Managers Compensation and Collusive Behaviour under Cournot Oligopoly

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Abstract

This paper shows that the existence of a concrete outside option for firms' high-skilled workers (for instance, the managers) may induce, under specific circumstances, oligopolistic firms to adopt restrictive output practises. In particular, the paper characterizes the conditions under which, in a Cournot oligopoly, existing firms behave more collusively than in a standard Cournot model without endogenous wages.

Keywords: Managers Compensation, Oligopoly, Collusion.

1 Introduction

In an economy where either the barriers to entry are not a serious obstacle or in which there is a way in which inactive workforce can become active - through the constitution of self-employed units, partnerships or entrepreneurial firms alike - every worker's outside option cannot be treated as exogenous. It should, indeed, be considered as part of the model variables, i.e., in economic words, be viewed as endogenous to the model itself. On the other hand, although there exists a well known stream of economic literature (See, amongst others, [1], [3], [4]). claiming that, under oligopoly, the delegation of sales decisions to managers can both be profitable for the firms' owners as well as boost every firm's output with respect to a standard Cournot model, an increase in managers' compensation due to the managers' leaving threat can also be thought to yield opposite results. Whilst in principle the incentive package for managers is responsibility of the owners, most frequently pay is set by executive compensation committees consisting of non-executive directors or by senior executives themselves. Since managers' compensation is a cost component for the company, it may well be argued that the “fat cats” phenomenon within companies reduces competition and output in a given market, when compared to a standard oligopoly model.
The model presented in this paper explores this possibility. It describes a noncooperative game among every company's owner and each hired manager. The latter has to decide whether to accept the offered compensation or just leave and setup a new business. As a result, the model proves that, under rather general circumstances, there may be an output restriction with respect to a standard Cournot model. Moreover, equilibrium executives' compensation turns out to be negatively related to the initial number of firms existing in the market. Although the relationship between senior executives' pay and company performance has become a source of controversy in recent years, the direct link found in our model between company's sales and managers' compensation is, usually, empirically confirmed ([2], [5]).

The paper is organized as follows. The next section briefly presents the game-theoretic structure of the model. Section 3 introduces a simple model specification to show the main paper findings. Section 4 concludes the paper.

2. The Model

The model describes an oligopolistic industry in which, at the beginning, only \( n \) agents (indicated as business initiators) possess the knowledge to produce a commodity with a given technology. To be effective, the technology requires both skilled and unskilled workers. Such an exclusive knowledge represents the only barrier to entry for other potential competitors (for instance the skilled workers, henceforth labelled as managers) assumed to need a specific on-the-job training to start a new business.

Thus, in the industry, the \( n \) initiators are assumed to set up \( n \) (identical) firms behaving a la Cournot and producing a homogenous commodity \( y \). The sequence of strategies described in the model is quite simple. Firstly, every \( n \)-th company decides how much commodity to produce (and thus, how many identical managers to hire), according to the usual profit maximization procedure. Secondly, the company has to fix each manager's remuneration, indicated as \( v \). Hence, a manager recruited by the firm can either decide to stay, accepting \( v \), or leave, to set up a competing company in the industry, thus earning a profit of at most \( \pi(n + 1) \). Every manager that has set up a firm continues the game exactly as before (i.e., first deciding \( y \) and then \( v \) for her or his recruited managers) and the game goes on in this way, with a potentially infinite number of stages. The solution concept used to solve the game is the standard subgame perfect Nash equilibrium. The definition that follows describes in detail an equilibrium of the game.

**Definition** An equilibrium of the game is a vector of quantities,

\[
\left( y_1^*, v^*(n + k), n + k, y_2^*(n + k), n + k, ..., y_{n+k}^*(n + k), n + k \right),
\]

where \( v^* \) represent every manager's equilibrium compensation and \( k \) the number of new firms entered the market in equilibrium, such that, for every \( i \)-th firm \( (i = 1, 2, ..., n + k) \), whatever the number of entrants \( (k = 0, 1, ..., \infty) \), it must be that:

\[
\pi \left[ y_i^*(v(n + k)), y_{-i}^*(v(n + k)) \right] \geq \pi \left[ y_i(v(n + k)), y_{-i}^*(v(n + k)) \right]
\]

while, for each manager recruited by a firm,

\[
v^*(n + k) \geq \pi \left[ y_i(v(n + k + 1)), y_{-i}^*(v(n + k + 1)) \right]
\]

where in all expressions above, \( y_{-i} \) indicates the vector of quantity selected by all firms different from
Definitions 1 imposes three conditions on an equilibrium of the game. Firstly, that no firm must find profitable to change its selected quantity; secondly, that, given the quantity chosen, no firm must have an incentive to change managers' equilibrium compensation; thirdly, that every hired manager must prefers to stay within the firm rather than setup a new business, otherwise the game would continue and an equilibrium would not be reached. It has to be noticed that the solution concept adopted here focuses on individual players' behaviour and excludes collective deviations from equilibrium.

The next section applies the equilibrium definition to a simple model specification, in order to show its main results.

3 A Simple Example

Let us assume that in a certain industry $n$ agents, initially disposing of the knowledge on how to produce a homogenous commodity $y$, decide to set up $n$ (identical) firms behaving à la Cournot. Let also the technology available to them be described by the following Cobb-Douglas constant returns to scale production function:

$$ y_i = m^\theta \cdot \ell^{(1-\theta)} $$

(1)

where respectively $m$ is the number of managers (or highly skilled workers) recruited by every $i$-th firm $(i = 1, ..., n)$ while $\ell$ is the number of unskilled workers. Let us assume, for simplicity, that $\theta = 1/2$. Let also every firm's fixed cost be equal to zero. Without loss of generality, the wage paid to unskilled workers is normalized to one, while $v$ denotes each manager's compensation. Moreover, let the market demand be linear and equal to:

$$ p(Y) = a - Y $$

(2)

Deriving by (1) every firm's cost function as:

$$ C_i(y_i) = 2\sqrt{v} \cdot y_i $$

(3)

it is straightforward to get every initial $i$-th firm's Cournot equilibrium quantity (for any arbitrary manager compensation) as:

$$ y_i^*(n) = \frac{a - 2\sqrt{v}}{n + 1} $$

(4)

and every $i$-th firm's equilibrium profit as:

$$ \pi_i^*(n) = \frac{(a - 2\sqrt{v})^2}{(n + 1)^2} $$

(5)

As explained above, the basic feature of the model is that, when managers are hired by a firm, they immediately acquire the specific knowledge to become potential competitors of the existing firms.
Hence, managers' compensation must be optimally decided by a firm knowing each manager's potential threat of leaving to set up, through the use of unskilled workers and other managers, a new production unit. When a manager leaves the firm, he or she will presumably set up a company of the same type as the one she or he is working for. Thus, if in the previous stages of the game $k$ firms have entered the market, from (5), the payoff of a leaving manager is at most equal to:

$$\pi_t(n + k + 1) = \left\{ \begin{array}{ll} (a - 2\sqrt{n + k + 1})^2 & (n + k + 2)^2 \\ 0 & \end{array} \right.$$  

(6)

where $v(n + k + 1)$ represents the wage that the leaving manager will pay to her or his managers. Expression (6) it is built under the presumption that every existing firm whose manager(s) has decide to leave, can easily find a substitute.

Now, we can apply definition 1 to find managers' equilibrium compensation and, hence, an equilibrium of the game. Firstly, we look for the level of compensation the firms have to pay to make a manager indifferent whether to stay in the firm as employee, earning that tends to infinity, given that $v(n + k) = 0$. Moreover, denoting as $d$ by the firms of the industry and, as a consequence, the corresponding managers' reservation wage equal to zero. Hence, managers' compensation must be optimally decided by a firm knowing each manager's potential to leave. This can be done by solving expression (6). In this way, at any stage of the game a firm knows that, if selected manager's compensation $v(n + k) \geq \pi_t(n + k + 1)$, the manager will stay, while, otherwise, she or he will leave. Secondly, it has to be found the number of firms $k$ that enter at the equilibrium, so to fully characterize the final compensation $v^*(n + k)$ and thus $v^*(v^*(n + k))$ for every firm active in the market. By using a standard backward induction procedure, we notice that under the specification of the model, two different cases arise. They are both illustrated below.

3.1 Infinite Number of Stages

This describes the case in which there exists a virtual unlimited availability of managers ready to be hired by the firms of the industry and, as a consequence, the corresponding managers' reservation wage is equal to zero. Moreover, denoting as $t$ the number of new entrants that makes every firm's profit (approximately) equal to zero, under the model specification, $\pi_t(n + t)$ equal to zero happens for $t$ that tends to infinity, given that $t$ must solve:

$$\pi_t(n + t) = \left\{ \begin{array}{ll} (a - 2\sqrt{n + t})^2 & (n + t + 1)^2 \\ 0 & \end{array} \right.$$  

(7)

provided that, when $\pi_t(n + t) = 0$, also $v(n + t) = \pi_t(n + t + 1) = 0$. Thus, when $(t - 1)$ firms have already entered the market, every manager will be indifferent whether to stay in the firm as employee, earning the corresponding wage $v(n + t - 1) = \pi_t(n + t) = 0$ or just leave. Firms at the previous stage, knowing this, need to decide whether to pay $v^*(n + t - 2) = \pi_t(v^*(n + t - 1))$ to their managers (where $v^*$ indicates, at every stage, the wage that respects (9)) or pay less, letting at least one manager leave to get a profit of $\pi_t(v^*(n + t - 1))$. Thus, since at the last stage a manager will necessarily be paid $v^*(n + t - 1) = \pi_t(n + t) = 0$, one stage before firms will find convenient to pay $v^*(n + t - 2)$ only if:

$$\pi_t(v^*(n + t - 2)) \geq \pi_t(v^*(n + t - 1))$$

The lemma presented below proves that, if every firm's profit is weakly positive and the next stage firm's optimal strategy is which to fix a compensation that respect (6), it will be always convenient for
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a firm at the previous stage to do the same, paying every manager just enough to keep her or him within the firm.

**Lemma 1** Under the model specification, if \( p(Y(n + k)) \geq AC_i(Y(n + k)) \) for any possible \( k = 1, 2, \ldots, n \), the following inequality holds for every firm \( i = 1, \ldots, n + k \):
\[
\pi_i^v(Y(n + k)) \geq \pi_i^v(Y(n + k + 1))
\]

**Proof.** (See Appendix).

Since Lemma 1 holds at any stage of the game, every firm's optimal strategy will be which of paying the minimal remuneration sufficient to induce its managers to stay within the firm, i.e., a wage that respects (6). By backward induction, at the first stage of the game, when the \( n \) initiators decide whether to let their managers stay (paying them accordingly) or leave (thus increasing the existing number of firms), they will certainly set \( v = v^*(n) \) and the equilibrium number of entrants will be \( k = 0 \). The equilibrium remuneration \( v^*(n) \) is thus obtainable by solving the following equation:

\[
v(n) = \pi_i^v(n + 1) = \frac{(a - 2\sqrt{v(n + 1)})^2}{(n + 2)^2}
\]

Expression (8) is a non linear difference equation that can be solved by iteration on the potential number of entrants \( k \). A straightforward substitution process yields:

\[
v(n) = \frac{(a(n + 3)(n + 4) - 2a(n + 4) - (n + k) + 4a(n + 5) - (n + k) - 8a^2 + (-1)^{k + 1}\sqrt{v(n + k - 1)})}{(n + 2)^2(n + 3) - (n + k)^2}
\]

Now, from (8) we know that, for \( k = t \rightarrow \infty \), \( v^*(n + t - 1) = \pi_i^v(n + t) = 0 \). Hence, putting \( v^*(n + t - 1) = 0 \) into expression (9) and rearranging, we obtain:

\[
v(n) = \lim_{t \rightarrow \infty} \left(\frac{\sum_{i=0}^{t-3}(-1)^i(2i^2)}{\prod_{i=0}^{t-3}(n + 3 + j) + (-1)^{t-2}(2^t - 2)}\right)^2.
\]

Interesting properties of expression (10) are that it is unique for every set of parameters values and that takes finite values, even for a finite and very low number of stage \( t \). Moreover, the value of \( v^*(n) \) is monotonically decreasing in the number \( n \) of initiators assumed to exists at the beginning of the game. Now, by inserting expression (8) into the equilibrium output, it ensues that, according to Definition 1, the unique equilibrium of the game is \( \{v^1, v^2, \ldots, v^n\} \).

In figure below, \( v(n) \) is plotted against different number of business initiators and compared to a given market clearing wage. Managers' compensation is decreasing with the number of firms assumed to exist in the market. Notice also that, since for a range of \( n \) equilibrium managers' compensation is higher than the market clearing level, there will always be, within this range, managers available to be hired by the firms. This endogenous availability of managers allows for their substitution when they decide to leave the firm and gives consistency to the game described above.
Proposition 1 Under the model assumptions, when the number of initiators is no greater than \( n \) the equilibrium number of managers \( m^* \) selected by every firm is less than it would be under standard neoclassical assumptions in the market for managers.

Proof. Since from (1) an efficient choice of managers implies \( m^* = \frac{y_i^*}{\sqrt{v_i^*}} \), and, from (4) Cournot equilibrium output \( y_i^* \) is monotonically decreasing in \( v_i \), it ensues that \( m^*(v_i(n)) < m^*(\bar{v}) \) whenever \( v_i'(n) > \bar{v} \), where \( \bar{v} \) indicates the neoclassical market clearing wage. Since \( v_i'(n) = \pi_i(v_i'(n + 1)) \) and, by lemma 2, the second member is monotonically decreasing in \( n \), there will certainly be a value of \( n = \bar{n} \) for which \( v_i^*(n) = \bar{v} \). Thus, for \( n < \bar{n} \), \( m_i^*(v_i^*(n)) < m^*(\bar{v}) \) and the result follows.

It can be noticed that equilibrium quantity \( y_i^*(v_i^*(n), n) \) is, for every firm, dependent on equilibrium managers' compensation. Thus, since \( v_i^*(n) \) is higher than \( \bar{v} \) for \( n < \bar{n} \), it turns out that, when \( n < \bar{n} \), \( y_i^*(v_i^*(n), n) < y_i^*(\bar{v}, n) \). This means that each firm is, within a given range of \( n \), more collusive in terms of output than under usual market clearing conditions. There also exists an initial number of firms for which \( y_i^*(v_i^*(n), n) \) exactly coincides with the perfectly collusive output choice under market clearing wage, i.e., that obtained when all firms cooperatively maximize their joint profit. These results are described in the next proposition.

Proposition 2 The output selected by every firm under Cournot equilibrium and managers' threat to leave is more collusive than under Cournot equilibrium and managers' competitive market for \( n = \bar{n} \), that is, \( y_i^*(v_i^*(n), n) < y_i^*(\bar{v}, n) \) for \( n < \bar{n} \). Moreover, there exists a level of \( n = n^* \) for which, \( y_i^*(v_i^*(n^*), n^*) = y_i^*(\bar{v}, n^*) \), where \( y_i^*(n^*) \) is the output resulting by cooperative agreement among firms.
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Proof. By proposition 1, for \( n < \tilde{n} \), \( v^*(n) \) is greater than \( \tilde{v} \). Since firm's equilibrium output is monotonically decreasing in \( v \) it follows that, for \( n < \tilde{n} \), \( y_i^*(v^*(n),n) < y_i^*(\tilde{v},n) \). Moreover, straightforward manipulations show that it is equal to:

\[
v^*(n) = \frac{a^2(n^5 + 23n^4 + 205n^3 + 881n^2 + 1818n + 1424)}{(n+2)^2(n+3)^2(n+4)^2(n+5)^2(n+6)^2(n+7)^2}
\]

Substituting for \( v^*(n) \) into \( y_i^*(n) \) yields:

\[
y_i^*(v^*(n),n) = \frac{a - 2\sqrt{v^*(n)}}{n+1}
\]  

(11)

while, the collusive quantity under market clearing managers' compensation \( \tilde{v} \) is:

\[
y_i^{\tilde{c}}(\tilde{v},n) = \frac{a - 2\sqrt{\tilde{v}}}{2n}
\]  

(12)

Thus, expression (11) is equal to (12), for \( n = \tilde{n} \), where \( n^* \) is the only positive solution of an equation that, for ease of brevity, is not presented here. It can be noticed that the higher the managers' market clearing wage and the lower will be the number of firms for which \( y_i^*(v^*(n),n) = y_i^{\tilde{c}}(\tilde{v},n) \).

A particular example of the result above is presented in figure 3.3. Given \( \tilde{v} \), for \( n = 220 \) collusive equilibrium quantity \( y_i^{\tilde{c}}(\tilde{v},n) \) coincides with \( y_i^*(v^*(n),n) \). Moreover, for \( n < \tilde{n} \), \( y_i^*(v^*(n),n) \) turns out to be even more collusive than \( y_i^{\tilde{c}}(\tilde{v},n) \). Moreover, since every firm's quantity \( y_i^*(v^*(n),n) \) is also a Nash equilibrium quantity, it will be stable against each firm's temptation to deviate from the equilibrium choice of output, differently to what normally happens under collusive agreement. It can be noticed that such a particular example of non-cooperative collusive solution can take place either through mergers among firms (when the number of initiators is greater than \( n^* \)) or through controlled departure of managers induced by a firm (when \( n \) is lower than \( n^* \)). The latter can specially be the case when the
initiators maintain a share of the new companies' control, a relatively widespread practise in high-tech industries. Anyway, the basic result of the model is that, whether the behaviour of existing firms in a market is less or more collusive than in usual Cournot models depends upon the number of business initiators that dispose of the basic know-how to set up a firm.

4. Concluding remarks

The paper has described, through a simple model, that companies owners' need to fix a level of compensation high enough to keep managers within the firm can give rise to a collusive choice of output stable against individual firm's deviations. The result holds when the depressive effect of leaving managers on firms' profit prevails on the positive effect due to a reduction of their compensation, and the existing number of firms is not very high. Furthermore, the model generates the empirically appealing property (see, for instance, [2] and [5]) that managers' compensation is decreasing with the number of firms existing in the market and, consequently, with their size.

References


Appendix

Proof of Lemma 1. As long as $p(Y^*(n+k)) \geq AC_i(y^*_i(n+k))$, the following inequality holds for every $i = 1,...,n+k$

$$\pi_i(v^*(n+k),n+k) \geq \pi_i(v^*(n+k+1),n+k+1)$$

The meaning of this expression is that, under positive profitability of existing firms, every company finds convenient to pay each manager the equilibrium wage $v^*(n+k)$ rather than let her or him go, starting a new negotiation with another manager. Let us prove the lemma by contradiction. Suppose that the inequality above does not hold, that is:

$$p(Y^*(n+k)) - 2\sqrt{v^*(n+k)}y^*_i(n+k) < p(Y^*(n+k+1)) - 2\sqrt{v^*(n+k+1)}y^*_i(n+k+1)$$
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This expression can be solved by iteration and, for each firm under the potential market entry of \( t \) firms, the following result ensues:

\[
v^*(n+k) > \frac{\left[p(Y^*(n+k))\right]^2}{4} - \frac{\left[p(Y^*(n+t))\right]^2}{4\left[y^*(n+t)^2\right]}\]

where \( t \) indicates the number of firms that can enter before every firm's profit is equal to zero and the game ends. Since in this case there are no entry costs, room potentially exists for an infinite number of entrants. Hence, taking the limit of expression above for \( t \) that tends to infinite, we get:

\[
v^*(n+k) > \frac{\left[p(Y^*(n+k))\right]^2}{4} - \lim_{t \to \infty}\frac{\left[p(Y^*(n+t))\right]^2}{4\left[y^*(n+k)^2\right]}\]

that can be rewritten as:

\[
2\sqrt{v^*(n+k)} \cdot y^*(n+k) > p(Y^*(n+k)) \cdot y^*(n+k) - \lim_{t \to \infty} \pi_i\left(v^*(n+t)\right)
\]

and then,

\[
2\sqrt{v^*(n+k)} \cdot y^*(n+k) > p(Y^*(n+k)) \cdot y^*(n+k) - 0
\]

from which:

\[
2\sqrt{v^*(n+k)} = AC\left(y^*(n+k)\right) > p(Y^*(n))
\]

that contradicts the assumption of every firm's weak positive profitability. This concludes the proof.

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