Effect of Rotation on Triple-Diffusive Convection  
in a Magnetized Ferrofluid with Internal Angular  
Momentum Saturating a Porous Medium  

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Abstract  
This paper deals with the theoretical investigation of the rotation in a magnetized ferrofluid with internal angular momentum, heated and soluted from below saturating a porous medium and subjected to a transverse uniform magnetic field. For a flat fluid layer contained between two free boundaries, an exact solution is obtained. A linear stability analysis theory and normal mode analysis method have been carried out to study the onset convection. The influence of various parameters on the onset of stationary convection such as rotation, medium permeability, solute gradients, non-buoyancy magnetization and internal angular momentum parameters (i.e. coupling parameter, spin diffusion parameter and heat conduction parameter) has been analyzed. The critical magnetic thermal Rayleigh number for the onset of instability is also determined numerically for sufficiently large value of buoyancy magnetization parameter $M_1$. The principle of exchange of stabilities is found to hold true for for the ferrofluid with internal angular momentum saturating a porous medium heated from below in the absence of rotation, coupling between vorticity and spin, microinertia and solute gradients. The oscillatory modes are introduced due to the presence of rotation, coupling between vorticity and spin, microinertia and solute gradients, which were non-existent in their absence. In this paper, an attempt is also made to obtain the sufficient conditions for the non-existence of overstability.  

Keywords: Triple- diffusive convection; Ferrofluid; Rotation; Internal angular momentum; Porous Medium, Solute Gradient; Magnetization
1. Introduction

Magnetic fluids or ferrofluids are colloidal suspension of fine ferromagnetic mono domain nano particles in non-conducting liquids. The ferromagnetic nanoparticles are coated with a surfactant to prevent their agglomeration. Rosensweig [12] in his monograph and review article provides a detailed introduction to this subject. Chandrashekher [15] has given a detailed account of thermal convection problems of Newtonian fluids. The theory of convective instability of ferrofluid begins with Finalyson [3] and is interestingly continued by Lalas and Carmi [7], Shliomis [9], Stile and Kagan [11], Venkatasubramanian and Kaloni [16] and Sunil et al; [17]. In the absence of an applied magnetic field, the particles in the colloidal suspensions are randomly oriented and thus the fluid has no net magnetization. When exposed to a magnetic field, Brownian rotational motions prevent complete alignment of the dipoles with the applied field. As a result when the applied field has a changing direction or magnitude, the magnetization is unable to track the field closely and becomes non-equilibrated. This non-equilibrium state of magnetization leads to the state of asymmetric stress. Rayleigh – Bénard convection in a ferromagnetic fluid layer with internal angular momentum permeated by uniform, vertical magnetic field with free-free, isothermal, spin-vanishing, magnetic boundaries has been considered by Abraham [1]. She observed that the micropolar ferromagnetic fluid layer heated from below is more stable as compared with the classical Newtonian ferromagnetic fluid. More recently, Sunil et al; [19] have studied the convection problems in a ferrofluid with internal angular momentum in a porous and non-porous medium. The effect of rotation on thermal convection in a micropolar fluids is important in certain chemical engineering and biochemical situations. Qin and Kaloni [23] have considered a thermal instability problem in a rotating micropolar fluid. They found that, depending upon the values of various micropolar parameters and the low values of the Taylor number, the rotation has a stabilizing effect. The effect of rotation on thermal convection in micropolar fluids has also been studied by Sharma and Kumar [13], whereas the numerical solution of thermal instability of rotating micropolar fluid has been discussed by Sastry and Datta [22] without taking into account the rotation effect in angular momentum equation. But we also appreciate the work of Bhattacharyya and Abbas [21] and Qin and Kaloni, they have considered the effect of rotation in angular momentum equation. More recently, Sunil et al., [18] have studied the effect of rotation on the thermal convection problems in ferrofluid.

In the standard Bénard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid, additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference, the temperature field and salt field. The solution behavior in the double-diffusive convection problem is more interesting than that of the single component situation in so
much as new instability phenomena may occur which is not present in the classical Bénard problem. When temperature and two or more component agencies, or three different salts, are present then the physical and mathematical situation becomes increasingly richer. Very interesting results in triply diffusive convection have been obtained by Pearlstein et al., [2]. The results of Pearlstein et al., are remarkable. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker [4] derive the equations for non-Boussinesq convection in a multi-component fluid and investigate the situation analogous to that of Pearlstein et al., but allowing for a density non linear in the temperature field. Lopez et al., [14] derive the equivalent problem with fixed boundary conditions and show that the effect of the boundary conditions breaks the perfect symmetry. In reality the density of a fluid is never a linear function of temperature, and so the work of Straughan and Walker applies to the general situation where the equation of state is one of the density quadratic in temperature. This is important, since they find that departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.,. A recent review of numerical techniques and their applications may be found in O’Sullivan et al; [10]. Oldenburg and Pruess [5] have developed a model for convection in a Darcy’s porous medium, where the mechanism involves temperature, NaCl, CaCl$_2$ and KCl. Solar ponds are a particularly promising means of harnessing energy from the Sun by preventing convective overturning in a thermohaline system by salting from below. A comprehensive review of the literature concerning convection in porous medium may be found in the book by Nield and Bejan [6].

Keeping in mind the importance of ferrofluids in various applications and in view of the above investigation, I intend to extend my work to effect of rotation on triple-diffusive convection in a magnetized ferrofluid with internal angular momentum saturating a porous medium. The understanding of the rotating ferrofluid stability problems plays an important role in microgravity environmental applications.

2. Mathematical formulation of the problem

Here we consider an infinite, horizontal layer of thickness ‘d’ of an electrically non-conducting incompressible thin rotating ferrofluid with internal angular momentum heated and salted from below saturating a porous medium. The temperature $T$ and solute concentrations $C^1$ and $C^2$ at the bottom and top
The continuity equation for an incompressible fluid is

\[ \nabla \cdot \mathbf{q} = 0 \]  

(1)

The momentum and internal angular momentum equations are

\[ \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} = -\nabla p + \rho \mathbf{g} + \nabla \cdot (\mathbf{H} \mathbf{B}) - \frac{1}{\kappa_a} (\zeta + \eta) \mathbf{q} + 2\zeta (\nabla \times \omega) + \frac{2\rho g}{\kappa_a} (\mathbf{q} \times \Omega) \]  

(2)

\[ \rho \left[ \frac{\partial \mathbf{q}}{\partial t} + \left( \frac{1}{\kappa_a} \right) (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = 2\zeta (\mathbf{q} \times \nabla \mathbf{q} - 2\omega) + \mu_0 (\mathbf{M} \times \mathbf{H}) + (\lambda + \eta') \nabla (\nabla \cdot \omega) + \eta' \nabla^2 \omega \]  

(3)

The temperature and solute concentration equations for an incompressible ferrofluid are

\[ [\rho_0 C_v, \nu, \mu_0 \mathbf{H}] \left( \frac{\partial \mathbf{T}}{\partial t} + (1-\epsilon)\rho_0 C_s \frac{\partial \mathbf{T}}{\partial t} + \mu_0 \mathbf{T} \left( \frac{\partial \mathbf{M}}{\partial t} \right) \right) \nu, \mathbf{H}, \mathbf{M} = \kappa_1 \mathbf{y}^2 \mathbf{T} + \delta (\nabla \times \omega). \nabla \mathbf{T} \]  

(4)

\[ [\rho_0 C_v, \nu, \mu_0 \mathbf{H}] \left( \frac{\partial \mathbf{T}}{\partial t} + (1-\epsilon)\rho_0 C_s \frac{\partial \mathbf{T}}{\partial t} + \mu_0 \mathbf{T} \left( \frac{\partial \mathbf{M}}{\partial t} \right) \right) \nu, \mathbf{H}, \mathbf{M} = \kappa_1' \mathbf{y}^2 \mathbf{C} \]  

(5)

\[ [\rho_0 C_v, \nu, \mu_0 \mathbf{H}] \left( \frac{\partial \mathbf{T}}{\partial t} + (1-\epsilon)\rho_0 C_s \frac{\partial \mathbf{T}}{\partial t} + \mu_0 \mathbf{T} \left( \frac{\partial \mathbf{M}}{\partial t} \right) \right) \nu, \mathbf{H}, \mathbf{M} = \kappa_1' \mathbf{y}^2 \mathbf{C} \]  

(6)

In terms of temperature \( T \) and the concentrations \( C \) and \( C' \), we suppose the density of the mixture is given by (known as density equation of state)

\[ \rho = \rho_0 [1 - \alpha (T - T_s) + \alpha' (C - C_s^1) + \alpha'' (C^2 - C_s^2)] \]  

(7)

Where \( \rho, \rho_0, \mathbf{q}, \omega, t, \mathbf{p}, \eta, \zeta, \lambda, \eta', \delta, I, \mu_0, \mathbf{B}, C_v, \mathbf{M}, K_s, \kappa_1, \kappa_1', \alpha, \alpha', \alpha'' \) are the fluid density, reference density, velocity, microrotation, time, pressure, shear kinematic viscosity coefficient, coupling viscosity coefficient or vortex viscosity, bulk spin viscosity coefficient, shear spin viscosity coefficient, micropolar heat conduction coefficient, moment of inertia (micronertia constant), magnetic permeability, magnetic induction, heat capacity at constant volume and
magnetic field, magnetization, thermal conductivity, solute conductivity, thermal expansion coefficient and concentration expansion coefficient analogous to the thermal expansion coefficient respectively. $T_a$ is the average temperature given by $T_a = \frac{(T_0 + T_1)}{2}$ where $T_0$ and $T_1$ are the constant average temperatures of the lower and upper surfaces of the layer and $C_{a1}$ and $C_{a2}$ are the average concentrations given by $C_{a1} = \frac{(C_{01} + C_{11})}{2}$ and $C_{a2} = \frac{(C_{02} + C_{12})}{2}$ where $C_{01}$, $C_{11}$ and $C_{02}$, $C_{12}$ are the constant average concentrations of the lower and upper surfaces of the layer. In writing equation (2), we also use the Boussinesq approximation by allowing the density to change only in the gravitational body force term. When the permeability of the porous material is low, then the inertial force becomes relatively insignificant as compared with the viscous drag when flow is considered. And as we know the $\frac{1}{\epsilon^2}(\mathbf{q} \cdot \nabla)\mathbf{q}$ term is generally small, so it seems best to drop it in numerical work.

A porous medium of very low permeability allows us to use the Darcy’s model. For a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy’s model. This is because the viscous drag force is negligibly small in comparison with Darcy’s resistance due to the large particle suspension. Darcy’s law governs the flow of ferromagnetic fluid through an isotropic and homogeneous porous medium. However, to be mathematically compatible and physically consistent with Navier- Stokes equations, Brinkman [8] heuristically proposed the introduction of the term $\frac{\mu}{\kappa_1} \nabla^2 \mathbf{q}$ (now known as the Brinkman term) in addition to the Darcian term $-\left(\frac{\eta}{\kappa_1}\right)\mathbf{q}$. But the main effect is through the Darcian term; the Brinkman term contributes to a very little effect for flow through a porous medium. Therefore, Darcy’s law is proposed heuristically to govern the flow of this ferrofluid saturating a porous medium.

The equation of magnetization relaxation (Rosensweig) is

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{M} = (\mathbf{\omega} \times \mathbf{M}) \cdot \frac{1}{\tau_m} (\mathbf{M} - \mathbf{M}_{eq})$$

(8)

Where $\tau_m$ is the Brownian relaxation time, $\mathbf{M}_{eq}$ is equilibrium magnetization described by Langevin formula (Shliomis).

In general, the presence of ferromagnetic fluid can distort an external field if magnetic interaction (dipole-dipole) takes place, but this is negligible for small particle concentration as is assumed here. We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{H}{H_0} \mathbf{M} (H, T, C_1, C_2).$$

(9)

The magnetic equation of state is linearized about the magnetic field, $H_0$, an average temperature, $T_a$, and average concentrations, $C_{a1}$ and $C_{a2}$ to become
where magnetic susceptibility, pyromagnetic coefficient and salinity magnetic coefficients are defined by
\[ \chi \equiv \left( \frac{\partial M}{\partial H} \right) H_0, T_3; \quad K_2 \equiv - \left( \frac{\partial M}{\partial T} \right) H_0, T_3; \quad K_3 \equiv \left( \frac{\partial M}{\partial C_1} \right) H_0, c_3^1 \quad \text{and} \quad K_4 \equiv \left( \frac{\partial M}{\partial C_2} \right) H_0, c_3^2 \]
respectively.

Here \( H_0 \) is the uniform magnetic field of the fluid layer when placed in an external magnetic field

\[ H = H_0^{\text{ext}} \hat{k}, \quad \text{where} \quad \hat{k} \quad \text{is a unit vector in the z direction,} \quad H = |H|, \quad M = |M| \quad \text{and} \quad M_0 = M( H_0, T_3, C_3^1, C_3^2) \]

The basic state is assumed to be quiescent state and is given by

\[ q = q_0 = (0,0,0), \quad \omega = \omega_b = (0,0,0), \quad \rho = \rho_b (z), \quad \rho = \rho_b (z), \quad T = T_b (z) = - \beta z + T_3, \quad C^1 = C_b^1 (z) = - \beta' z + C^1_3 (z) = \beta'' z + C^2_3 (z), \quad \beta = (0-T_1)/d, \quad \beta' = (C^1_3 - C^1_b)/d, \]

\[ H_0 = \frac{K_b \beta z}{1+x^2} + \frac{K_b \beta z}{1+x^2} + \frac{K_b \beta^2 z}{1+x^2} \quad \hat{k}, \quad M_b = \frac{M_0 + K_b \beta z}{1+x^2} - \frac{K_b \beta^2 z}{1+x^2} \quad \hat{k} \quad \text{and} \quad H_0 + M_0 = H_0^{\text{ext}} \]

where the subscript \( \cdot \) denotes the basic state.

3. The perturbation equations and normal mode analysis method

We now examine the stability of the basic state, and assume that the perturbation quantities are small. We write

\[ q = q_b + q', \quad \omega = \omega_b + \omega', \quad \rho = \rho_b + \rho', \quad p = \rho_b (z) + \rho', \quad T = T_b (z) + \theta, \quad C^1 = C_b^1 (z) + \gamma, \quad C^2 = C_b^2 (z) + \gamma', \quad H = H_b (z) + H' \quad \text{and} \quad M = M_b (z) + M' \]

where \( q' = (u, v, w), \quad \omega' = (\omega_1, \omega_2, \omega_3), \quad \rho', \quad \theta, \quad \gamma, \quad \gamma', \quad H', \quad M' \) are perturbation in ferrofluid filter velocity \( q \), spin \( \omega \), pressure \( p \), temperature \( T \), concentrations \( C^1 \) and \( C^2 \), magnetic field intensity \( H \), and magnetization \( M \), respectively. The change in density \( \rho' \), caused mainly by the perturbations \( \theta, \gamma, \gamma' \) in temperature and concentrations, respectively, is given by

\[ \rho' = - \rho_0 (\alpha \theta - \alpha' \gamma - \alpha'' \gamma'') \]

Then, the linearized perturbation equations (by neglecting second-order small quantities) of the micropolar ferromagnetic fluid give the following dimensionless equations:

\[
\{1 - \frac{\alpha (1+\alpha)}{k_1} (D_2 - a^2)\} \{D^2 - a^2\} W = a \sqrt{\alpha} \left\{ \left[ (M_1 - M_3) D \phi_3^* - (1+M_1 - M_3) T^* \right] + a \sqrt{\alpha} \left( (M_1'' - M_3') D \phi_3^* + (1-M_1' + M_3') C^{1*} \right) + a \sqrt{\alpha} \left( (M_1''' - M_3'') D \phi_3^* \right) \right\}
\]
where the following non dimension quantities and non dimensionless parameters are introduced:

\[ t' = \frac{v t}{d^2}, \quad W' = \frac{W}{v}, \quad \phi_1' = \frac{(1+\gamma) \kappa_1 a \sqrt{\gamma}}{\kappa_2 \rho \sigma \beta d^2}, \quad \phi_2' = \frac{(1+\gamma) \kappa_1 a \sqrt{\gamma}}{\kappa_4 \rho \sigma \beta d^2}, \quad \phi_3' = \frac{(1+\gamma) \kappa_1 a \sqrt{\gamma}}{\kappa_4 \rho \sigma \beta d^2}, \quad \phi_3' = \frac{\Omega_s d^2}{g} \]

\[ R = \frac{g a \beta a^4 \rho \sigma}{\nu k_1}, \quad S_1 = \frac{g a \beta a^4 \rho \sigma}{\nu k_1}, \quad S_2 = \frac{g a \beta a^4 \rho \sigma}{\nu k_1}, \quad T' = \frac{k_1 a \sqrt{\gamma}}{\rho \sigma \beta d^2}, \quad C' = \frac{k_1 a \sqrt{\gamma}}{\rho \sigma \beta d^2}, \quad \Gamma, \quad C_{2''} = \frac{k_1 a \sqrt{\gamma}}{\rho \sigma \beta d^2}, \quad a = k d, \quad z' = \frac{z}{d}, \quad D = \frac{d}{d z'}, \quad P_2 \rho C_1 = \frac{\nu k_1}{k_1}, \quad P_1 = \frac{\nu k_1}{k_1}, \quad P_1 = \frac{\nu k_1}{k_1}, \quad \phi_3' = \frac{\Omega_s d^2}{g} \]

\[ M_1 = \frac{\mu_0 \kappa_2 \beta}{(1+\chi)a \rho \sigma g}, \quad M_2 = \frac{\mu_0 \kappa_2 \beta}{(1+\chi)a \rho \sigma g}, \quad M_3 = \frac{\mu_0 \kappa_2 \beta}{(1+\chi)a \rho \sigma g}, \quad M_4 = \frac{\mu_0 \kappa_2 \beta}{(1+\chi)a \rho \sigma g}, \quad M_5 = \frac{\mu_0 \kappa_2 \beta}{(1+\chi)a \rho \sigma g}, \quad N_1 = \frac{\zeta}{\eta}, \quad N_2 = \frac{\eta}{\rho c_2 d^2}, \quad N_3 = \frac{\rho c_2 d^2}{I} \]

4. Exact solution for free boundaries

Here the simplest boundary conditions chosen, namely free-free, no-spin, isothermal with infinite magnetic susceptibility \( \chi \) in the perturbed field keep the problem analytically tractable and serve the purpose of providing a qualitative insight into the problem. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus the exact solution of Eqs. (15)-(23) subject to the boundary conditions are

\[ W' = \frac{D}{\Omega} W = \frac{T}{C} = \frac{\phi_1}{C} = \phi_2 = \phi_3 = 0 \quad \text{at} \quad z = \pm \frac{1}{2}, \quad (24) \]

is written in the form

\[ W'^* = A_1 e^{\alpha z} \cos \pi z, \quad T'^* = B_1 e^{\alpha z} \cos \pi z, \quad D \phi_1'^* = C_1 e^{\alpha z} \cos \pi z, \quad \Omega'^* = D_1 e^{\alpha z} \cos \pi z, \]

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$$D \phi_2^* = E e^{\sigma t} \cos \pi z^*, \, \phi_1^* = \left( \frac{C_2}{\pi} \right) e^{\sigma t} \sin \pi z^*, \, C_1^* = F e^{\sigma t} \cos \pi z^*$$

$$C_2^* = G e^{\sigma t} \cos \pi z^*, \, D \phi_3^* = H e^{\sigma t} \cos \pi z^*, \, \phi_3^* = \left( \frac{H_1}{\pi} \right) e^{\sigma t} \sin \pi z^*$$  \hspace{1cm} (25)

Where $A_1$, $B_1$, $C_1$, $D_1$, $E_1$, $F_1$, $G_1$, and $H_1$ are constants and $\sigma$ is the growth rate, in general, a complex constant. For existence of non-trivial solutions, the determinant of the coefficients of $A_1$, $B_1$, $C_1$, $D_1$, $E_1$, $F_1$, $G_1$, and $H_1$ must vanish.

This determinant on simplification yields

$$i T_5 \sigma_5^5 + T_4 \sigma_4^4 - i T_3 \sigma_3^3 - T_2 \sigma_2^2 + i T_1 \sigma_1 + T_0 = 0.$$  \hspace{1cm} (26)

Here

$$T_5 = \frac{b}{e^2} \left[ 1 \right],$$

$$T_4 = \frac{b}{e^2} \left[ \left( bL_4 L_1 (L_3 + L_2) + L_3 L_2 \right) \left( 1 + N_1 + N_3 b \right) \right] + \left( 4 N_1 + N_3 b \right) \right],$$

$$T_3 = \frac{b^2}{e^2} \left( L_1 (L_4 L_1 + L_2 + L_3) \left( 1 + N_1 + N_3 b \right) \right] + \left( 4 N_1 + N_3 b \right) \right],$$

$$T_2 = \frac{b^3}{e^2} \left( \frac{N_4 N_5}{b^2} \right) \left( x S_1 \right) + \left( 4 N_4 + N_5 b \right) \right],$$

$$T_1 = \frac{b^4}{e^2} \left( \frac{N_4 N_5}{b^2} \right) \left( x S_1 \right) + \left( 4 N_4 + N_5 b \right) \right],$$

$$T_0 = \frac{b^5}{e^2} \left( \frac{N_4 N_5}{b^2} \right) \left( x S_1 \right) + \left( 4 N_4 + N_5 b \right) \right],$$

where

$$R_1 = \frac{R}{\pi^2}, \, S_1 = \frac{S_1}{\pi^2}, \, S_2 = \frac{S_2}{\pi^2}, \, x = \frac{a^2}{\pi^2}, \, I_1, \pi^2 I_1, \sigma = \frac{\sigma}{\pi^2}, \, N_1 = \pi^2 N_1, \, N_2^* = \pi^2 N_2, \, b = 1 + x, \, D_3 = \pi^2 k_1,$$

$$L_1 = (1 + x M_3) L_1 = \left[ \begin{array}{ccc} 1 + x M_3 \end{array} \right], \, L_2 = \left[ \begin{array}{ccc} 1 + x M_3 + x M_3 M_1 (1 - M_2) \end{array} \right], \, L_3 = \left[ \begin{array}{ccc} 1 + x M_3 + x M_3 M_1 (1 - M_2) \end{array} \right], \, L_4 = \left[ \begin{array}{ccc} 1 + x M_3 + x M_3 \end{array} \right].$$

The coefficients $T_2$ and $T_1$ are quite lengthy and not needed in the discussion.

5. Results and discussion

5.1 The case of stationary convection

When the instability sets in as stationary convection in the case $M_2 \equiv 0$, $M_2^* \equiv 0$, the marginal state will be characterized by $\sigma_1 = 0$ [14], then the Rayleigh number $R_1$ is given by

$$R_1^* = \frac{1}{M_2^*} \left[ 1 + x M_3 \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right] \left[ 1 + x M_3 + x M_3 M_1 (1 - M_2) \right]$$.  \hspace{1cm} (28)
which expresses the modified Rayleigh number $R_3$ as a function of dimensionless wave number $x$, buoyancy magnetization parameter $M_1$, the non-buoyancy magnetization parameter $M_3$, Taylor number $T_A$, medium permeability parameter $D_a$, (Darcy number), solute gradient parameters $S_1$ and $S_2$, ratio of the salinity effect on magnetic field to pyromagnetic coefficient $M_5$, coupling parameter $N_1$, (coupling between vorticity and spin effects), spin diffusion parameter $N_3$ and micropolar heat conduction parameter $N_5$, (coupling between spin and heat fluxes).

The classical results in respect of Newtonian fluids can be obtained as the limiting case of present study. Setting $N_3 = 0$ and $S_1 = 0$, and keeping $N_3$ arbitrary in equation (28), we get

$$R_3 = \frac{(1 + x)(1 + xM_3) + T_A D_a}{x(1 + xM_1)};$$

which is the expression for the Rayleigh number of ferrofluid in a porous medium in the presence of rotation. Setting $M_3 = 0$ in equation (29), we get

$$R_3 = \frac{(1 + x) + T_A D_a}{x};$$

the classical Rayleigh Bénard result in a porous medium for the for the Newtonian fluid case.

To investigate the effect of solute gradients, non-buoyancy magnetization coefficient, coupling parameter, spin parameter, and micropolar heat conduction parameter, we examine the behavior of

$$\frac{dR_1}{dS_1}, \frac{dR_1}{dS_2}, \frac{dR_1}{dT_A}, \frac{dR_1}{dM_3}, \frac{dR_1}{dN_1}, \frac{dR_1}{dN_3}, \frac{dR_1}{dN_5}$$

analytically.

Equation (56) gives

$$\frac{dR_1}{dS_1} = \frac{1 + xM_3 + x M_1 M_5 \frac{1}{M_5} \left( \frac{1}{N_3} - 1 \right)}{L_2 \left( 4 N_1 + b(N_5' - \frac{2N_1 N_5}{c}) \right)},$$

$$\frac{dR_1}{dS_2} = \frac{1 + xM_3 + x M_1 M_5 \frac{1}{M_5} \left( \frac{1}{N_3} - 1 \right)}{L_2 \left( 4 N_1 + b(N_5' - \frac{2N_1 N_5}{c}) \right)},$$

$$\frac{dR_1}{dT_A} = \frac{bL_1 (1 + N_1) \left( 4 N_1 + b \left( N_5' - \frac{2N_1 N_5}{c} \right) \right)}{xL_2 \left( 1 + N_1 \right) \left( 4 N_1 + b \left( N_5' - \frac{2N_1 N_5}{c} \right) \right)},$$

This shows that, for a stationary convection, the rotation and solute gradients have stabilizing effect, if

$$N_3' > 2 N_1 N_5'.$$

In the absence of micropolar viscous effect (coupling parameter $N_3$), stable solute gradients always have stabilizing effect, on the system. Equation (28) also yields...
\[
\frac{dR_1}{dD_0} = \frac{bL_1(1+2(4N_1+4N_2)N_s\beta)(1+2N_1)N_1-b-T_{A_1}(D_0)^2}{x\alpha^2 L_2 \left( \frac{4+4N_1}{D_0} \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right)},
\]

\[\text{(34)}\]

Which is negative if

\[N_3' > \frac{2N_1N_5^s}{\epsilon}, \quad T_{A_1} < \frac{1}{D_0^2}\]

This shows that the medium permeability has a destabilizing effect when condition (35) holds. In the absence of the coupling parameter \(N_1=0\) and rotation \(T_{A_1} = 0\), the medium permeability always has a destabilizing effect on the onset of convection. Thus, in a ferrofluid heated and soluted from below saturating a porous medium, there is a competition between the destabilizing role of medium permeability and stabilizing role of the coupling parameter and rotation. Thus, the destabilizing behavior of medium permeability is virtually unaffected by magnetization parameters but is significantly affected by angular parameters, \(N_1, N_3, N_5'\), and Taylor number, \(T_{A_1}\). Equation (28) also yields

\[
\frac{dR_1}{dM_3} = \frac{(1-N_0) \left( \left( 4 N_1 + b(1+2N_1)N_s\beta \right) \left( 1+2N_1 \right) N_2T_{A_1} - bL_1 \left( \frac{4+4N_1}{D_0} \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 1+\chi M_3 + \chi M_4 \left( 1-M_4 \right) \right) + \left( \chi \alpha \right) \left( M_4 \right) - \left( \chi \alpha \right) \left( M_3 \right) \left( M_4 \right) }{(1+\chi M_3 + \chi M_4 \left( 1-M_4 \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 1+\chi M_3 + \chi M_4 \left( 1-M_4 \right) \right) + \left( \chi \alpha \right) \left( M_4 \right) - \left( \chi \alpha \right) \left( M_3 \right) \left( M_4 \right) )}
\]

\[\text{(35)}\]

Which is negative, if

\[N_3' > \frac{2N_1N_5^s}{\epsilon}, \quad \frac{1}{D_0} > \frac{b}{\epsilon} N_1, \quad M_1 M_5 > M_1' \quad \text{and} \quad M_1 M_5 > M_1''\]

\[\text{(36)}\]

This shows that the non-buoyancy magnetization has a destabilizing effect when conditions (36) hold. In the absence of micropolar viscous effect \(N_1=0\) and the effect on magnetization due to salinity \((M_1'=0\) and \(M_1''=0\)), the non-buoyancy magnetization always has a destabilizing effect on the system. It follows from equation (28) that

\[
\frac{dR_1}{dN_1} = \frac{bL_1 \left( 4N_1(1+2N_1)N_s\beta \right) \left( 1+2N_1 \right) N_1 \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 1+\chi M_3 + \chi M_4 \left( 1-M_4 \right) \right) + \left( \chi \alpha \right) \left( M_4 \right) - \left( \chi \alpha \right) \left( M_3 \right) \left( M_4 \right) )}{x\alpha \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 4 N_1 + b(N_3 - \frac{2N_1N_5^s}{\epsilon}) \right) \left( 1+\chi M_3 + \chi M_4 \left( 1-M_4 \right) \right) + \left( \chi \alpha \right) \left( M_4 \right) - \left( \chi \alpha \right) \left( M_3 \right) \left( M_4 \right) )}
\]

\[\text{(37)}\]

which is always positive if

\[N_3' > \frac{2N_1N_5^s}{\epsilon}, \quad \frac{1}{D_0} > \frac{b}{\epsilon}, \quad T_{A_1} < \frac{1}{D_0^2}, \quad N_5' > 2\epsilon\]

\[\text{(38)}\]

This shows that coupling parameter has a stabilizing effect, when condition (38) hold. In a non-porous medium, equation (37) yield that \(\frac{dR_1}{dN_1}\) is always positive, implying that thereby the stabilizing effect of coupling parameter. Thus the
medium permeability and porosity have a significant role in developing condition for the stabilizing behavior of coupling parameter. Equation (28) gives

$$\frac{dR_m}{dN_5} = \frac{2b^2 N_1}{e} \left[ \left( \frac{1 + x M_2}{\phi} \right)^2 \left( \frac{1 + N_1}{\phi} \right)^2 \left( 1 + x M_2 \right) + T_{\mathcal{R}} \left( \frac{1 + N_1}{\phi} \right) \right] - \frac{2b^2 N_1}{e} \left( \frac{1 + x M_2}{\phi} \right)^2 \left( \frac{1 + N_1}{\phi} \right) \left( x^2 \left( 1 + x M_2 \right) + b \left( \frac{1 + x M_2}{\phi} \right) \right) + \left( \frac{1 + x M_2}{\phi} \right) \left( \frac{1 + N_1}{\phi} \right) \left( x^2 \left( 1 + x M_2 \right) + b \left( \frac{1 + x M_2}{\phi} \right) \right)$$

which is always positive if

$$\frac{1}{b_a} > \frac{b}{e^3}$$

This shows that the heat conduction always has a stabilizing effect when condition (39) holds. Here we also observe that in a non-porous medium, \( \frac{dR_m}{dN_5} \) is always positive, implying thereby the stabilizing effect of heat conduction parameter. For sufficiently large values of \( M_1 \) [Finlayson, 1970], we obtain the results for the magnetic mechanism operating in a porous medium.

\[ R_m = R_1 M_1 \]

\[ = \left[ \left( \frac{1 + x M_2}{\phi} \right)^2 \left( \frac{1 + N_1}{\phi} \right)^2 \left( 1 + x M_2 \right) + T_{\mathcal{R}} \left( \frac{1 + N_1}{\phi} \right) \right] - \frac{2b^2 N_1}{e} \left( \frac{1 + x M_2}{\phi} \right)^2 \left( \frac{1 + N_1}{\phi} \right) \left( x^2 \left( 1 + x M_2 \right) + b \left( \frac{1 + x M_2}{\phi} \right) \right) + \left( \frac{1 + x M_2}{\phi} \right) \left( \frac{1 + N_1}{\phi} \right) \left( x^2 \left( 1 + x M_2 \right) + b \left( \frac{1 + x M_2}{\phi} \right) \right) \]

(40)

here \( R_m \) is the magnetic thermal Rayleigh number. As a function of \( x \), \( R_m \) given by equation (40) attains its maximum when

\[ P_0 x^6 + P_1 x^5 + P_2 x^4 + P_3 x^3 + P_4 x^2 + P_5 x + P_6 = 0. \]  

(41)

The coefficients \( P_0, P_1, P_2, P_3, P_4, P_5, P_6 \) being quite lengthy, have not been written here and are evaluated numerical calculation.

The values of critical wave number for the onset of instability are determined numerically using Newton-Raphson method by the condition \( \frac{dR_m}{dx} = 0 \). With \( x_1 \) determined as a solution of equation (40), equation (41) will give the required critical magnetic thermal Rayleigh number \( N_c \) which depend upon \( M_3, D_a, S_1, S_2 \) and parameters \( N_1, N_3 \) and \( N_5 \).
5.2 The case of overstability

The present section is devoted to find the possibility that the observed instability may really be overstability. Since we wish to determine the Rayleigh number for the onset of instability through state of pure oscillations, it is sufficient to find conditions for which (26) will admit of solutions with $\sigma_i$ real. Equating real and imaginary parts of (26) and eliminating $R_1$ between them, we obtain

$$A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0,$$  \hspace{1cm} (42)

Where, $c_1 = \alpha_0^2$. Since $\sigma_i$ is real for overstability, the three values of $c_1(= \alpha_0^2)$ are positive. The product of roots of equation (42) is $-\frac{A_0}{A_3}$, where

$$A_3 = -\frac{b}{e^2} + \frac{L_2^2L_3^2}{L_1^2} \left\{ \left( L_1 + \frac{1 + N_2}{D_0} \right) L_2 \right\} \left( 1 - M_2 \right) + \frac{2N_1}{e} L_2 \left( b N_3 \right) + \frac{1}{D_0} \left( b^2 + \frac{2D_3^{T_A}}{b(1 + N_1)^2} \right) \left[ (1 - M_3')L_1L_3 - (1 - M_2')L_2L_3 \right],$$  \hspace{1cm} (43)

$$A_0 = \left( b + \frac{2N_1}{e} \right) L_2L_3 \left\{ \left( L_1 + \frac{1 + N_2}{D_0} \right) L_2 \right\} \left( 1 - M_2 \right) + \frac{2N_1}{e} L_2 \left( b N_3 \right) \left( b^2 + \frac{2D_3^{T_A}}{b(1 + N_1)^2} \right) \left[ (1 - M_3')L_1L_3 - (1 - M_2')L_2L_3 \right] - \left( 1 - M_3' \right) L_1L_3 \left\{ \left( b^2 + \frac{2D_3^{T_A}}{b(1 + N_1)^2} \right) \left[ (1 - M_3')L_1L_3 - (1 - M_2')L_2L_3 \right] \right\},$$  \hspace{1cm} (44)

The coefficients $A_2$ and $A_3$ being quite lengthy and not needed in the discussion of overstability, has not been written here.

Since $\sigma_i$ is real for overstability, the three values of $c_1(= \alpha_0^2)$ are positive. The product of roots of equation (42) is $-\frac{A_0}{A_3}$, and if this is to be negative, then $A_3$ and $A_0$ are of the same sign. Now, the product is negative if

$$N_3 \left( 1 - M_2 \right) > \frac{4N_1N_2}{e}, \quad \frac{N_3}{L_3} > \frac{1}{D_0} > T_{A_1},$$

i.e. if $N_3 \left( 1 - M_2 \right) \left( L_2 - L_3 \right) > \frac{2L_2 N_1 N_2}{e}, \quad L_1 > N_1 L_2 \frac{L_2}{D_0} \quad \text{and} \quad L_2 > L_3,$

which implies that

$$N_3 > \max \left\{ \frac{4N_1N_2}{e(1 - M_2)} \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right), \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right) \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right) \right\}, \quad T_{A_1} < \frac{1}{D_0}.$$

Thus, for $\frac{1}{N_1} > P_{r'} > N_3 > \max \left\{ P_{S_1}, \frac{P_{r'}}{P_{S_2}} + \frac{D_0}{e} \right\},$ and

$$N_3 > \max \left\{ \frac{4N_1N_2}{e(1 - M_2)} \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right), \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right) \left( \frac{1}{P_{S_1} + eP_{S_2}M_2} \right) \right\},$$

and for $T_{A_1} < \frac{1}{D_0}$ overstability cannot occur and the principle of the exchange of stabilities is valid. Hence, the above conditions are the sufficient conditions for

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the non existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

5. Conclusions

In this paper, linear stability of triple-diffusive convection in a magnetized ferrofluid layer heated and soluted from below saturating a porous medium with internal angular momentum has been investigated. The analysis is restricted to a physical situation in which the magnetization induced by temperature and concentration variations is small compared to that induced by the external magnetic field. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. In conclusion, we see that convection can encourage in ferrofluid with internal angular momentum by means of spatial variation in magnetization, which is induced when the magnetization of the fluid depends on the temperature and solute concentrations and a uniform temperature gradient and a uniform solute gradient are established across the layer. This problem represents thermal-salinity-micro rotational – mechanical interaction in porous medium arising through the stress tensor, salinity and micro rotation. We have investigated the effect of various parameters like medium permeability, solute gradients, non-buoyancy magnetization, rotation, coupling parameter, spin diffusive parameter and heat conduction on the onset of convection has been analyzed analytically and numerically. The destabilizing behavior of medium permeability and stabilizing behavior of solute gradients, rotation are virtually unaffected by magnetic parameters but are significantly affected by angular momentum parameters. The presence of coupling between vorticity and spin effects (viscous effect), microinertia, solute gradient and rotation may bring overstability in the system. Thus oscillatory modes are introduced due to the presence of the viscous effect, microinertia and solute gradients, which were non-existent in their absence. Finally, we conclude that the angular momentum parameters, rotation and solute gradient have a profound influence on the onset of convection in porous medium.

References

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