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Types of Degrees in Bipolar Fuzzy Graphs

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Abstract

A bipolar fuzzy graph is a generalization of graph theory by using bipolar fuzzy sets. The bipolar fuzzy sets are an extension of fuzzy sets. This paper introduces an effective degree of a vertex, a (ordinary) degree of a vertex in bipolar fuzzy graph as analogous of fuzzy graph, a semiregular bipolar fuzzy graph, and a semicomplete bipolar fuzzy graph. Further, this paper gives some propositions.

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Keywords: Bipolar fuzzy graph, effective degree of a vertex, semiregular bipolar fuzzy graph, semicomplete bipolar fuzzy graph

1 Introduction

The notion of fuzzy sets was introduced by Zadeh [12] as a way of representing uncertainty and vagueness. As a generalization of fuzzy sets, Zhang ([13, 14]) introduced the concept of bipolar fuzzy sets. A bipolar fuzzy set has a pair of positive and negative membership values range is [-1, 1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership value (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership value [-1, 0) of an element indicates that the element somewhat satisfies the implicit counter-property. In many domains, it is important to be able to deal with bipolar information. It is distinguished that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to assign a fuzzy graph model. Rosenfeld [11] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [8]. The fuzzy relations between two fuzzy sets were also considered by Rosenfeld [11] and developed the structure of fuzzy graphs, obtained analogous of several graph theoretical concepts. Bhattacharya gave some remarks on fuzzy graphs in [5]. Bhutani and Rosenfeld [6] discussed strong arc in fuzzy graphs. The concept of closed neighborhood degree and its extension in fuzzy graphs was introduced by Basheer Ahamed et. al. [4]. The extension of fuzzy set theory, that is, bipolar fuzzy set theory gives more precision, flexibility, and compatibility to the system as compared to the classical and fuzzy models. Recently, Akram [1] introduced bipolar fuzzy graphs by combining bipolar fuzzy set theory and graph theory. Akram and Dudek [2] studied regular bipolar fuzzy graphs, and Akram [3] also discussed bipolar fuzzy graphs with applications. In this paper, an effective degree of a vertex, a (ordinary) degree of a vertex in bipolar fuzzy graph as analogous of fuzzy graph, a semiregular bipolar fuzzy graph, and a semicomplete bipolar fuzzy graph are introduced. Finally, some propositions with suitable examples are examined.

2 Preliminaries

In this section, some basic definitions and its related results are recalled.

Definition 2.1 [12] Let V be a nonempty set. A *fuzzy subset* of V is mapping $\mu: V \to [0, 1]$, where $[0, 1]$ denotes the set $\{t \in R: 0 \leq t \leq 1\}$.

Definition 2.2 [12] Let V and W be any two sets, and let μ and ν be fuzzy subsets of V and W, respectively. A *fuzzy relation* ρ from the fuzzy subset μ into the fuzzy subset ν is a fuzzy subset of VxW such that $\rho(v, w) \leq \mu(v) \wedge \nu(w)$ for all $v \in V$, $w \in W$. That is, for ρ to be a fuzzy relation, we require that the degree of membership of a pair of elements never exceeds the degree of membership of either of the elements themselves. Note that a fuzzy relation on a finite and nonempty set V is a function $\rho: V \times V \to [0, 1]$; a fuzzy relation ρ is symmetric if $\rho(v, w) = \rho(w, v)$ for all $v, w \in V$.

Definition 2.3 ([13, 14]) Let X be a nonempty set. A bipolar fuzzy (sub) set B in X is an object having the form $B = \{x, \mu^P(x), \mu^N(x) : x \in X\}$ where $\mu^P : X \to [0,1]$ and $\mu^N : X \to [-1,0]$ are mappings.

The positive membership degree $\mu^P(x)$ is used to denote the satisfaction degree of an element *x* to the property corresponding to a bipolar fuzzy set *B*, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element *x* to some implicit counter-property corresponding to a bipolar fuzzy set *B*. If $\mu^P(x) \neq 0$ and $\mu^N(x) = 0$, it is the situation that *x* is regarded as having only positive satisfaction for *B*. If $\mu^P(x) = 0$ and $\mu^N(x) \neq 0$, it is the situation that *x* does not satisfy the property of *B* but somewhat satisfies the counter property of *B*. It is possible for an element *x* to be such $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$, when the membership function of the property overlaps that of its counter property over some portion of *X*. For the sake of simplicity, the symbol $B = (\mu^P, \mu^N)$ is used for the bipolar fuzzy set $B = \{x, \mu^P(x), \mu^N(x) : x \in X\}$.

Definition 2.4 ([13, 14]) Let *X* be a nonempty set. Then, we call a mapping $A = (\mu_A^P, \mu_A^N) : X \times X \to [-1,1] \times [-1,1]$ a bipolar fuzzy relation on *X* such that $\mu^P(x,y) \in [0,1]$ and $\mu^N(x,y) \in [-1,0]$

At this juncture, let us recall some basic definitions in graph theory [7], a graph is an ordered pair $G^* = (V, E)$, where *V* is the nonempty set of vertices of G^* and *E* is the set of edges of G^*. Two vertices *x* and *y* in an undirected graph G^* are said to be adjacent or neighbors in G^* if $\{x,y\}$ is an edge of G^*. A simple graph is an undirected graph that has no loops and/or no more than one edge between any two different vertices. The neighborhood of a vertex *v* in a graph G^* is the induced subgraph of G^* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted *N(v)*. The degree *deg(x)* of vertex *x* (simply *d(x)*) is the number of edges incident on *x* or equivalently, $deg(x) = |N(x)|$. The set of neighbors, called a (open) neighborhood *N(x)* for a vertex *x* in a graph G^*, consists of all vertices adjacent to *x* but not including *x*, that is, $N(x) = \{y \in V : xy \in E\}$. When *x* is also included, it is called a closed neighborhood, denoted $N[x]$, that is, $N[x] = N(x) \cup \{x\}$. A regular graph is a graph where each vertex has the same number of neighbors, that is, all the vertices have the same open neighborhood degree. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge.

Let us recall some basic definitions in fuzzy graph theory [4, 9, 10], a fuzzy graph $G = (V, \mu, \rho)$ is a nonempty set *V* together with a pair of functions $\mu : V \to [0,1]$ and $\rho : V \times V \to [0,1]$ such that $\rho(x,y) \leq \mu(x) \wedge \mu(y)$, for all $x, y \in V$, where $\mu(x) \wedge \mu(y)$ denotes the minimum of $\mu(x)$ and $\mu(y)$. A fuzzy graph *G* is called a *strong fuzzy graph* if $\rho(x,y) = \mu(x) \wedge \mu(y), \forall \{x,y\} \in V \text{x} V$. A fuzzy graph *G* is *complete* if $\rho(x,y) = \mu(x) \wedge \mu(y)$, $\forall x, y \in V$. Note that if $\rho(x, y) > 0$ we call *x* and *y* *neighbors* and we say that *x* and *y* *lie on* $\{x, y\}$. The order and size of a fuzzy graph *G* are defined as $O(G) = \sum_{x \epsilon V} \mu(x)$ and $S(G) = \sum_{\{x,y\} \epsilon E} \rho(\{x,y\})$, respectively. An edge *{x,y}* of a fuzzy graph *G* is called an *effective edge* if $\rho(\{x,y\}) = \mu(x) \wedge \mu(y)\}$. $N(x) = \{y \in V : \rho(\{x,y\}) = \mu(x) \wedge \mu(y)\}$ is called the neighborhood of *x*, and $N[x] = N(x) \cup \{x\}$ is called the *closed neighborhood* of *x*. The degree of a vertex can be generalized in different ways for a fuzzy graph. The *effective degree* of a vertex *x* is defined as the sum of the membership value of the effective edges incident with *x*, and is denoted by $d_E(x)$. That is, $d_E(x) = \sum_{\{x,y\}\epsilon E} \rho_E(\{x,y\})$. The neighborhood degree of a vertex is defined as the sum of the membership value of the neighborhood vertices of *x*, and is denoted by $d_N(x)$.

Now, some definitions in bipolar fuzzy graph which can be found in [1-3] are summarized.

Definition 2.5 A *bipolar fuzzy graph* with an underlying set V is defined to be a pair $G = (A, B)$, where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that $\mu_B^P(\{x,y\}) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(\{x,y\}) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all $\{x,y\} \in E$. Here, A the bipolar fuzzy vertex set of V and B the bipolar fuzzy edge set of E.

Note that B is symmetric bipolar fuzzy relation on A. We use notion *xy* for an element of E. Thus $G = (A, B)$ is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$.

The definition 2.5 can be written as, a *bipolar fuzzy graph* $G = (V, E, \mu, \rho)$ where V is a nonempty vertex set, $E \subseteq V \times V$ is an edge set, μ is bipolar fuzzy set on *V*, and ρ is bipolar fuzzy set on *E*, that is, $\mu(x) = (\mu^P(x), \mu^N(x)), \mathit{and}$ $\rho(xy) = (\rho^P(xy), \rho^N(xy))$ such that $\rho^P(xy) \le \min(\mu^P(x), \mu^P(y))$ and $\rho^N(xy) \ge \min(\mu^N(x), \mu^N(y))$ for all $x, y \in V$ and $xy \in E$. Here, *xy* means that $\{x,y\}$, an undirected edge. The notation $G = (V, E, \mu, \rho)$ is used for bipolar fuzzy graph G, where V is nonempty vertex set, E is edge set, μ is fuzzy subset of V, and ρ is fuzzy subset of E.

Definition 2.6 A bipolar fuzzy graph $G = (V, E, \mu, \rho)$ is said to be *strong* if $\rho^P(xy) = \min\{\mu^P(x), \mu^P(y)\}\ \mathit{and}\ \rho^N(xy) = \max\{\mu^N(x), \mu^N(y)\}$ for all $xy \in E$.

Definition 2.7A bipolar fuzzy graph $G = (V, E, \mu, \rho)$ is said to be *complete* if $\rho^P(xy) = \min\{\mu^P(x), \mu^P(y)\}\ \mathit{and}\ \rho^N(xy) = \max\{\mu^N(x), \mu^N(y)\}$ for all $x, y \in V$.

Definition 2.8 Let G be a bipolar fuzzy graph. The neighborhood of a vertex x in G is defined by $N(x) = (N_\mu(x), N_\gamma(x))$ where $N_\mu(x) = \{y \epsilon V: \mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y)\}$ and $N_\gamma(x) = \{y \epsilon V: \mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y)\}$.

Definition 2.9 Let G be a bipolar fuzzy graph. The *(open) neighborhood degree* of a vertex x in G is defined by $\deg(x) = (deg_\mu(x),\ deg_\gamma(x))$, where $deg_\mu(x) = \sum_{y \in N(x)} \mu_A^P(y)$and $deg_\gamma(x) = \sum_{y \in N(x)} \mu_A^N(y)$.

Note: $\mu_B^P(xy) > 0, \mu_B^N(xy) < 0\ for\ xy \in E;\ \mu_B^P(xy) = \mu_B^N(xy) = 0\ for\ xy \notin E$

Definition 2.10 Let G be a bipolar fuzzy graph. The *closed neighborhood degree* of a vertex x in G is defined by $\deg[x] = (deg_\mu[x],\ deg_\gamma[x])$ where $deg_\mu[x] = \sum_{y\in N(x)} \mu_A^P(y) + \mu_A^P(x)$ and $deg_\gamma[x] = \sum_{y\in N(x)} \mu_A^N(y) + \mu_A^N(x)$.

Definition 2.11 A bipolar fuzzy graph $G = (V, E, \mu, \rho)$ is said to be (k_1, k_2)-*regular* if $d(v) = (k_1, k_2)$, where $k_1 = d^P(v)\ and\ k_2 = d^N(v)$.

Hereafter, the neighborhood degree of a vertex v is denoted by $d_N(v)$ and the closed neighborhood degree of a vertex v is denoted by $d_N[v]$.

3. Order and size in bipolar fuzzy graph

In this section, the order of bipolar fuzzy graph, which is a pair of positive order and negative order of bipolar fuzzy graph and the size of bipolar fuzzy graph, which is a pair of positive size and negative size of bipolar fuzzy graph are presented. These ideas are analogous of order and size in fuzzy graph [10].

Definition 3.1 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph. The *order* of bipolar fuzzy graph, denoted $O(G)$, is defined as $O(G) = \left(O^P(G), O^N(G)\right)$, where$O^P(G) = \sum_{v\epsilon V} \mu^P(v)$; $O^N(G) = \sum_{v\epsilon V} \mu^N(v)$. Similarly, the *size* of bipolar fuzzy graph, denoted $S(G)$, is defined as $S(G) = \left(S^P(G), S^N(G)\right)$, where $S^P(G) = \sum_{vw\epsilon E} \rho^P(vw)$; $S^N(G) = \sum_{vw\epsilon E} \rho^N(vw)$.

Proposition 3.2 In a bipolar fuzzy graph $G = (V, E, \mu, \rho)$, the following inequalities hold: (a) $O^P(G) \geq S^P(G)$; (b) $O^N(G) \leq S^N(G)$.

Proof Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph. By definition 3.1, we have $O^P(G) = \sum_{v\epsilon V} \mu^P(v) \geq \sum_{vw\epsilon E} \rho^P(vw) = S^P(G)$. This implies$O^P(G) \geq S^P(G)$. Similarly,$O^N(G) = \sum_{v\epsilon V} \mu^N(v) \leq \sum_{vw\epsilon E} \rho^N(vw) = S^N(G)$. This implies $O^N(G) \leq S^N(G)$. □

Example 3.3 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph, where $V = \{v_1, v_2, v_3, v_4\}, E = \{v_1v_2, v_1v_4, v_2v_3, v_2v_4, v_3v_4\}$ with $\mu(v_1) = (0.7, -1), \mu(v_2) = (0.6, -0.8), \mu(v_3) = (0.8, -0.6), \mu(v_4) = (0.4, -0.5)$; $\rho(v_1v_2) = (0.6, -0.5), \rho(v_1v_4) = (0.3, -0.3), \rho(v_2v_3) = (0.5, -0.6), \rho(v_2v_4) = (0.4, -0.5), \rho(v_3v_4) = (0.4, -0.5)$. By usual calculations, we get, $O^P(G) = 2.5, S^P(G) = 2.2 \Rightarrow O^P(G) > S^P(G)$. Similarly, $O^N(G) = -2.9, S^N(G) = -2.4 \Rightarrow O^N(G) < S^N(G)$.

Corollary 3.4 In a regular bipolar fuzzy graph G, (a) $O^P(G) = S^P(G)$; (b) $O^N(G) = S^N(G)$.

4. Types of degrees in bipolar fuzzy graphs

Akram[3] introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in bipolar fuzzy graph as generalization of neighborhood degree and closed neighborhood degree of a vertex in fuzzy graph. This section introduces an effective degree of a vertex and a degree of a vertex in bipolar fuzzy graph as analogous of an effective degree and a degree of a vertex in fuzzy graph. Also, introduces a semiregular bipolar fuzzy graph and a semicomplete bipolar fuzzy graph. Further, some of their results are investigated.

Definition 4.1 let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph. An edge is called effective if $\rho^P(vw) = \min\{\mu^P(v), \mu^P(w)\}\, and\ \rho^N(vw) = \max\{\mu^N(v), \mu^N(w)\}$ for all $xy \in E$. It is denoted by $\rho_E(vw) = (\rho_E^P(vw), \rho_E^N(vw))$. The *effective degree of a vertex v* in bipolar fuzzy graph G, denoted by $d_E(v)$, is defined as $d_E(v) = \sum_{vw\epsilon E}(\rho_E^P\,(vw), \rho_E^N(vw))$.

Definition 4.2 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph. The *ordinary degree (simply degree)) of a vertex* '*v*' in bipolar fuzzy graph G, denoted by $d(v)$, is defined as $d(v) = \sum_{vw\epsilon E}(\rho^P\,(vw), \rho^N(vw))$.

Example 4.3 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.6, -0.7), \mu(v_3) = (0.8, -0.6); \rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.4, -0.6), \rho(v_2v_3) = (0.6, -0.5)$. By usual calculations, $d(v_1) = (0.9, -1.2)$, $d(v_2) = (1.1, -1.1)$, $d(v_3) = (1.0, -1.1)$. Here v_1v_2 is the only effective edge. The effective degrees are $d_E(v_1) = (0.5, -0.6)$, $d_E(v_2) = (0.5, -0.6)$, $d_E(v_3) = (0, 0\ )$.
Note: $d_E(v_3) = (0, 0\ )$ means that there is no effective edge incident on v_3.

Definition 4.4 A bipolar fuzzy graph $G = (V, E, \mu, \rho)$ is said to be *semiregular* if all vertices have same closed neighborhood degrees. We say that G is $(k_1, k_2) - semiregular$ if $d_N[v] = (k_1, k_2), \forall v \in V, and\ k_1, k_2 \in R$, where R is real number set.

Example 4.5 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.6, -0.7), \mu(v_3) = (0.8, -0.6); \rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.4, -0.6), \rho(v_2v_3) = (0.6, -0.5)$. By usual calculations, the (open) neighborhood degrees are $d_N(v_1) = (1.4, -1.3)$, $d_N(v_2) = (1.3, -1.2)$, $d_N(v_3) = (1.1, -1.3)$. Therefore, it is not regular. But, $d_N[v_1] = (1.9, -1.9)$, $d_N[v_2] = (1.9, -1.9)$, $d_N[v_3] = (1.9, -1.9)$. It is a semiregular bipolar fuzzy graph. Also, it is known as $(1.9, -1.9)$ - semiregular bipolar fuzzy graph.

Definition 4.6 A bipolar fuzzy graph $G = (V, E, \mu, \rho)$is said to be semicomplete if it satisfies complete (crisp) graph condition, but G does not satisfy strong bipolar fuzzy graph condition.

Example 4.7 Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.6, -0.7), \mu(v_3) = (0.8, -0.6)$; $\rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.4, -0.6), \rho(v_2v_3) = (0.6, -0.5)$. Here, all vertices are joined together, but all edges are not strong. Hence, it is a semicomplete bipolar fuzzy graph.

Proposition 4.8 In a strong bipolar fuzzy graph $G = (V, E, \mu, \rho)$, $d(v) = d_E(v)$ for all $v \in V$.

Proof Let $G = (V, E, \mu, \rho)$ be a strong bipolar fuzzy graph.By definition 4.2, we have $d(v) = \sum_{vw\epsilon E}(\rho^P\,(vw), \rho^N(vw))$ … (1)
Since G is strong bipolar fuzzy graph, all edges are strong edges, therefore the equation (1) can be written as $d(v) = \sum_{vw\epsilon E}(\rho_E^P\,(vw), \rho_E^N(vw)) = d_E(v), \forall v \in V$, by definition 4.1. □

Example 4.9 Let $G = (V, E, \mu, \rho)$ be a strong bipolar fuzzy graph, where $V = \{v_1, v_2, v_3, v_4\}, E = \{v_1v_2, v_1v_4, v_2v_3, v_2v_4, v_3v_4\}$ with $\mu(v_1) = (0.7, -1), \mu(v_2) = (0.6, -0.8), \mu(v_3) = (0.8, -0.6), \mu(v_4) = (0.4, -0.5)$; $\rho(v_1v_2) = (0.6, -0.8), \rho(v_1v_4) = (0.4, -0.5), \rho(v_2v_3) = (0.6, -0.6)$, $\rho(v_2v_4) = (0.4, -0.5), \rho(v_3v_4) = (0.4, -0.5)$. By usual computations, $d(v_1) = (0.9, -0.8)$, $d(v_2) = (1.5, -1.6)$, $d(v_3) = (0.9, -1.1)$, $d(v_4) = (1.1, -1.3)$; $d_E(v_1) = (0.9, -0.8)$, $d_E(v_2) = (1.5, -1.6)$, $d_E(v_3) = (0.9, -1.1)$, $d_E(v_4) = (1.1, -1.3)$. Thus, $d(v) = d_E(v) for\ any\ v \in V$.

Proposition 4.10 Every complete bipolar fuzzy graph is semiregular bipolar fuzzy graph.

Proof Let $G = (V, E, \mu, \rho)$be a complete bipolar fuzzy graph.
Since G is complete bipolar fuzzy graph, all edges are strong and all vertices are connected together. By the closed neighborhood degrees of all vertices, we have $d_N[v]'s$ are equal for all $v \in V$. Hence, G is semiregular bipolar fuzzy graph. □
Note: Every complete bipolar fuzzy graph need not be regular bipolar fuzzy graph.

Example 4.11 Let $G = (V, E, \mu, \rho)$ be a complete bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.6, -0.7), \mu(v_3) = (0.8, -0.6)$; $\rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.5, -0.6), \rho(v_2v_3) = (0.6, -0.6)$. By usual calculations, the (open) neighborhood degrees are $d_N(v_1) = (1.4, -1.3)$, $d_N(v_2) = (1.3, -1.2)$,

$d_N(v_3) = (1.1, -1.3)$. Therefore, it is not regular. But, $d_N[v_1] = (1.9, -1.9)$, $d_N[v_2] = (1.9, -1.9)$, $d_N[v_3] = (1.9, -1.9)$. Thus, it is a semiregular bipolar fuzzy graph. Also, it is known as $(1.9, -1.9)$ - semiregular bipolar fuzzy graph.

Proposition 4.12 In any complete bipolar fuzzy graph $G = (V, E, \mu, \rho)$, $d_N[v] = O(G), \forall v \in V$. That is, the closed neighborhood degree of any vertex is equal to the order of bipolar fuzzy graph.

Proof Since G is complete bipolar fuzzy graph, it is easy to check that from proposition 4.10,

$$d_N[v] = \left(\sum_{w \in N[v]} \rho^P(vw), \sum_{w \in N[v]} \rho^N(vw) \right)$$

$$= \left(\sum_{w \in N(v)} \rho^P(vw) + \mu^P(v), \sum_{w \in N(v)} \rho^N(vw) + \mu^N(v) \right)$$

$$= \left(\sum_{v \epsilon V} \mu^P(v), \sum_{v \epsilon V} \mu^N(v) \right) = O(G) \text{ for all } v \in V. \square$$

Proposition 4.13 In a regular bipolar fuzzy graph $G = (V, E, \mu, \rho)$, closed neighborhood degree, order of bipolar fuzzy graph and size of bipolar fuzzy graph are equal.

Proof Since G is regular, the degrees of all vertices are same and also all vertices have same bipolar membership values and connected one another.
By proposition 4.12, we have $d_N[v] = O(G), \forall v \in V$. … (2)
By definition 3.1, we have $S(G) = \left(\sum_{vw \in E} \rho^P(vw), \sum_{vw \in E} \rho^N(vw) \right)$
$= \left(\sum_{v \epsilon V} \mu^P(v), \sum_{v \epsilon V} \mu^N(v) \right) = O(G), \forall v \in V$. … (3)
From the equations (2) and (3), we get $d_N[v] = O(G) = S(G)$ for all $v \in V$. $\square$

Proposition 4.14 In a regular bipolar fuzzy graph $G = (V, E, \mu, \rho)$, ordinary degrees, effective degrees and neighborhood degrees are equal for any vertex in G. That is, $d(v) = d_E(v) = d_N(v), \forall v \in V$.

Proof Since G is regular, we have
$\left(\rho^P(vw), \rho^N(vw)\right) = \left(\min\left(\mu^P(v), \mu^P(w)\right), max\left(\mu^N(v), \mu^N(w)\right)\right), \forall vw \in E$...(4)
By definition 2.9, definition 4.1, definition 4.2 inclined with equation (4), we get $d(v) = d_E(v) = d_N(v), \forall v \in V$. $\square$

Example 4.15 Let $G = (V, E, \mu, \rho)$ be a regular bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.5, -0.6), \mu(v_3) = (0.5, -0.6)$; $\rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.5, -0.6), \rho(v_2v_3) = (0.5, -0.6)$. By usual calculations:The (ordinary) degrees are $d(v_1) = (1.0, -1.2)$, $d(v_2) = (1.0, -1.2)$, $d(v_3) = (1.0, -1.2)$. The effective degrees are $d_E(v_1) = (1.0, -1.2)$, $d_E(v_2) = (1.0, -1.2)$, $d_E(v_3) = (1.0, -1.2)$. The (open) neighborhood degrees are $d_N(v_1) = (1.0, -1.2)$, $d_N(v_2) = (1.0, -1.2)$, $d_N(v_3) = (1.0, -1.2)$. The closed neighborhood degrees are $d_N[v_1] = (1.5, -1.8)$, $d_N[v_2] = (1.5, -1.8)$, $d_N[v_3] = (1.5, -1.8)$. We have $d(v) = d_E(v) = d_N(v), \forall v \in V$.

Proposition 4.16 Every complete bipolar fuzzy graph is strong bipolar fuzzy graph.
Proof Since G is complete bipolar fuzzy graph, all edges in G are strong and all vertices are joined together. Obviously, G is strong bipolar fuzzy graph. □
Note: Converse need not be true

Proposition 4.17 Every semicomplete bipolar fuzzy graph is semiregular bipolar fuzzy graph.
Proof Let G be semicomplete bipolar fuzzy graph. By definition 4.6, all vertices in G are connected together, but the edges are not necessarily strong. For the closed neighborhood degree computation, the strong edge condition would not affect. Therefore, the closed neighborhood degrees of all vertices are same. Hence, G is semiregular. □
Converse of the proposition 4.17 is also true. Proof is obvious.

Example 4.18 Let $G = (V, E, \mu, \rho)$ be a semicomplete bipolar fuzzy graph, where $V = \{v_1, v_2, v_3\}, E = \{v_1v_2, v_1v_3, v_2v_3\}$ with $\mu(v_1) = (0.5, -0.6), \mu(v_2) = (0.6, -0.7), \mu(v_3) = (0.8, -0.6); \rho(v_1v_2) = (0.5, -0.6), \rho(v_1v_3) = (0.4, -0.6), \rho(v_2v_3) = (0.6, -0.5)$. By usual calculations, the closed neighborhood degrees are $d_N[v_1] = (1.9, -1.9), d_N[v_2] = (1.9, -1.9), d_N[v_3] = (1.9, -1.9)$. Therefore, G is semiregular.

5 Conclusions

In this paper, the effective degree of a vertex and degree of a vertex in bipolar fuzzy graph, semicomplete bipolar fuzzy graph and semiregular bipolar fuzzy graph are introduced and some of their propositions are examined. Based on these ideas, we can extend our research work to other graph theory areas by using bipolar fuzzy graph; also construct a network model for bipolar fuzzy graph and establish algorithm oriented solution.

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