A New Approach on Solving Intuitionistic Fuzzy Linear Programming Problem

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Abstract
In this paper we define division operation of Triangular Intuitionistic Fuzzy number (TIFN) using \( \alpha, \beta \) – cut and a scoring function to rank TIFNs. An accuracy function to defuzzify TIFN is also introduced. Based on this new approach, the solution of Intuitionistic Fuzzy Linear Programming is obtained. Finally, an illustrative example is given to verify the developed approach.

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Keywords: Triangular Intuitionistic fuzzy number, Scoring function, Accuracy function, Linear programming problem

1. Introduction
Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set [1]. Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [2] using membership and non–membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by S.Mahapatra& T.K.Roy in [5], by considering the six tuple number itself. Here we have developed division operation on six tuple TIFN using \( \alpha, \beta \) – cut method. Most of the authors used the
membership and non-membership values of TIFNs for ranking. A ratio ranking method of TIFN is developed by Deng-Feng-Li in [3]. Ranking methods based on probabilities and hesitations is defined by L. Shen et al in [8]. Scoring function of a fuzzy number intuitionistic fuzzy value is defined by X.F. Wang in [9]. We have defined ranking of TIFNs using integral value by considering six tuple TIFNs in [6].

The aim of this paper is to propose division of TIFN using $\alpha, \beta –$ cut, score function and accuracy function of TIFNs. Based on the score functions we compare two TIFNs and it is applied to solve Intuitionistic Fuzzy Variable Linear Programming Problem. An accuracy function is developed to defuzzify TIFN.

2. Preliminaries

2.1 Definition [1]: Given a fixed set $X = \{x_1, x_2, x_3, \ldots, x_n\}$, an intuitionistic fuzzy set (IFS) is defined as $A = (\{x_i, t_A(x_i), f_A(x_i)\} / x_i \in X)$ which assigns to each element $x_i$ a membership degree $t_A(x_i)$ and a non-membership degree $f_A(x_i)$ under the condition $0 \leq t_A(x_i) + f_A(x_i) \leq 1$, for all $x_i \in X$.

2.2 Definition [5]:

A fuzzy number $\tilde{A}$ ($a_1, a_2, a_3$) is a Triangular Fuzzy Number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
$$

where $a_1, a_2, a_3$ are real numbers.

2.3 Definition [5] ($a, \beta$) – Level intervals or ($a, \beta$) – cuts:

A set of ($a, \beta$) – cut generated by IFS $\tilde{A}^l$, where $a, \beta \in [0,1]$ are fixed numbers such that $a + \beta$ defined as $\tilde{A}^l_{\alpha,\beta} = \{(x, \mu_{\tilde{A}^l}(x), \vartheta_{\tilde{A}^l}(x)) : x \in X, \mu_{\tilde{A}^l}(x) \leq a, \vartheta_{\tilde{A}^l}(x) \leq \beta, \alpha, \beta \in [0,1]\}$.

($a, \beta$) – level interval or ($a, \beta$) – cut denoted by $\tilde{A}^l_{\alpha,\beta}$ is defined as the crisp set of elements of $x$ which belong to $\tilde{A}^l$ at least to the degree $a$ and which does belong to $\tilde{A}^l$ at most to the degree $\beta$.

2.4 Definition [5]:

A Triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^l$ is an intuitionistic fuzzy set in $R$ with the following membership function $\mu_{\tilde{A}^l}(x)$ and non-membership function $\vartheta_{\tilde{A}^l}(x)$

$$
\mu_{\tilde{A}^l}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}, \quad \vartheta_{\tilde{A}^l}(x) = \begin{cases} 
\frac{a_2-x}{a_2-a_1'}, & a_1' \leq x \leq a_2 \\
\frac{x-a_2}{a_3'-a_2}, & a_2 \leq x \leq a_3' \\
1, & \text{otherwise}
\end{cases}
$$

where $a_1' \leq a_1 \leq a_2 \leq a_3$ and $\mu_{\tilde{A}^l}(x) + \vartheta_{\tilde{A}^l}(x) \leq 1$, $\forall x \in R$. This TIFN is denoted by.
\[\tilde{A}^I = (a_1, a_2, a_3; a'_1, a'_2, a'_3) = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\]

Fig.1 Membership and non-membership functions of TIFN

2.5 Arithmetic operations of Triangular Intuitionistic Fuzzy Number based on \((\alpha, \beta)\) - cuts method:[5]

i) If \(\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(\tilde{B}^I = \{(b_1, b_2, b_3); (b'_1, b'_2, b'_3)\}\) are two TIFNs, then their sum
\[\tilde{A}^I + \tilde{B}^I = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3)\}\] is also a TIFN.

ii) If \(\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(\tilde{B}^I = \{(b_1, b_2, b_3); (b'_1, b'_2, b'_3)\}\) are two TIFNs, then
\[\tilde{A}^I - \tilde{B}^I = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a'_1 - b'_3, a'_2 - b'_2, a'_3 - b'_1)\}\] is also a TIFN.

iii) If \(\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(\tilde{B}^I = \{(b_1, b_2, b_3); (b'_1, b'_2, b'_3)\}\) are two TIFNs, then their product
\[\tilde{A}^I \times \tilde{B}^I = \{(a_1b_1, a_2b_2, a_3b_3); (a'_1b'_1, a'_2b'_2, a'_3b'_3)\}\] is also a TIFN.

iv) If TIFN \(\tilde{A}^I = (a_1, a_2, a_3; a'_1, a'_2, a'_3) = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(y = ka\) (with \(k > 0\)) then \(\gamma^I = k\tilde{A}^I\) is a TIFN \(\{(ka_1, ka_2, ka_3); (ka'_1, ka'_2, ka'_3)\}\)

v) If TIFN \(\tilde{A}^I = (a_1, a_2, a_3; a'_1, a'_2, a'_3) = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(y = ka\) (with \(k < 0\)) then \(\gamma^I = k\tilde{A}^I\) is a TIFN \(\{(ka_1, ka_2, ka_3); (ka'_1, ka'_2, ka'_3)\}\)

3. Proposed Division of two TIFNs based on \((\alpha, \beta)\) - cuts method

If \(\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}\) and \(\tilde{B}^I = \{(b_1, b_2, b_3); (b'_1, b'_2, b'_3)\}\) are two positive TIFNs, then \(\frac{\tilde{A}^I}{\tilde{B}^I}\) is also a TIFN
\[\frac{\tilde{A}^I}{\tilde{B}^I} = \left\{\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \left(\frac{a'_1}{b'_3}, \frac{a'_2}{b'_2}, \frac{a'_3}{b'_1}\right)\right\}\]

Proof: Let \(z = \frac{x}{y}\) be the transformation with the membership functions and non-membership functions of TIFNs. Then \(\tilde{Z}^I = \frac{\tilde{A}^I}{\tilde{B}^I}\) can be found by \((\alpha, \beta)\) – cuts method:

- **\(\alpha\)-cut for membership function of \(\tilde{A}^I\)** is \([a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]\), \(\forall \alpha \in [0,1]\) i.e., \(x \in [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]\)
Now expressing division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ & $\tilde{B}^I$, we first divide the $\alpha$ - cuts of $\tilde{A}^I$ & $\tilde{B}^I$ using interval arithmetic

$$\tilde{A}^{\alpha} = \left[ a_1 + \alpha(a_2 - a_1), a_2 - \alpha(a_2 - a_1) \right], \forall \alpha \in [0,1] \text{ i.e., } y \in \left[ a_1 + \alpha(a_2 - b_1), a_2 - \alpha(a_2 - b_1) \right]$$

$$\tilde{B}^{\alpha} = \left[ b_1 + \alpha(b_2 - b_1), b_2 - \alpha(b_2 - b_1) \right]$$

$$= \left[ a_1 + \alpha(a_2 - a_1), a_2 - \alpha(a_2 - a_1) \right]$$

To find the membership function $\mu_{\tilde{A}^I}(x)$, we equate to $x$ both the first and second component, which gives

$$x = \frac{a_1 + \alpha(a_2 - a_1)}{b_2 - \alpha(b_2 - b_1)} \text{ and } x = \frac{a_2 - \alpha(a_2 - a_1)}{b_1 + \alpha(b_2 - b_1)}$$

Now expressing $\alpha$ in terms of $x$ and setting $\alpha = 0$ and $\alpha = 1$, we get

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{b_3x - a_1}{a_2 - a_1} + \frac{(b_3 - b_2)x}{a_3 - b_1x}, & \frac{a_1}{b_3} \leq x \leq \frac{a_2}{b_2} \\ \frac{a_2 - \alpha(a_2 - a_1)}{b_1 + \alpha(b_2 - b_1)} \end{cases}$$

• $\beta$-cut for non-membership function of $\tilde{A}^I$ is

$$[a_2 - \beta(a_2 - a_1), a_2 + \beta(a_3 - a_2)], \forall \beta \in [0,1]$$

i.e., $x \in [a_2 - \beta(a_2 - a_1), a_2 + \beta(a_3 - a_2)]$

• $\beta$-cut for non-membership function of $\tilde{B}^I$ is

$$[b_2 - \beta(b_2 - b_1), b_2 + \beta(b_3 - b_2)], \forall \beta \in [0,1]$$

i.e., $y \in [b_2 - \beta(b_2 - b_1), b_2 + \beta(b_3 - b_2)]$

To calculate division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ & $\tilde{B}^I$, we now divide the $\beta$ - cuts of $\tilde{A}^I$ & $\tilde{B}^I$ using interval arithmetic

$$\tilde{A}^{\beta} = \left[ a_2 - \beta(a_2 - a_1), a_2 + \beta(a_3 - a_2) \right]$$

$$\tilde{B}^{\beta} = \left[ b_2 - \beta(b_2 - b_1), b_2 + \beta(b_3 - b_2) \right]$$

$$= \left[ a_2 - \beta(a_2 - a_1), a_2 + \beta(a_3 - a_2) \right]$$

To find the non-membership function $\mu_{\tilde{A}^I}(x)$, we equate to $x$ both the first and second component, which gives

$$x = \frac{a_2 - \beta(a_2 - a_1)}{b_2 + \beta(b_3 - b_2)} \text{ and } x = \frac{a_2 + \beta(a_3 - a_2)}{b_2 - \beta(b_2 - b_1)}$$

Now expressing $\beta$ in terms of $x$ and setting $\beta = 0$ and $\beta = 1$, we get
Intuitionistic fuzzy linear programming problem

\[ \theta_{\tilde{A}}(x) = \begin{cases} \frac{a_2 - b_2x}{(a_2 - a'_1) + (b_2 - b'_{2})x}, & \frac{a'_1}{b'_3} \leq x \leq \frac{a_2}{b_2} \\ \frac{b_2x - a_2}{(a_3 - a_2) + (b_2 - b'_1)x}, & \frac{a_2}{b_2} \leq x \leq \frac{a'_3}{b'_1} \end{cases} \]

Hence the division rule is proved for membership and non-membership functions.

Thus

\[ \frac{\tilde{A}^l}{\tilde{B}^l} = \left\{ \left( \frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right), \left( \frac{a'_1}{b'_3}, \frac{a'_2}{b'_2}, \frac{a'_3}{b'_1} \right) \right\} \]

is also a TIFN.

4. Proposed Score function and Accuracy function

Let \( \tilde{A} = \left\{ (a_1, a_2, a_3); (a'_1, a_2, a'_3) \right\} \) be a TIFN, then we define a Score function for membership and non-membership values respectively as

\[ S(\tilde{A}^l) = \frac{a_1 + 2a_2 + a_3}{4} \quad and \quad S(\tilde{A}^l) = \frac{a_1 + 2a_2 + a_3}{4} \]

Let \( \tilde{A}^l = \left\{ (a_1, a_2, a_3); (a'_1, a_2, a'_3) \right\} \) be a TIFN, then we define a Score function of \( \tilde{A}^l \), to defuzzify the given number.

4.1 Ranking using Score function:

Let \( \tilde{A}^l \) and \( \tilde{B}^l \) be any two TIFNs. Then

\[ S(\tilde{A}^l) \leq S(\tilde{B}^l) \]

\[ S(\tilde{A}^l) \geq S(\tilde{B}^l) \]

\[ S(\tilde{A}^l) = S(\tilde{B}^l) \]

4.2 Theorem:

Let \( \tilde{A}^l \) and \( \tilde{B}^l \) be any two TIFNs. Then

\[ S(\tilde{A}^l) \leq S(\tilde{B}^l) \]

\[ S(\tilde{A}^l) \geq S(\tilde{B}^l) \]

we get

\[ S(\tilde{A}^l) + S(\tilde{B}^l) \leq S(\tilde{A}^l) + S(\tilde{B}^l) \]

i.e.,

\[ \frac{S(\tilde{A}^l) + S(\tilde{B}^l)}{2} \leq \frac{S(\tilde{B}^l) + S(\tilde{B}^l)}{2}, \quad \text{i.e.,} \quad H(\tilde{A}^l) \leq H(\tilde{B}^l) \]

Hence the proof.

5. Intuitionistic Fuzzy Linear Programming

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined in [4] as

\[ (IFLP) \max \tilde{Z}^l = \sum_{j=1}^{n} \tilde{c}^l_j \tilde{x}_j^l \quad \text{Subject to} \quad \sum_{j=1}^{n} \tilde{a}^l_{ij} \tilde{x}_j^l \leq \tilde{b}^l_i \\
i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, m, \quad \text{where} \quad \tilde{A}^l = \left( \tilde{a}^l_{ij} \right), \quad \tilde{c}^l, \tilde{b}^l, \tilde{x}^l \quad \text{are} \ (m \times n), \ (1 \times n), \ (m \times 1), \ (n \times 1) \text{ intuitionistic fuzzy matrices consisting of Triangular Intuitionistic} \]
Fuzzy Numbers (TIFN).

5.1 Standard Form[6] The objective function should be of maximization form 

\( IFLP \) \( \max \ Z^l = \sum_{j=1}^{n} c_j^l \bar{x}_j^l \)  \hspace{1cm} \text{(5.1)} \\

Subject to 

\[
\begin{align*}
\alpha_{11}^l \bar{x}_1^l + \alpha_{12}^l \bar{x}_2^l + \cdots + \alpha_{1n}^l \bar{x}_n^l + \bar{x}_{n+1}^l &= \bar{b}_1^l \\
\alpha_{21}^l \bar{x}_1^l + \alpha_{22}^l \bar{x}_2^l + \cdots + \alpha_{2n}^l \bar{x}_n^l + \bar{x}_{n+2}^l &= \bar{b}_2^l \\
\vdots \quad \vdots \\
\alpha_{m1}^l \bar{x}_1^l + \alpha_{m2}^l \bar{x}_2^l + \cdots + \alpha_{mn}^l \bar{x}_n^l + \bar{x}_{n+m}^l &= \bar{b}_m^l 
\end{align*}
\]

\( \bar{x}_1^l, \bar{x}_2^l, \ldots, \bar{x}_n^l, \bar{x}_{n+1}^l, \ldots, \bar{x}_{n+m}^l \geq 0 \)  \hspace{1cm} \text{(5.2)}

5.2 Intuitionistic fuzzy optimum feasible solution[6]

Let \( X \) be the set of all intuitionistic fuzzy feasible solutions of (5.1). An intuitionistic fuzzy feasible solution \( \bar{x}_0^l \in X \) is said to be an intuitionistic fuzzy optimum solution to (5.1), if 

\( \bar{c}_l \bar{x}_0^l \geq \bar{c}_l \bar{x}_l^l \)  \hspace{1cm} \text{for all} \ \bar{x}_l^l \in X, \text{where} \ \bar{c}_l = (\bar{c}_1^l, \bar{c}_2^l, \bar{c}_3^l, \ldots, \bar{c}_n^l), \text{and} \ \bar{c}_l \bar{x}_l^l = \bar{c}_1^l \bar{x}_1^l + \bar{c}_2^l \bar{x}_2^l + \cdots + \bar{c}_n^l \bar{x}_n^l .

6. Numerical Illustration

Solve  \( \max \ \bar{Z}^l = \bar{5}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l \)

Subject to  \( \bar{4}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l \leq \bar{12}_1^l \),  \( \bar{1}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l \leq \bar{6}_1^l \),  \( \bar{x}_1^l, \bar{x}_2^l \geq 0 \)  \hspace{1cm} \text{where}

\( \bar{c}_1^l = \bar{5}_1^l = \{(4,5,6); (4,5,6.1)\} \quad \bar{c}_2^l = \bar{3}_1^l = \{(2,5,3,3,2); (2,3,3,5)\} \)

\( \alpha_{11}^l = \bar{4}_1^l = \{(3,5,4,4,1); (3,4,5)\} \quad \alpha_{12}^l = \bar{3}_1^l = \{(2,5,3,3,5); (2,4,3,3,6)\} \)

\( \alpha_{21}^l = \bar{1}_1^l = \{(0,8,1,2); (0,5,1,2,1)\} \quad \alpha_{22}^l = \bar{3}_1^l = \{(2,8,3,3,2); (2,5,3,3,2)\} \)

\( \bar{b}_1^l = \bar{1}_2^l = \{(11,12,13); (11,12,14)\} \quad \bar{b}_2^l = \bar{6}_1^l = \{(5,5,6,7,5); (5,6,8,1)\} \)

Solution: Rewriting the problem in standard form:

Max  \( \bar{Z}^l = \bar{5}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l \)

Subject to  \( \bar{4}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l = \bar{12}_1^l \),  \( \bar{1}_1^l \bar{x}_1^l + \bar{3}_1^l \bar{x}_2^l = \bar{6}_1^l \),

\( \bar{x}_1^l, \bar{x}_2^l \geq 0 \). Here the co-efficient of \( \bar{S}_1^l, \bar{S}_2^l \) are given by

\( \bar{1}_1^l = \{(1,1,1); (1,1,1)\} \) and \( \bar{0}_1^l = \{(0,0,0); (0,0,0)\} \).

Initial Iteration: Basic variables are \( \bar{s}_1^l = \bar{1}_2^l, \bar{s}_2^l = \bar{6}_1^l \)

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
C_B & BV & \bar{x}_1^l & \bar{x}_2^l & \bar{s}_1^l & \bar{s}_2^l & \bar{b}_1^l & \bar{b}_2^l & \text{ratio} \\
\hline
\bar{0}_1^l & \bar{s}_1^l & \bar{4}_1^l & \bar{3}_1^l & \bar{1}_1^l & \bar{6}_1^l & \bar{1}_2^l & \bar{6}_1^l & \bar{1}_2^l / \bar{4}_1^l = \bar{3}_1^l \\
\hline
\bar{0}_1^l & \bar{s}_2^l & \bar{1}_1^l & \bar{3}_1^l & \bar{0}_1^l & \bar{1}_1^l & \bar{6}_1^l & \bar{6}_1^l / \bar{1}_1^l = \bar{6}_1^l \\
\hline
\bar{z}_j^l & \bar{0}_1^l & \bar{0}_1^l & \bar{0}_1^l & \bar{0}_1^l \\
\hline
\bar{c}_j^l - \bar{z}_j^l & \bar{5}_1^l & \bar{3}_1^l & \bar{0}_1^l & \bar{0}_1^l & - \\
\hline
\end{array}
\]
Since all $c_i - \bar{z}_j \geq 0$, the solution is not optimal. $\bar{x}_i$ is the entering variable, since the most positive value corresponds to the $\bar{x}_i$ column. Then the ratio is calculated. Using the division procedure and Scoring function as defined in the sections(2 & 3) of this paper, we get the following results:

\[
\bar{n}_i = \{(2.68, 3, 3.71); (2.2,3.467)\} = \bar{3}_i
\]

\[
\bar{n}_j = \{(2.75, 6, 9.375); (2.38,6,16.2)\} = \bar{6}_j
\]

i) Score function $S(\bar{3}_i) = 3.0975$ & $S(\bar{3}_j) = 3.2175$

iv) Score function $S(\bar{6}_i) = 6.03125$ & $S(\bar{6}_j) = 7.645$

Since $S(\bar{3}_i) < S(\bar{6}_i)$ & $S(\bar{3}_j) < S(\bar{6}_j)$, we get $\bar{3}_i < \bar{6}_i$.

So $\bar{s}_i$ is the leaving variable.

**First Iteration:** Basic variables are $\bar{x}_i = \bar{3}_i$, $\bar{s}_i = \bar{3}_i$

<table>
<thead>
<tr>
<th>C_B</th>
<th>BV</th>
<th>$\bar{x}_i$</th>
<th>$\bar{x}_2$</th>
<th>$\bar{s}_1$</th>
<th>$\bar{s}_2$</th>
<th>$\bar{b}^l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{5}_i$</td>
<td>$\bar{x}_i$</td>
<td>$\bar{1}_i$</td>
<td>0.75$^l$</td>
<td>0.25$^l$</td>
<td>$\bar{0}_i$</td>
<td>$\bar{3}_i$</td>
</tr>
<tr>
<td>$\bar{0}_i$</td>
<td>$\bar{s}_2$</td>
<td>$\bar{0}_i$</td>
<td>2.25$^l$</td>
<td>$-0.25^l$</td>
<td>$\bar{1}_i$</td>
<td>$\bar{3}_i$</td>
</tr>
<tr>
<td>$\bar{z}_j^l$</td>
<td>$\bar{5}_j$</td>
<td>$\bar{3}_j^l$</td>
<td>1.25$^l$</td>
<td>$\bar{0}_i$</td>
<td>$\bar{1}_i$</td>
<td>$\bar{3}_i$</td>
</tr>
<tr>
<td>$\bar{c}_j^l - \bar{z}_j^l$</td>
<td>$\bar{0}_i$</td>
<td>$-0.75^l$</td>
<td>$-1.25^l$</td>
<td>$\bar{0}_i$</td>
<td>$\bar{0}_i$</td>
<td></td>
</tr>
</tbody>
</table>

Where the Triangular intuitionistic representation for each element based on arithmetic operations is listed below:

$c_i = \bar{5}_i = \{(4,5,6); (4,5,6.1)\}$

$a_i = \{(0.85, 1.1.17); (0.6, 1.167)\} = \bar{1}_i$

$a_i = \{(0.24,0.25,0.29); (0.2,0.25,0.67)\} = \bar{0}_i$

$a_i = \{(0,0,0); (0,0,0)\} = \bar{0}_i$

$a_i = \{(-0.37,0,1.15); (0,0,0)\} = \bar{0}_i$

$a_i = \{(1.8,2.25,2.59); (1.3,2.25,2.74)\} = \bar{2}_i$

$a_i = \{(-0,0,0); (0,0,0)\} = \bar{0}_i$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\bar{x}_i$

$a_i = \{(3.4,5,7.02); (2.4,5,10.18)\} = \bar{5}_i$

$a_i = \{(2.44,3.75,6); (1.84,3.75,7.32)\} = \bar{3}_i$

$a_i = \{(0.96,1.25,1.74); (0.8,1.25,4.08)\} = \bar{1}_i$

$a_i = \{(0,0,0); (0,0,0)\} = \bar{0}_i$

REPRESENTATION OF EACH ELEMENT IN THE ROW $\bar{z}_j$

$a_i = \{(-3.02,0.26); (-6.187,0.3.7)\} = \bar{0}_i$

$a_i = \{(0,0,0); (0,0,0)\} = \bar{0}_i$
\[ \bar{a}_{42}^1 = -\{(-0.76,0.75,0.76);(-1.66,0.75,1.66)\} = -0.75 \]
\[ \bar{a}_{43}^1 = -(0.96,1.25,1.74);(0.8,1.25,4.08)\} = -1.25 \]

**Conclusion of the problem:**
Since all the elements in the row \( \bar{c}_j^1 \) are less than or equal to zero, the solution obtained is optimal i.e., Max \( \bar{z}_1^1 = \bar{T}^1 \) when \( \bar{x}_1^1 = \bar{3}^1, \bar{x}_2^1 = \bar{0}^1, \bar{s}_1^1 = \bar{0}^1, \bar{s}_2^1 = \bar{3}^1 \)

**Conclusion**
Our procedure gives the better solution. The problem is taken from the paper [4] with minor changes. In [4], D.Dubey & A.Mehra approaches by crisp optimization technique and obtained the results. We have defined a more general division of TIFN. Thereafter, a ranking function based on Score function has been proposed keeping central thought that the same has to be used in solving IFLP in which the data parameters are TIFNs. The solution methodology is illustrated through an example. In future it is proposed to obtain a better method to defuzzify TIFNs.

**References**

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