Design and Analysis Bidirectional Chaotic Synchronization of Rossler Circuit and Its Application for Secure Communication

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Abstract

Synchronization is considered as the complete coincidence of the states of individual systems. Such a regime can result from an interaction between systems or subsystems, as well as from the influence of external noisy or regular fields. In this paper, we have performed the design and numerical simulation of the synchronization between two identical coupled Rossler circuits and applied to a security system of communication. We have demonstrated in simulations that chaotic systems can be synchronized and this technique can be applied to signal masking communications by using MATLAB and MultiSIM programs. All simulations results performed on Rossler system, verify the applicable of secure communication.

Keywords: Chaos, chaotic synchronization, Rossler circuit, secure communication.

1 Introduction

Chaotic behaviour has been found in many typical iterated maps such as the logistic map, Gaussian map, Hénon map etc. In various physical systems, including the unidirectional Chaotic Synchronization of Non-Autonomous Circuit [5], Nonlinear Dynamics of Chaotic Attractor of Chua Circuit [4] and Synchronization FitzHugh-Nagumo Neuronal System [6].
One of the pioneers of synchronization is probably the Dutch scientist Christiaan Huygens. In the 17th century he described an observation of two pendulum clocks, both attached to the same beam that was supported by two chairs, that always end up swinging in opposite direction independent of their starting positions. Even when he applied a disturbance the two clocks showed anti-phase synchronized motion within half an hour [10, 2, 8].

Synchronization of chaotic oscillators in particular became popular when Pecora and Carroll published their observations of synchronization in unidirectionally coupled chaotic systems [8]. Their results were remarkable since chaos can be seen as a form of instability while synchronization implies stability of the error dynamics [10]. The research in synchronization of couple chaotic circuits is carried out intensively and some interesting applications such as communications with chaos have come out of that research [9].

In this paper, a simple electronic system of two coupled circuits in the development scheme of chaos-based secure communication system has been used. First, we analyzed and simulated a mathematical model of Rossler circuit in the form of a system ordinary differential equations (ODEs). Furthermore, the bidirectional coupling method is applied to synchronize Rossler circuit. Finally, chaotic masking communication circuits and their simulations of the Rossler circuit are realized also MATLAB and MultiSIM.

2 Mathematical Model of Chaotic Rossler Circuit

One of the most well-known autonomous nonlinear systems [1, 3, 7, 11]. The one nonlinearity in the circuit is a piecewise linear function made from op amp U4A with diode, 3 resistors and a diode. The Rossler electronic circuits are describe by the following equations [1]:

\[
\begin{align*}
\frac{dx}{dt} &= -\alpha (\Gamma x + \beta y + \lambda z) \\
\frac{dy}{dt} &= -\alpha (-x - \gamma y + 0.02 z) \\
\frac{dz}{dt} &= -\alpha (-g(x) + z)
\end{align*}
\]

The piecewise linear function \( g(x) \) is defined by:

\[
g(x) = \begin{cases} 
0 & x \leq 3 \\
\mu(x - 3) & x > 3
\end{cases}
\]

Where time factor \( \alpha \) is \( 10^4 \text{ s}^{-1} \), \( \Gamma \) is 0.05, \( \beta \) is 0.5, \( \lambda \) is 1, \( \mu \) is 15 and the circuit contains a variable resistor that can be used to change the value of \( \gamma \). The relation between the value \( R_0 \) of the variable resistor and \( \gamma \) is \( R/R_0 \), with
$R=10k\Omega$.

$R_0=R_1$ is a control parameter which exhibit bifurcation and chaotic dynamics.

The complete implementation of the Rossler chaotic circuit design using MultiSIM software is shown in Figure 3. The function of nonlinear resistor as see in Figure 3 are implemented with the analog operational amplifier. By comparing Figure 1 and Figure 2 a good qualitative agreement between the numerical integration of (1) and (2) by using MATLAB, and the circuits simulation by using MultiSIM, can be concluded.

![Phase Space Rossler Attractor](image1.png) ![Time series circuit Rossler](image2.png)

(a) Phase Portrait of $y$ versus $x$ (b) Time Series of signal $y$

Figure 1: Numerical simulation results for $R_0=60\ k\Omega$, with MATLAB.

![Phase Space Rossler Attractor](image3.png) ![Time series circuit Rossler](image4.png)

(a) Phase Portrait of $y$ versus $x$ (b) Time Series of signal $y$

Figure 2: Numerical simulation results for $R_0=60\ k\Omega$, with MultiSIM.
3 Bidirectional Chaotic Synchronization and Circuit’s Mathematics Synchronization

There are two main forms of coupling. In the case of unidirectional or Master-Slave scheme, the master is the guide or reference system and the slave is driven system which is dependent on the master. In the case of bidirectional coupling two systems interact and are coupled with each other creating a mutual synchronization. The following master-slave (bidirectional coupling) configuration, as described below:

\[
\begin{align*}
\frac{dx}{dt} &= -\alpha (\Gamma x_1 + \beta y_1 + \lambda z_1) + g(x_2 - x_1) \\
\frac{dy}{dt} &= -\alpha (-x_1 - \gamma y_1 + 0.02 z_1) \\
\frac{dz}{dt} &= -\alpha (-g(x_1) + z_1)
\end{align*}
\]

\[g(x_i) = \begin{cases} 
0 & x_i \leq 3 \\
\mu(x_i - 3) & x_i > 3 
\end{cases}\]
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\[
\begin{align*}
\text{Slave} \\
\frac{dx_2}{dt} &= -\alpha(\Gamma x_2 + \beta y_2 + \lambda z_2) + g_e(x_1 - x_2) \\
\frac{dy_2}{dt} &= -\alpha(-x_2 - \gamma y_2 + 0.02 z_2) \\
\frac{dz_2}{dt} &= -\alpha(-g(x_2) + z_2) \\
g(x_2) &= \begin{cases} 
0 & x_2 \leq 3 \\
\mu(x_2 - 3) & x_2 > 3
\end{cases}
\end{align*}
\]

where \( g_c = 1/R_c \), \( C \) is the coupling strength, \( R_c = R_f \) is the variable resistor and \( C \) is Capacitance in \( x \) signal (Figure 5). The asymptotic synchronized situation is defined as:

\[
\lim_{t \to \infty} |x_1(t) - x_2(t)| = 0
\]

The dynamic error system is defined as follows:

\[
\begin{align*}
e_x &= x_1 - x_2 \\
e_y &= y_1 - y_2 \\
e_z &= z_1 - z_2
\end{align*}
\]

The time derivative of this error signal is:

\[
\begin{align*}
\dot{e}_x &= \dot{x}_1 - \dot{x}_2 \\
\dot{e}_y &= \dot{y}_1 - \dot{y}_2 \\
\dot{e}_z &= \dot{z}_1 - \dot{z}_2
\end{align*}
\]

By substituting (3) and (4) into (7), we have the following error dynamics:

\[
\begin{align*}
\dot{e}_x &= -\alpha(\Gamma x_1 + \beta y_1 + \lambda z_1) + g_e(x_2 - x_1) - (-\alpha(\Gamma x_2 + \beta y_2 + \lambda z_2)) + g_c(x_1 - x_2) \\
&= -\alpha \Gamma (x_1 - x_2) - \alpha \beta (y_1 - y_2) - \alpha \lambda (z_1 - z_2) + 2 g_e(x_1 - x_2)
\end{align*}
\]

\[
\begin{align*}
\dot{e}_y &= -\alpha(-x_1 - \gamma y_1 + 0.02 z_1) - (-\alpha(-x_2 - \gamma y_2 + 0.02 z_2)) \\
&= \alpha x_1 + \alpha \gamma y_1 - \alpha 0.02 z_1 - \alpha x_2 - \alpha \gamma y_2 + \alpha 0.02 z_2
\end{align*}
\]

\[
\begin{align*}
\dot{e}_z &= -\alpha(-g(x_1) + z_1) - (-\alpha(-g(x_2) + z_2)) \\
&= \alpha g(x_1) - \alpha z_1 - \alpha g(x_2) + \alpha z_2)
\end{align*}
\]

where \( e_g = g(x_1) - g(x_2) \)

With the objective to demonstrate synchronization, we analyze the stability for the dynamic error system. Thus, we propose the following candidate function to Lyapunov function:

\[
V = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2)
\]
Derive equation (9), is obtained:
\[ V = e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z \]  
\[ \dot{V} = e^T \begin{bmatrix} \alpha \lambda - 2g_c & \alpha \beta & \alpha \lambda \\ -\alpha & -\alpha \gamma & 0.02 \alpha \\ -\alpha \mu & 0 & \alpha \end{bmatrix} e \]
\[ \dot{V} = -e^T A e < 0 \]

Which is a negative definite function. It means that the dynamic error system (8) is asymptotically stable and, therefore, each one of synchronization errors, \( e_x \), \( e_y \) and \( e_z \), tends to zero as \( t \) tends to infinite. If synchronization errors tends to zero, then the states from slave system tend to those from master system, which means that they synchronize.

### 4 Numerical Simulations

#### Simulation in Matlab

Numerical simulations are used to describe the dynamics of the phenomenon of bidirectional synchronization circuit Rossler equation (3-4) with fourth-order Runge-Kutta method. In bidirectional (mutual) coupling, both drive and response subsystems are connected in such a way that they mutually influence each other’s behavior. First synchronization between identical systems is considered. We consider coupling through \( g_c = 1/R_c \cdot C \) It can be seen in Figure 4. That synchronization occurs if \( R_c \) does not exceed 1Ω.

(a) \( R_c = 10 \Omega \)
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Figure 4: Bidirectional chaotic synchronization phase portrait and error $x_1$-$x_2$ numerical results

Synchronization numerically appears for a coupling strength $R_c \leq 1\Omega$ as shown in Figure 4(b). For different initial condition, if the resistance coupling strength $R_c \gg 1\Omega$, the synchronization cannot occur as shown in Figure 4 (a), the synchronization occurs when $R_c \leq 1\Omega$ with errors $e_s = x_1 - x_2 \to 0$ implies the complete synchronization for this resistance coupling strength as shown in Figure 4 (b).

Analog Circuit Simulation in MultiSIM

Simulation results show that the two systems synchronize well. Figure 5 shows the circuit schematic for implementing the unidirectional synchronization of coupled Rossler systems. We use 741 op-amps, appropriate valued resistors, one diode and capacitors for MultiSIM simulations. Figure 6 also shows MultiSIM simulation results of this circuit.
Synchronization numerically appears for a coupling strength $R_c \leq 1\Omega$ as shown in Figure 6 (d). For different initial condition, if the resistance coupling strength $R_c \gg 1\Omega$, the synchronization cannot occur as shown in Figure 6 (a)-(c), the synchronization occurs when $R_c \leq 1\Omega$ with errors $e_i = x_1 - x_2 \rightarrow 0$ implies the complete synchronization for this resistance coupling strength as shown in Figure 6 (d).
5 Applications to Secure Communication Systems

In chaos-based secure communication schemes, information signals are masked or modulated (encrypted) by chaotic signals at the transmitter and the resulting encrypted signals are sent to the corresponding receiver across a public channel (unsafe channel). Perfect chaos synchronization is usually expected to recover the original information signals. In other words, the recovery of the information signals requires the receiver’s own copy of the chaotic signals which are synchronized with the transmitter ones. Thus, chaos synchronization is the key technique throughout this whole process [12].

The sinusoidal wave signals of amplitude 1 V and frequency 2 kHz is added to the generated chaotic $x$ signal and the $S(t) = x + i(t)$ is fed into the receiver. The chaotic $x$ signal is regenerated allowing a single subtraction to retrieve the transmitted signal, $[x + i(t)] - xr = i'(t)$, if $x = xr$. Figure 8 shows the circuit schematic for implementing the Rossler circuit Chaotic Masking Communication.

MultiSIM simulation results for several different frequencies are shown in Figure 7. Figure 7 shows the MultiSIM simulation results for masking signal communication system by varying the input signal’s frequency. The red signal describes the wave information signal $i(t)$, the green signal describes the transmitted chaotic masking signal $S(t)$ and the purple signal describes the retrieved signal $i'(t)$.

The simulation results shows that circuit autonomous Rossler is an excellent for chaotic masking communication when the frequency information is at intervals of 0.2 kHz – 9kHz. Otherwise, when the frequency information is more than 9 kHz or less than 0.2 kHz, the chaotic masking communication is not occur.

![Figure 7: MultiSIM outputs of Rossler circuit masking communication systems. (a). Information frequency 4 kHz (b). Information frequency 0.1 kHz (c). Information frequency 10 kHz](image)

6 Conclusion

We propose a communication scheme for secure communications based on synchronization of chaotic systems. The scheme implies the use of two system variables, the one serves for chaos synchronizations and the other is used for signal transmission and recovering. We show that the synchronization error for the novel scheme is smaller when $R_c \leq 1 \, \Omega$ and complete synchronization occurs.

In this paper, it has been shown that Rossler circuit can be used in a communication security system at a frequency information interval of 0.2 kHz - 9 kHz. When the frequency information is more than 9 kHz or less than 0.2 kHz. The chaotic masking communication is not occur.
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References


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