Numerical Simulation Chaotic Synchronization
of Chua Circuit and Its Application
for Secure Communication

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Abstract. The Chua circuit is among the simplest non-linear circuits that show most complex dynamical behavior, including chaos which exhibits a variety of bifurcation phenomena and attractors. In this paper, Chua attractor’s chaotic oscillator, synchronization and masking communication circuits were designed and simulated. The Chua system is addressed suitable for chaotic synchronization circuits and chaotic masking communication circuits using Matlab® and MultiSIM® software. Simulation results are used to visualize and illustrate the effectiveness of Chua chaotic system in synchronization and application of secure communication.

Keywords: Chaotic synchronization, Chua circuit, bifurcation, double scroll attractor, secure communication.

1. Introduction

In recent years chaos theory has attracted much interest in both the academic area and engineering study. One of the great achievements of the chaos theory is the application in secure communications. Chaotic signals depend very sensitively on
initial conditions, have unpredictable features and noise like wideband spread spectrum. So, it can be used in various communication applications because of their features of masking and immunizing information against noise. The chaos communication fundament is the synchronization of two chaotic systems under suitable conditions if one of the systems is driven by the other. Since Pecora and Carrol [1],[2] have demonstrated that chaotic systems can be synchronized, the research in synchronization of couple chaotic circuits is carried out intensively and some interesting applications such as communications with chaos have come out of that research.

In this work we use a simple electronic system to develop a scheme for chaos secure communication with two coupled Chua circuits. First, we analyze separately each oscillator to study their dynamic behavior when a parameter of control is changed, and then we investigate the synchronization effect in the coupled circuits. Bifurcations of the output voltage are constructed using a resistance as a control parameter. While using two channels, we may send an information signal via one of the channels and recover the signal via another channel. We will show that this scheme can improve synchronization in a system with coexisting attractors. Finally secure communications with chaos is demonstrated experimentally using the novel communication scheme.

2. Chaotic Dynamics of Chua Circuit

Chua's circuit [3-8], shown in Fig. 1, is a simple oscillator circuit which exhibits a variety of bifurcations and chaos. The circuit contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor, and a single nonlinear resistor NR. Applying Kirchoff’s law, the Chua's circuit is described by three differential equations:
Numerical simulation chaotic synchronization

\[ C_1 \frac{dv_{c1}}{dt} = G(v_{c2} - v_{c1}) - g(v_{c1}) \]
\[ C_2 \frac{dv_{c2}}{dt} = G(v_{c1} - v_{c2}) + i_L \]
\[ L \frac{di_L}{dt} = -v_{c2} \]

(1)

where
- \( v_{c1} \) = the voltage over the capacitor \( C_1 \),
- \( v_{c2} \) = the voltage over the capacitor \( C_2 \),
- \( i_L \) = the current through the inductance,
- \( C \) = capacitance the capacitor,
- \( L \) = inductance the inductor, and
- \( G \) = conductance the linear resistor.

Fig. 2. Chua’s nonlinear resistor function

\[ g(v_{c_i}) = g(v_x) = m_0 v_x + \frac{1}{2} (m_1 - m_0) \| v_x + B_x \| - \| v_x - B_x \| \]

(2)

where \( m_1 \) and \( m_0 \) are the slopes in the inner and outer regions, respectively, and \( \pm B_x \) denote the breakpoints. The resistor \( R \) is a potentiometer and is used to tune the oscillator over a range of bifurcation values.

The values of the two capacitors, inductor and resistor are chosen from the computer simulations confirmed by Matsumoto [3]. \( E_{sat} \) is the saturation voltage of the operational amplifier. It is determined by the power supplies and the internal structure of the op amps. The nonlinear resistor consists of two negative resistors connected in parallel. We choose \( R_1 \) large enough so that the op amp is not loaded.
say around 220Ω. $R_2$ is chosen to be equal to $R_1$ so as to make the analysis simple. The break points (boundary points for the attractor) are calculated such that, the attractor (the state in which the system settles) remains in the negative resistance region (the region in which the current is inversely proportional to the voltage) so that the attractor is bounded see Fig. 3. The detail design of the nonlinear resistor is given by Kennedy [4].

The constant $m_0$, $m_1$, and $B_p$ can be easily computed [4].

$$m_0 = \frac{R_5}{R_1R_6}, m_1 = \frac{R_5 + 1}{R_1R_6},$$

$$B_p = \frac{R_5}{R_1 + R_6} E_{sat},$$

We present numerical simulation to illustrate the dynamical behavior of Chua’s circuit from system (1). For numerical simulation of chaotic systems defined by a set of differential equations such as Chua’s circuit, different integration techniques can be used in simulation tools. In the MATLAB numerical simulations, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used.

According to these numerical simulations, the circuit’s chaotic dynamics and double-scroll attractors are shown in Fig. 3. For showing the dynamics of the system (1) change, the parameter set given as fixed parameters, see Table 1 and $R$ as varied parameters.

### TABLE I. Chua’s Circuit Parameters

<table>
<thead>
<tr>
<th>Element</th>
<th>Description</th>
<th>Value</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>Resistor</td>
<td>220 Ω</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Resistor</td>
<td>220 Ω</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R_3$</td>
<td>Resistor</td>
<td>2.2kΩ</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R_4$</td>
<td>Resistor</td>
<td>22kΩ</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R_5$</td>
<td>Resistor</td>
<td>22kΩ</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R_6$</td>
<td>Resistor</td>
<td>3.3kΩ</td>
<td>± 5%</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Capacitor</td>
<td>10nF</td>
<td>± 5%</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Capacitor</td>
<td>100nF</td>
<td>± 5%</td>
</tr>
<tr>
<td>$L$</td>
<td>Inductor</td>
<td>18mH</td>
<td>± 10%</td>
</tr>
<tr>
<td>$E_{sat}$</td>
<td>Power Supply Op amp</td>
<td>9 V</td>
<td>± 5%</td>
</tr>
<tr>
<td>$R$</td>
<td>Potentiometer</td>
<td>varied</td>
<td>± 5%</td>
</tr>
<tr>
<td>$UA$</td>
<td>TL082CD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$UB$</td>
<td>TL082CD</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
By reducing the variable resistor $R$ in Fig. 1 from 2 kΩ towards zero, Chua's circuit exhibits a sequence of bifurcations from limit cycle equilibrium through a Hopf bifurcation and period-doubling sequence to a Rossler-type attractor, from a Rossler-type attractor changes to the Double Scroll strange attractor, and from the Double Scroll strange attractor changes to large limit cycle as illustrated in Fig. 3.

Notice that varying $R$ in this way causes the size of the attractors to change: the period-one orbit is large, period-two is smaller, the Rossler-type attractor is smaller again, and the Double Scroll shrinks considerably before it dies.
3. Synchronization of The Chua Circuit

Synchronization between chaotic systems has received considerable attention and led to communication applications. With coupling and synchronizing identical chaotic systems methods, a message signal sent by a transmitter system can be reproduced at a receiver under the influence of a single chaotic signal through synchronization. This paper presents the study of numerical simulation of chaos synchronization for chaotic Chua attractor. Drive system and response system were constructed.

Synchronization of chaotic motions among coupled dynamical systems is an important generalization from the phenomenon of the synchronization of linear system, which is useful and indispensable in communications. The idea of the methods is to reproduce all the signals at the receiver under the influence of a single chaotic signal from the driver. Therefore, chaos synchronization provides potential applications to communications and signal processing [9]-[14]. However, to build secure communications system, some other important factors, need to be considered. Simulations of synchronization of Chua system are presented as shown in Fig. 4.

The control values \( R \) of the two systems are different, the control value \( R \) of the drive system is 1.85 kohm, and the control value \( R \) of the response system is 1.75 kohm. Simulation results show that the two systems synchronize well. Fig. 4 shows the circuit schematic for implementing the Synchronization of Chua system. We use TL082CD op-amps, appropriate valued resistors, inductor and capacitors for MultiSIM® simulations. Fig. 4 also shows MultiSIM® simulation results of this circuit.
Fig. 4. MultiSIM® Circuit and simulations: (a) Synchronization of Chua’s attractor Circuit; (b) Drive and response system chaotic signals before synchronization; (c) The phase portrait of unsynchronized case; (d) Drive and response system chaotic signals after synchronization; (e) X-Xr Synchronization.

4. Application for Secure Communication Systems

Due to the fact that output signal can recover input signal, it indicates that it is possible to implement secure communication for a chaotic system. The presence of
the chaotic signal between the transmitter and receiver has proposed the use of chaos in secure communication systems. The design of these systems depends as we explained earlier on the self synchronization property of the Chua attractor. Transmitter and receiver systems are identical except for their control value $R$, in which the transmitter system is 1.85 kohm and the receiver system is 1.75 kohm as shown in Fig. 4.

It is necessary to make sure the parameters of transmitter and receiver are identical for implementing the chaotic masking communication [9]-[14]. In this masking scheme, a low-level message signal is added to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. Thus, the message has been perfectly recovered by using the signal masking approach through synchronization in the Chua attractor. Computer simulation results have shown that the performance of Chua attractor in chaotic masking and message recovery. The square wave signal is added to the generated chaotic $x$ signal, and the $S(t) = x + i(t)$ is feed into the receiver. The chaotic $x$ signal is regenerated allowing a single subtraction to retrieve the transmitted signal, $(x+i(t)) - x_r = i'(t)$, If $x = x_r$. Fig. 5 shows the circuit schematic for implementing the Chua attractor’s Chaotic Masking Communication. Fig. 6 shows MultiSIM® simulation results of this Chaotic Masking Circuit.

![Fig. 5. Chua attractor chaotic masking communication circuit.](image-url)
5. Conclusion

This paper focuses on the chaotic oscillator circuit and the identical synchronization of the Chua’s attractor and its applications in signal masking communications. In this paper, Chua’s chaotic circuit system is studied in detail by varying mostly the control parameter $R$. The system has rich chaotic dynamics behaviors. We have demonstrated in simulations that chaos can be synchronized and applied to secure communications. We suggest that this phenomenon of chaos synchronism may serve as the basis for little known Chua attractor to achieve secure communication. Chaos synchronization and chaos masking were realized using MultiSIM® programs.

References


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