fuzzy deviation. Storage space constraint is satisfied stochastically and the least allowed probability on this constraint is a fuzzy number which is definable. All random parameters are independent, storage spaces parameters follow normal distribution and demand follows distribution. In this model the amount of constant order is calculated based on the economical order amount. In this paper fuzzy constraints by defuzzification and fuzzy stochastic constraints of storage space by programming of fuzzy stochastic constraints
change into crisp limitation and the output model which is a multi-objectives programming model can be solved by fuzzy logic technique. Finally an illustrated example is given and solved by using LINGO software package. The rest of the paper is organized as follows: first briefly classification of ordering system is presented in first part then in second part we will do the same about symbols and signs. In part 3 models and assumption are discussed. In part 4 we will deal with the development of models and in part 5 methodologies is discussed. In part 6 a digital example is given and in final part conclusion and future research will follow. In chapter 4 we talked about developing the models and in chapter 5 solving methodology were discussed. In chapter 6 a digital example was presented and in final chapter conclusion Problem definition, models and their assumptions are presented in section 2. In section 3, developing the model is addressed. Solution methodology is proposed in section 4. Section 5 presents Experimental results. Finally, Conclusion and future works appear in section 6.

2. Model formulation

The following notations and assumption are used to develop models:

2.1. Notations

\( n \): number of items
\( A \): available storage space
\( B \): available total budgetary cost
\( \sim \): fuzzy symbol

Decision variables and parameters for the ith (i = 1; …; n) item are:

\( h_i \): Annual holding cost per unit item
\( SS_i \): Safety stock per unit item
\( r_i \): Reorder point per unit item
\( \lambda_{L_i} \): rate of demand per unit item within lead-time L
\( \lambda_{L_i} / \lambda_{L_i} \): Average demand per unit item within lead-time L
\( \pi_i \): Annual shortage cost per unit item
\( b(r_i) \): Average shortage quantity per unit item
\( PC_i \): Purchase cost per unit item
2.2. Model and assumptions [4]:
We suppose (r,Q) ordering system with following cost functions:
Cost function of (r,Q) system is:
\[ C(r) = hss + \pi \frac{D}{Q} \bar{b}(r) \]  
(1)
And assumptions of these models are:
• Demand functions are assumed to be exponential: \( D \sim \text{Exp} (\lambda) \)
• \( r \) and \( Q \) is independence
• Extra demand consider as lost sales.

3. Developing the model
To develop the (r,Q) model, we present crisp model and then fuzzy-stochastic model. Afterwards we propose two objectives and six constrains. Model objectives consist of:

3.1. Minimizing costs:
We use the following formula for this objective
Where:

\[ SS_i = r_i - \frac{1}{\lambda_{li}} \]  

\[ \bar{b}(r_i) = \int_{r_i}^{\infty} (x - r_i) \lambda e^{-\lambda x} dx \]  

And

\[ \Rightarrow \bar{b}(r_i) = \frac{1}{\lambda} e^{-\lambda r_i} \]  

3.2. Minimizing risk level:
Risk level gets over zero demand be more than order point "r". We used the following relation for minimizing the risk level:

\[ Min : P(D_L > r) = P((D_L - \frac{1}{\lambda_{li}}) > (r_i - \frac{1}{\lambda_{li}})) \]  

Because stochastic demand is possible, risk level is directly related to the shortage and changes as shortage changes. So when it increases, shortage increases too in the same way. To reach to this purpose we have:

\[ Z_2 : e^{-\lambda r_i} \]  

3.3. Limitations
Review of subject literature related to inventory control model shows the use of various constraints. In this study we propose 6 constraints as follows:

1. Constraint of budgetary:

\[ \sum_{i=1}^{n} (r_i - \frac{1}{\lambda_{li}}) \leq B \]  

2. Constraint of allowed shortage quantities:

\[ \sum_{i=1}^{n} \frac{1}{\lambda_{li}} e^{-\lambda_{li} r_i} \leq N_i \]  

3. Constraint of storage spaces:

\[ \sum_{i=1}^{n} a_i I_{max, i} \leq A \]  

Where:

\[ I_{max, i} = r_i - \frac{1}{\lambda_{li}} + Q_i \]  

4-Constraint of service level: for this purpose we have:

\[ P(D_{li} \leq r_i) \geq P_{0i} \]

Thus we applied the following equation for this:
5- In this paper we assumed \( Q_i = Q_i^W \), thus we used EOQ formula as a constraint in this model:

\[
Q_i = \sqrt{\frac{2D_iA_i'}{h_i}}
\]  

(12)

Where:

6-Finally, because probability rate is a number in range \([0,1]\), we considered the following constraint:

\[
0 \leq e^{-\lambda_i r_i} \leq 1
\]  

(13)

Thus the developed crisp model of (r, Q) is:

\[
\begin{align*}
\text{Min} & \quad Z_1 = \sum_{i=1}^{n} h_i(r_i - \frac{1}{\lambda_i L_i}) + \left[\pi_i + (S_i - PC_i)\right]D_i e^{-\lambda_i r_i} \\
\text{Min} & \quad Z_2 = e^{-\lambda_i r_i} \\
\text{S.t.:} & \quad \sum_{i=1}^{n} (r_i - \frac{1}{\lambda_i L_i})PC_i \leq B \\
& \quad (1 - e^{-\lambda_i r_i}) \geq P_0 \\
& \quad \sum_{i=1}^{n} a_i(r_i - \frac{1}{\lambda_i L_i} + Q_i) \leq A \\
& \quad \frac{1}{\lambda_i} e^{-\lambda_i r_i} \leq N_i \\
& \quad Q_i = \sqrt{\frac{2D_iA_i'}{h_i}} \\
& \quad 0 \leq e^{-\lambda_i r_i} \leq 1 \\
& \quad r_i \geq 0, \quad i = 1,2,..,n
\end{align*}
\]  

(14)

3.4. Fuzzy stochastic model

This section is presented to develop model in fuzzy environment, available shortage level, supposed to be a fuzzy triangular number and order as \( \tilde{B} \) and \( \tilde{N_i} \) as well. The storage space is proposed as a fuzzy normal stochastic parameter with fuzzy mean \( \tilde{m}_i \) and fuzzy variance \( \tilde{\sigma}_A^2 \):

\[
\tilde{A} \sim N(\tilde{m}_i, \tilde{\sigma}_A^2)
\]  

(15)

And allowed minimum probability is a fuzzy number as \( \tilde{P}_A \).
As a result fuzzy-stochastic model is:
\[
\text{Min} Z_i = \sum_{t=1}^{n} b_i (r_t - 1/\lambda_{ti}) + \left[ \pi_i + (S_i - PC_i) \right] D_i e^{-\lambda_{ti}} / \lambda_{ti}
\]
\[
\text{Min} Z_i = e^{-\lambda_{ti}}
\]
\[
\text{S.t. : } \sum_{i=1}^{n} (r_t - 1/\lambda_{ti}) PC_i \leq \bar{B}
\]
\[
(1-e^{-\lambda_{ti}}) \geq P_c
\]
\[
\tilde{P}(\sum_{i=1}^{n} a_i (r_t - 1/\lambda_{ti} + Q_i) \leq \tilde{A} \approx N(m_i, \sigma^2) \geq \tilde{A}
\]
\[
1/\lambda_{ti} e^{-\lambda_{ti}} \leq \tilde{N}_i
\]
\[
Q_i = \sqrt{2D_i A_i / \lambda_{ti}}
\]
\[
0 \leq e^{-\lambda_{ti}} \leq 1
\]
\[
r_t \geq 0, \quad i = 1, 2, ..., n
\]

4. Solution methodology

Developed fuzzy-stochastic model can be solved if the model changes to a crisp model and by using one of multi-objective programming techniques, it can be like these steps:

4.1. First step: Difuzzification of fuzzy constraints

If the programming model is like this [23]:
\[
\text{Max} Z = \sum_{i=1}^{n} c_i x_i
\]
\[
\text{s.t. : } \sum_{i=1}^{n} (s_i, l_i, r_i) x_i \leq (t_i, u_i, v_i)
\]
\[
x_i \geq 0 \quad (i \in N_m) \quad (j \in N_n)
\]

Where supposed fuzzy number from the left side of the first number of mean number with one degree \(\mu = 1\) in membership function, and second number is the distance between the first number and the left bound and third number is the distance between the first number and right bound. For solving the model and difuzzification constraints:
4.2. Converting the Fuzzy-Stochastic Constraint to a crisp constraint

4.2-1. Fuzzy-Chance Constrained Programming

To this end, we used Fuzzy-Chance Constrained Programming [27]. A crisp CCP problem is a particular type of Stochastic Programming which is of the form, (CCP):

\[
\begin{align*}
\text{Min} & \sum_{j=1}^{n} c_j x_j \\
\text{S.t.:} & \\
\sum_{j=1}^{n} (s_{ij} - l_{ij}) x_{ij} & \leq t_i - u_i \\
\sum_{j=1}^{n} (s_{ij} - r_{ij}) x_{ij} & \leq t_i - v_i \\
x_i & \geq 0, \quad i = 1,2,\ldots,n
\end{align*}
\]

(18)

Where at least one of \(C_j, a_{ij}, d_i\) is a random variable. We define a FCCP problem as a CCP problem of the above form where at least one of \(C_j, a_{ij}, d_i\) is a fuzzy random variable (FRV) and \(p_i\) is a fuzzy number. Regarding the special state of \(i^{th}\) constraints, problem CCP is:

\[
\tilde{P}(\sum_{j=1}^{n} a_{ij} x_j \leq \tilde{b}_i) \geq \tilde{P}, a_{ij} \in R
\]

(20)

"\(b_i\)" is a normally distributed FRV (in fact we may consider any type of FRV), whose mean and variance are fuzzy numbers denoted by \(\tilde{m}_{bi}\) and \(\tilde{\sigma}_{bi}\).

Let

\[
\begin{align*}
\tilde{m}_{bi}[\alpha] & = [m_{bi}(\alpha), m_{bi}^*(\alpha)] \\
\tilde{\sigma}_{bi}^2[\alpha] & = [\sigma_{bi}^2(\alpha), \sigma_{bi}^2(\alpha)] \\
\tilde{P}[\alpha] & = \left[ P_s(\alpha), P_s(\alpha) \right]
\end{align*}
\]

(21)

If \(\tilde{b}_i\) is a normally distributed FRV then the inequality (22) is equivalent to:
The crisp conversion of CCP for each $\alpha \in [0, 1]$ is:

$$\left\{ \begin{array}{ll}
\sum_{j=1}^{n} a_j x_j - m_{hv}(\alpha) \\
\sigma_{hv}(\alpha)
\end{array} \right\} \leq 1 - P_i^*(\alpha) \quad (22)
$$

The problem can be solved by using crisp stochastic programming method.

4.3. Second step: problem solving of crisp Multi-objective programming

This section will change the fuzzy-stochastic model to a crisp multi-objective problem by using one of multi-objective programming methods, we can approach the purpose, and therefore we use fuzzy-logic method.

4.3.1. Fuzzy -logic method for problem solving of multi-objective decision-making (MODM) [38]

Try to suppose this multi-objective decision-making:

$$\text{Max} \ Z = f_i(x_1, x_2, \ldots, x_n), i = 1, 2, \ldots, k$$

s.t. \( g_i(x_1, x_2, \ldots, x_n) \leq b_i, i = 1, 2, \ldots, m \)

\[ \Rightarrow \text{Max} \ \alpha \]

$$\text{S.t : } \alpha \leq \mu(Z_i), i = 1, 2, \ldots, k \quad (24)$$

$$g_i(x_1, x_2, \ldots, x_n) \leq b_i, i = 1, 2, \ldots, m$$

So, "n" is the number of variables, "m" is the number of constraints and "k" is the number of object functions.

At first we constitute the following table (table1):
Then, calculate the degree of membership \( \mu(z_i) \) function, like (Fig.1):}

![Membership function \( z_i \)](image)

So as \( U_i \) is the best value and \( L_i \) is the worst value and \( \Delta_i \) is tolerance of per unit of \( i \) function.

\[
\Delta_i = U_i - L_i \quad (25) \\
\mu(Z_i) = \begin{cases} 
0, & \text{where: } Z_i \leq L_i \\
\frac{Z_i - L_i}{U_i - L_i}, & \text{where: } L_i \leq Z_i \leq U_i \\
1, & \text{where: } Z_i \geq U_i
\end{cases} \quad (26) \\
\alpha = \min(\mu(Z_1), \mu(Z_2), \ldots, \mu(Z_k)) \quad (27) \\
\alpha \leq \mu(Z_i), i = 1, 2, \ldots, k \quad (28)
\]
Where $\alpha =$ optimization percent

\[
\Rightarrow \alpha \leq \mu(Z_i) = \alpha \leq \frac{Z_i - L_i}{U_i - L_i} \quad (29)
\]

\[
\Rightarrow Z_i \geq U_i - \Delta_i (1 - \alpha)
\]

If $\alpha$ is not of the same value: so we have:

\[
\begin{align*}
\max \sum \alpha_i \\
\alpha_i \leq \mu(Z_i), & i = 1, 2, ..., k
\end{align*} \quad (30)
\]

5. Numerical example

In this part we show the application of proposed model when the decision maker indentifies the order-quantity ($r, Q$). In other word this numerical example clarifies the application of model to minimize the total cost and risk level considering the reorder points level.

We assumed that:

\[
D_{z_1} \sim \text{Exp}\left(\frac{1}{20}\right), \quad D_{z_2} \sim \text{Exp}\left(\frac{1}{15}\right) \quad h_1 = 500, \quad h_2 = 600,
\]

\[
D_1 = 300, \quad D_2 = 200, \quad S_1 = 7300000, \quad S_2 = 13000000,
\]

\[
N_1 = (15, 15, 20), \quad N_2 = (8, 8, 9), \quad a_1 = 6.5, \quad a_2 = 7, \quad P_{01} = 0.7
\]

\[
\pi_1 = 1730000, \quad \pi_2 = 2300000, \quad PC_1 = 6570000, \quad PC_2 = 11700000,
\]

\[
\bar{B} = (350000000, 400000000, 450000000),
\]

\[
\bar{P}_d = (0.83, 0.85, 1), \quad \alpha = 0.7, \quad P_{02} = 0.8,
\]

\[
\bar{A} \sim N\left(1500, 1700, 1800\right), (20, 24, 31)^2
\]

So:
Fuzzy-chance constrained multi-objective programming applications

\[
\begin{align*}
\text{Min } Z_1 &= 500(r_1 - 20) + \frac{[2460000][300e^{-0.05r_1}]}{0.05Q_1} + \\
&\quad 600(r_2 - 150) + \frac{[3600000][200e^{-0.066r_1}]}{0.0665Q_2} \\
\text{Min } Z_2 &= e^{-0.05r_1} \\
\text{Min } Z_3 &= e^{-0.066r_1} \\
\end{align*}
\]

\(S1:\)
\[
6570000(r_1 - 20) + 1170000(r_2 - 15) \leq (350000000, 400000000, 450000000) \\
[1 - e^{-15r_1}] \geq 0.7 \\
[1 - e^{-20r_1}] \geq 0.8 \\
\tilde{P}\left\{\left[6.5(r_1 - 15) + Q_1\right] + \left[8(r_2 - 20) + Q_2\right]\right\} \leq \left\{\left[(1500, 1700, 1800), (20, 24, 30)r_1\right]\right\} \\
\geq (0.83, 0.85, 1.00) \\
20e^{-0.5r_1} \leq (15, 15, 20) \\
15e^{-0.066r_1} \leq (8, 8, 9) \\
Q_1 = 11 \\
Q_2 = 15 \\
0 \leq e^{-0.5r_1} \leq 1 \\
r_i \geq 0 \quad ; \quad i = 1, 2, ..., n
\]

Considering the above mentioned methodology, the crisp multi-objective is as follows.
Consequently, we have a crisp multi-objective programming that can be solved by fuzzy logic method and its results are presented in table 2. Considering table 2 and defining membership functions, we have:

\[
\begin{align*}
\text{Min } Z_1 &= 500(r_1 - 20) + \frac{[2460000]300e^{-0.05r_1}}{0.05Q_1} + \\
&\quad 600(r_2 - 150) + \frac{[3600000]200e^{-0.066r_2}}{0.0665Q_2} \\
\text{Min } Z_2 &= e^{-0.05r_1} \\
\text{Min } Z_3 &= e^{-0.066r_2} \\
\text{S.t.:} \\
&6570000(r_1 - 20) + 1170000(r_2 - 15) \leq 450000000 \\
&6570000(r_1 - 20) + 1170000(r_2 - 15) \leq 400000000 \\
&6570000(r_1 - 20) + 1170000(r_2 - 15) \leq 350000000 \\
&(1 - e^{-15}) \geq 0.7 \\
&(1 - e^{-20r_2}) \geq 0.8 \\
&6.5r_1 + 8r_2 \leq 1710.468 \\
&20e^{-0.5} \leq 15 \\
&20e^{-0.5} \leq 20 \\
&15e^{-0.066r_2} \leq 8 \\
&15e^{-0.066r_2} \leq 9 \\
&Q_1 = 11 \\
&Q_2 = 15 \\
&0 \leq e^{-\lambda r_i} \leq 1 \\
r_i \geq 0 \quad ; \quad i = 1, 2, \ldots, n
\end{align*}
\]
As a result, the obtained model should be like following model solved by Lingo package and its results are illustrated in table3:

\[ \text{Max } \alpha \]
\[ \text{S.t.:} \]
\[ Z_1 \geq 613733 + 534226 \alpha \]
\[ Z_2 \geq 0.090719 + 0.730191 \alpha \]
\[ Z_3 \geq 0.0814 + 0.54402 \alpha \]
\[ 6570000 (r_1 - 20) + 11700000 (r_2 - 15) \leq 450000000 \]
\[ 6570000 (r_1 - 20) + 11700000 (r_2 - 15) \leq 400000000 \]
\[ 6570000 (r_1 - 20) + 11700000 (r_2 - 15) \leq 350000000 \]
\[ (1 - e^{-15r_1}) \geq 0.7 \]
\[ (1 - e^{-20r_2}) \geq 0.8 \]
\[ 6.5r_1 + 8r_2 \leq 1710.468 \]
\[ 20e^{-0.005} \leq 15 \]
\[ 20e^{-0.005} \leq 20 \]
\[ 15e^{-0.006} \leq 8 \]
\[ 15e^{-0.006}r_2 \leq 9 \]
\[ Q_1 = 11 \]
\[ Q_2 = 15 \]
\[ 0 \leq e^{-\lambda r} \leq 1 \]
\[ r_i \geq 0, i = 1, 2, ..., n \]
Table 2, Lingo output

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduce Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.2034</td>
<td>0.0000</td>
</tr>
<tr>
<td>R1</td>
<td>27.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>R2</td>
<td>25.0000</td>
<td>0.2329 E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slack or Surplus</th>
<th>Dual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>754553.75</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.0003</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.2870E+09</td>
</tr>
<tr>
<td>6</td>
<td>0.2370E+09</td>
</tr>
<tr>
<td>7</td>
<td>0.1870E+09</td>
</tr>
<tr>
<td>8</td>
<td>0.40749E-01</td>
</tr>
<tr>
<td>9</td>
<td>0.79405E-02</td>
</tr>
<tr>
<td>10</td>
<td>1334.96</td>
</tr>
<tr>
<td>11</td>
<td>13.7979</td>
</tr>
<tr>
<td>12</td>
<td>5.91569</td>
</tr>
<tr>
<td>13</td>
<td>0.20341</td>
</tr>
<tr>
<td>14</td>
<td>0.79658</td>
</tr>
</tbody>
</table>


Table 3. Results of solving the functions

<table>
<thead>
<tr>
<th>$r_2$</th>
<th>$r_1$</th>
<th>$Z_{33}$</th>
<th>$Z_{32}$</th>
<th>$Z_{31}$</th>
<th>$Z_{23}$</th>
<th>$Z_{22}$</th>
<th>$Z_{21}$</th>
<th>$Z_{12}$</th>
<th>$Z_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>41</td>
<td>0.113</td>
<td>0.11</td>
<td>0.113</td>
<td>0.12</td>
<td>0.128</td>
<td>0.12</td>
<td>8273</td>
<td>8273</td>
</tr>
<tr>
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6-Conclusion and future works

In this paper, a traditional inventory control model with two objectives of minimizing costs and risk level and budgetary limitation and also warehouse, number of allowed shortages and minimum level of allowed performance were developed and discussed. Developed models are nonlinear and some of its parameters are considered as fuzzy ones. Warehouse limitation which changed to a crisp constraint using fuzzy chance constraint programming in fuzzy–chance environment was discussed and finally a crisp multi objectives model was solved by fuzzy logic method. In future research we can use some assumptions about demand distribution and warehouse space like erlang. Also we can consider some objectives and other limitation s Instead of triangular fuzzy we can use trapezoid type and also we can use optimization function method and lexicograph from other solving method.
of multi objectives programming like geometric programming. The obtained results also
can be compared with each other.

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Received: August, 2011