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Construction of Reliable Renewal System from Unreliable Elements Using Stochastic Entropy

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Abstract

A problem of a construction of a reliable renewal system from unreliable elements is considered in terms of thermodynamical approach. Conditions of the phase transition which divide a situation when it is possible to build reliable system with unreliable elements from a situation when it is impossible are obtained.

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Keywords: a stochastic entropy, a phase transition, a thermodynamical approach, a renewal system

1 Introduction

A problem of a construction of reliable systems from unreliable elements was formulated by J. von Neumann [1], [2]. Comparing specifics of a functioning of natural and artificial automata von Neumann draw an attention that alive automata and particularly a human brain work with high reliability in spite of comparatively small reliability of their elements. Is it possible to model this specific of alive organisms by artificial automata? Is it possible and if yes how to construct a reliable automat from unreliable elements? Is it possible to decrease error threshold up to fixed meaning?

Von Newmann devoted a large attention to a role of an error in a logic and in a physical instrument of the logic - a synthesis of an automata. So the error is considered not as exclusive event or as a result or as a cause of some incorrectness but as an essential part of a considered process. Von Newmann thought that the error is to be considered in terms of thermodynamical methods as it is made with an information in papers of L. Szilard and C. Shannon.

A lot of von Newmann ideas as yet did not receive their necessary development. One of them is an idea about an interdependence between a level of a complexity with a possibility of a system to a self-reproduction, to an existence of a critical level of a complexity so that below this level a system degenerates and above - receives a possibility to the self-reproduction.

In this paper we attempt to analyze this question by thermodynamical methods using entropy concept. Such approach in the theory of renewal systems is developed in articles of P. Rocchi [3], [4]. The authors of this paper attempted to consider this problem by means of classical theory of renewal systems [5]. But an application of thermodynamical methods allows to analyze it from more general viewpoint and in much more simple manner. So here a problem of a construction of a reliable renewal system from unreliable elements is considered in terms of thermodynamical approach. Conditions of the phase transition which divide a situation when it is possible to build the reliable system with unreliable elements from a situation when it is impossible are obtained.

2 Main results

Following P. Rocchi approach [4] make a following consideration. Denote a probability that a system works by P_f and a probability that the system recovers by P_r. The following equality $P_f+P_r=1$ is true and these probabilities are connected with an entropy as follows: $H_f=\ln P_f,\ H_r=\ln P_r$. Consequently we obtain $P_f=1-P_r=1-\exp(H_r)$ and so the equality $H_f=\ln(1-\exp(H_r))$. Put $-\delta=H_r<0$, then the following relations

$$\delta\to\infty\implies H_f=\ln(1-\exp(-\delta))\sim\ -\exp(-\delta)\to 0,$$

$$\delta\to\ 0\implies H_f=\ln(1-\exp(-\delta))\sim\ \ln\delta\to-\infty$$

take place. Assume that regenerated system consists of n identical and independent subsystems with the entropy $\bar{H}_r$ each of them. Then the equality

$H_r = n\bar{H}_r,\ \bar{H}_r = c$ takes place and so

$$\lim_{n\to\infty} H_f(nc) = \lim_{n\to\infty} \ln(1-\exp(nc)) = 0,\ \lim_{n\to\infty} P_f = 1.$$

Assume that each of regenerated subsystems satisfies the equality $\bar{H}_r = cn^{-a},\ a > 0$, then $H_r = n\bar{H}_r = cn^{1-a},\ H_f = \ln(1-\exp(cn^{1-a}))$. Consequently for $n \to \infty$

$$H_f \sim \exp(-cn^{1-a}) \to 0,\ \Longrightarrow\ P_f \to 1,\ a < 1,$$

$$H_f \sim \ln cn^{1-a} \to -\infty,\ \Longrightarrow\ P_f \to 0,\ a > 1.$$

So for $a = 1$ we have a phenomenon identical to phase transition in models of physical statistics.

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