

Mathematical Assessment on the Robustness of PLS Mechanism Against Adverse Selection and Moral Hazard Problems

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Abstract

Most of conventional banks conduct their banking and financial systems under interest scheme, which is vulnerable to asymmetric information problems. This paper provides a mathematical assessment on the robustness of profit and loss sharing (PLS) based contract against asymmetric information problems. In particular we show that PLS scheme has an inherent mechanism to overcome adverse selection and moral hazard problems. Specifically, it will be shown that PLS contracts is relatively immune to the problems. This is due to the so called *incentive compatible* mechanism provided by PLS contracts. We also propose a risk-pooling mechanism to seize the risk exposure.

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1 Introduction

Asymmetric information problems occurs in financial contract when a borrower possesses information that cannot be accessed by the banks as a lender. Conventional banking system always uses either interest rate instrument to address asymmetric information problems by raising interest rate on loans or the application of collateral.

Study on asymmetric information problems has been initiated through modern approach since 1970s by Akerlof [1] and since then has been paid much attention in macroeconomic and banking issues. A book by Bebczuk [2] assesses comprehensively the asymmetric information problems in financial market. It is shown that the poor as borrowers can encumber a high transaction cost due to these problems. Analytically it is demonstrated how interest based financial scheme eliminates the poor and small size enterprises from the market by high interest rate instrument and collateral requirement. Impacts of asymmetric information problems on capital and labor markets, as well as on health and insurance industries can be referred to Hillier [8]. Some further studies on problem-solving proposed mainly either by embracing interest and collateral or by empirical treatments. Capra *et al.* [3] offered an incentive model by invoking interest and collateral simultaneously to measure borrower's *ex ante* level of risk. Jung *et al.* [9] applied a multiperiod framework in determining optimal incentive of a contract to undertake moral hazard problem, while Edelberg [5] proposed two-period model with interest and collateral instruments to evaluate adverse selection and moral hazard problems in automobile credit market. Furthermore, Fernandez-Olmos *et al.* [7] performed a simulation of two-sided moral hazard model on PLS based agricultural activity. It is revealed that conditional PLS contract may deteriorate the incentive of agent to improve the quality of products. Computational approach was employed by Veghes [14] to examine an asymmetric information dependent economic model. Screening and incentive effects on credit market due to asymmetric information was quantified by McIntosh & Wyndick [11].

Theoretically it has been proven that PLS mechanism is just and more efficient than interest based system (see, for instance, Sugema *et al.* [13]). This paper is aimed to mathematically assess the immunity property toward asymmetric information problems which inherently attached in PLS mechanism.

2 Adverse Selection Problem

Adverse selection is one of fundamental problems in the conventional debt market as a consequence of asymmetric information. The literature on this problem is relatively abundant and has many important implications for practical as well as policy purposes. It has been shown in [12] that adverse selection

can lead to credit rationing in a way that the case of increasing interest rate. The classic paper [10] examines the problem of a firm investing in a project and the market portfolio. They show that the amount of funds invested in the project can be interpreted as a signal of its quality, which enable us to partially overcome the adverse selection problem by separating the 'good' and 'bad' projects in term of their level of self-financing.

Adverse selection problem is formally introduced by Akerlof on his seminal contribution on asymmetric information [1]. He illustrate the case of market for lemon in which a buyer of used car faces two quality sellers; good and bad quality cars. The sellers have better information about the actual condition of the cars and all cars will be claimed as good cars. None of bad quality car owner will honestly tell the true condition as they expect higher price. Thus the owner has an incentive to inflate the quality of the car. On the other side, the buyers are being disadvantaged by the behavior of the seller. They simply cannot distinguish between good and bad quality cars. They will know the actual quality of the car only after being used for quite some time. There is a risk that the good quality car that are paid for turn out to be a bad car.

The same problem is faced by lenders in the credit market where it is difficult to distinguish between good and bad quality borrowers since they have an incentive to hide every 'bad' information that likely to make their application turned down by the bank. Thus, banks tend to have complete knowledge about every applicants. This is exactly the asymmetric information problem.

To assess the implications of adverse selection on the borrower-lender relationship in the conventional credit market, we shall use a simple model with several assumptions as follows. We consider two types of stochastic production technology, X and Y . The probabilities of success of both projects are θ_X and θ_Y , respectively, and thus their probabilities of fail are $1 - \theta_X$ and $1 - \theta_Y$. The pay-offs R of both projects are respectively provided by

$$R(X) := \begin{cases} R_X & ; \text{ if success} \\ 0 & ; \text{ if fail} \end{cases}, \quad R(Y) := \begin{cases} R_Y & ; \text{ if success} \\ 0 & ; \text{ if fail.} \end{cases} \quad (1)$$

We assume that technology X has lower probability of success but higher revenue compared to that of technology Y , i.e., $\theta_X < \theta_Y$ and $R_X > R_Y$. We also assume that X and Y share the same expected revenue:

$$E[R(X)] = E[R(Y)] =: E[R],$$

where

$$E[R(X)] = \theta_X R_X, \quad E[R(Y)] = \theta_Y R_Y. \quad (2)$$

By (2), it is easy to see that type X technology is riskier because it has a lower probability of success and hence higher probability of failure. Suppose

that to run the project, each technology requires K unit of capital and would be fully borrowed from a bank. The credit contract involves limited liability on the part of borrower in the sense that the borrower is not forced to use personal assets to pay for capital and interest owed. Thus, the bank bears all the risk. Assume that the bank is risk neutral and has the opportunity to invest in a safe instrument, such as government bond, with interest rate r .

In the next subsections we consider three cases, namely financial system without asymmetric information, conventional system with asymmetric information and PLS system with asymmetric information. We follow [2] to analysis the first two systems.

2.1 Financial System without Asymmetric Information

First, we consider the case where the technology chosen by borrower is fully observable by the bank in the sense that the revenue and probability can be verified perfectly. In this case, there is no asymmetric information as both borrower and lender have access to the same information. Thus the bank can differentiate which type of technology is chosen. Because of that, the bank can differentiate the price of capital between the two. By certainty equivalent, the interest rate r_X charged to a riskier borrower (type X) can be derived as follows. Participation constraints imposes

$$(1+r)K = \theta_X(1+r_X)K, \quad (1+r)K = \theta_Y(1+r_Y)K,$$

which then give

$$1+r_X = \frac{1+r}{\theta_X}, \quad 1+r_Y = \frac{1+r}{\theta_Y}. \quad (3)$$

Since $0 < \theta_X < \theta_Y < 1$, it must be the case that

$$r < r_Y < r_X. \quad (4)$$

In other words, the riskier the borrower the higher the interest rate. Because the interest is adjusted in accordance to the risk, the riskier borrower is penalized with a higher rate. Note carefully, however, the expected repayment to the bank for both cases are the same, i.e.,

$$\begin{aligned} E[R^B(X)] &= \theta_X(1+r_X)K = \theta_X \frac{(1+r)K}{\theta_X} = (1+r)K, \\ E[R^B(Y)] &= \theta_Y(1+r_Y)K = \theta_Y \frac{(1+r)K}{\theta_Y} = (1+r)K. \end{aligned}$$

It means that the higher interest rate charged to the riskier borrower is just enough to compensate the risk. Although the interest rate on type X borrower is higher, the probability of success is lower, so that the bank actually cannot

take benefit in the form of a higher repayment from risk taking activities on the part of the borrower, i.e., the expectation of the repayments R^B from both borrower are the same. The incomes π for both borrowers are also equal:

$$\begin{aligned} E[\pi(X)] &= E[R(X) - R^B(X)] = E[R] - (1+r)K, \\ E[\pi(Y)] &= E[R(Y) - R^B(Y)] = E[R] - (1+r)K. \end{aligned}$$

Thus under symmetric information, there is no 'marginal' benefit for a borrower from taking a higher risk because it is correctly priced by the bank.

2.2 Conventional System with Asymmetric Information

The situation will be different if the borrower can 'hide' the information about which technology is chosen. The bank experiences difficulties in distinguishing between type X and type Y borrowers. Knowing that interest rate for type Y borrower is lower, type X borrower has an incentive to give a false information about which technology he is applying. If the strategy is successful, the expected repayment to the bank is only

$$E[R^B(X) | r_Y < r_X] = \theta_X(1+r_Y)K < (1+r)K,$$

which reveals that the expected repayment is lower than it should be and the income of type X borrower becomes higher. There is an incentive for this borrower to 'mimic' the characteristic of type Y borrower.

Because the bank suffer 'loses' from lending to riskier borrower and it is not possible to differentiate borrower based on risk exposure, then the bank will apply a 'single' loan rate r_K to all borrowers. Suppose the bank knows the proportion of risky borrower γ in the population, that is $0 < \gamma < 1$. The single loan rate r_K is now determined such that

$$1+r = \gamma\theta_X(1+r_K) + (1-\gamma)\theta_Y(1+r_K). \quad (5)$$

It says that the expected repayment should be equal to the weighted average of expected repayment from both type borrowers. The first term of the right hand side of (5) is expected repayment from riskier borrowers and the second term is from the others. The above equation can be expressed as

$$1+r_K = \frac{1+r}{\gamma\theta_X + (1-\gamma)\theta_Y}. \quad (6)$$

Recalling $0 \leq \theta_X < \theta_Y \leq 1$, the denominator of the r.h.s. of (6) satisfies

$$0 < \gamma\theta_X + (1-\gamma)\theta_Y < \gamma\theta_Y + (1-\gamma)\theta_Y = 1,$$

which implies that $r < r_K$. Furthermore (3) and (6) provides

$$r_X - r_K = \frac{(1+r)(1-\gamma)(\theta_Y - \theta_X)}{\theta_X[\gamma\theta_X + (1-\gamma)\theta_Y]} > 0,$$

which shows that $r_K < r_X$. We can similarly show that $r_Y < r_K$. By arranging all the inequalities including (4) we discover that

$$r < r_Y < r_K < r_X, \quad (7)$$

showing that the less risky borrower is over-charged while riskier one is under-charged. Under this arrangement, the riskier borrower will be the most eager to obtain loan. Meanwhile, some of good borrowers may quit from the market. Therefore, the bank at risk that the loan portfolio is actually dominated by risky borrowers. This mimics the lemon problem asserted by [1]. In other words, interest based loan market inherently suffers adverse selection problem.

2.3 PLS System with Asymmetric Information

One of the distinct characteristic of *sharia* finance is the existence of profit and loss sharing (PLS) financial contract. However, one should note that PLS does not exclusively belong to *sharia* banking system. One can find such contract in any financial transaction especially in agricultural sector [4]. In this section it will be shown that PLS system can solve adverse selection problem commonly observed in the conventional banking systems.

As before we still assume two types of borrower, type X and Y , and the bank experiences asymmetric information. The bank lends money under PLS arrangement in a way that the final cash flow will be distributed between bank and borrower according to a predetermined 'share'. If the project succeeds, the bank claims a share of α from the cash flow. If the project fails, the bank claims nothing. Bank has an opportunity cost of placing the fund in save assets with rate of return ρ .

Using that argument, for any project under technology i , where i is either X or Y , the following expected revenue equality must be true:

$$(1 + \rho)K = \alpha E[R(i)], \quad (8)$$

which can be restated as

$$\alpha = \frac{(1 + \rho)K}{\theta_i R_i}. \quad (9)$$

Based on (9), we can say that the share depends on the opportunity cost ρ , the amount of capital invested K , the probability of success θ_i , and the return when succeed R_i . The higher the opportunity cost, the higher the share set by the bank. The higher the ratio of investment to expected cash

flow (K/R_i), the higher the share, which is very plausible. More importantly, the lower the probability (or the higher the risk), the higher the share. Thus, the share is adjusted to the risk, return, capital invested, and the opportunity cost. Intuitively, this is a more comprehensive pricing model compared to the interest rate because more 'variables' are taken into consideration.

Since we assume that the expected returns of type X and type Y technologies are the same, i.e., $E[R(X)] = E[R(Y)]$, as well as the size of loan K , then (9) provides a universal α such that

$$\alpha = \frac{(1 + \rho)K}{\theta_X R_X} = \frac{(1 + \rho)K}{\theta_Y R_Y}. \quad (10)$$

Thus, the bank will apply a single share (10) to all projects provided the bank is risk neutral. This is unique to the case where $E[R(X)] = E[R(Y)]$. This is exactly what we are referring to *incentive compatible* contract. What is important for the bank is that the repayment should compensate the opportunity cost of capital. It is up to the borrower to take higher or lower risk technology. This implies that there is no incentive for type X borrower to mimic the characteristic of the lower risk type. Although the risk is taken into account in setting the share, the cash flow (revenue) is also taken into consideration. Therefore, as long as the risk is compensated by the revenue the share remains the same. The share is actually determined by expected cash flow, that is the product of probability and revenue. In effect, as long as the expected cash flow is the same there would be no adverse selection.

Under this scheme the expected revenues received by the bank from a risky and a less risky borrowers are, respectively, given by

$$E[R^B(X)] = \alpha \theta_X R_X = (1 + \rho)K = \alpha \theta_Y R_Y = E[R^B(Y)],$$

suggesting that the expected revenue of the bank is the same in both cases. Consequently, if the bank is risk neutral both projects are equally preferred. In such circumstances, there is no incentive for the bank to validate which technology is actually applied by borrowers. All borrowers are charged with the same price of capital preventing borrowers to lie about the actual risk exposure. Thus, profit and loss sharing scheme is naturally free from adverse selection problems. Note that the expected profits to a risky and a less risky borrowers are also the same. This fact indicates that there is no incentive for the borrowers to distort the technology actually applied. Regardless the technology they applied, the expected pay-off is just the same.

A critical point which can be drawn is laid on the assumption that cash flow is perfectly verifiable. If not, then the problem of cheating arises. The labor owner can hide some of the proceeds or even claim that the project actually fails. One way to solve this problem is by monitoring and auditing which can be costly [6]. One can clearly see that cheating and adverse selection are two different kinds of problem. We should address the problem separately.

2.4 Risk Pooling in Intermediated Financing

The other problem concerns with risk exposure. As mentioned before lenders are exposed to risk of loosing capital, while borrowers lose nothing. This is a typical 'limited liability' problem which also occurs in interest based financial contracts. The problem is that the decision on the degree of risk taken is on the hand of borrower. On the other hand, the lender has to bear the risk. This is problematic in a direct financing. It should not be a problem for intermediated financing through bank. Therefore we shall consider the two cases of financing: direct and intermediated financing.

First is the case of direct financing. With this we mean that individual capital owner get involved directly in financing the project. Suppose that there is large enough population of capital owner, say 10000 people, and each are endowed with 100 unit of capital. This implies that each capital owner can only finance one project. This assumption leads the following interesting results. If all borrowers are type X , amongst 10000 capital owners, there will be 6000 who lost the capital totally and there will be only about 4000 whose capital increases to 275 units. This can be considered as a brutal situation. Let's compare with type Y projects; there will be 2000 who lost the capital and there will be 8000 whose capital increases to 137.5 units. This is less brutal, but still brutal.

The point is that although statistically each capital owner has the same expected return, that is 110 units, but with direct financing scheme, the income distribution among capital owner is greatly affected. Regardless of which project is being financed, there will be big losers as well as gainers. The only difference is in the proportion of looser-gainer and how big loss and gain are.

One way to solve this problem is through an intermediated financing. Suppose that all capital owners deposit their capital in an islamic bank. The bank then allocates the capital to finance individual projects, collects the share from successful projects, and then distributes the proceeds equally to depositor. With this mechanism, all depositors can be warranted to earn almost certain return. The following is the proof.

Suppose there are n capital owners and each puts K units of capital in a PLS-based bank. The fund is allocated to n borrowers of type X and Y , whose proportions in the population are γ and $1 - \gamma$, respectively. Total repayment R^B collected by the bank from successful projects is

$$R^B = R^B(X) + R^B(Y) = \alpha n (\gamma R_X + (1 - \gamma) R_Y).$$

By assuming that the bank does not employ any intermediation cost and evenly distributes the shares among n capital owners, then the return R^C of each capital owner is

$$R^C = \alpha (\gamma R_X + (1 - \gamma) R_Y) \tag{11}$$

with the expected return

$$E[R^C] = \alpha(\gamma E[R(X)] + (1 - \gamma)E[R(Y)]) = \alpha E[R]. \quad (12)$$

We can see that the bank provides risk pooling and risk absorbing mechanism. The bank faces the risk that some projects will fail, but the return from successful projects will be enough to compensate the failure. The risk is fully absorbed by the bank and is not transmitted to depositors. The law of big number suggests no difference of whether the projects are less risky or not.

3 Moral Hazard Problem

Moral hazard problem arises when a debtor decides to falsify a riskier project after a contract has been signed and the loan has been disbursed in pursuing a higher pay off for the borrower but at the same time leaving the lender to face a higher probability of default. This action is deemed to be dishonest as it creates hazard to the lender.

As analyzed in [2], conventional credit contract is prone to moral hazard problem. However, an incentive compatible mechanism can be developed under such a contract to prevent the problem by means of setting up an interest rate that makes "high return" project more profitable to borrower. Let consider two different production technology H and L , where $R_H < R_L$ and $\theta_H > \theta_L$. Unlike in the adverse selection problem, we assume here that the expected revenue of project H (high) is higher than that of project L (low), i.e., $\theta_H R_H > \theta_L R_L$. By participation constraints we have $r < r_H < r_L$. Let suppose that borrower has incentive to hide the actual type of his project and engage in project L with lower expected return. As borrower can choose the project in which the capital invested, the bank needs to make sure that project H is more alluring than project L in the borrower's perspective. Technically, the bank should prevail an interest rate r_K such that the borrower's expected profit of project H is greater than that of project L , i.e., $E[\pi(H)] > E[\pi(L)]$. This condition is always satisfied provided that $r_K < \bar{r}$, where \bar{r} denotes the maximum level of interest rate in order for the contract consistent with the objective, i.e.,

$$\bar{r} = \frac{\theta_H(R_H - K) - \theta_L(R_L - K)}{(\theta_H - \theta_L)K}. \quad (13)$$

Therefore, alongside with condition $r < r_H < r_L$, we argue that the single loan rate r_K should be implemented if and only if $r_K < \bar{r}$ and

$$r < r_H < r_K < r_L. \quad (14)$$

If the bank sets an interest rate above the maximum level, moral hazard arises: riskier project becomes more attractive for the borrower. Thus, conventional

credit contract is prone to interest rate hikes. Monetary authorities in many cases resort to excessive contraction policy which results in a jump in interest rate. In such a situation, moral hazard increases putting the balance sheet of commercial banks in jeopardy.

Subsequently, it will be shown that PLS scheme is relatively superior as opposed to the conventional one in mitigating moral hazard problem. This is due to its ability to produce an incentive compatible mechanism that push business operators to choose least risky projects. Unlike the conventional one, such mechanism can actually work effectively in all monetary conditions. The formal proof of this superiority is as follows. As in conventional case, business operator obtains K units of capital from a *sharia* bank. If the project succeeds, the operator is required to surrender a fraction α of their cash flow to the bank. If the project fails, the bank receives nothing. See the following table. Under PLS arrangement it is possible for the bank to make project H is always more attractive for the operator all the time. Indeed this is one of the main advantages of PLS arrangement over the conventional one.

Suppose that the bank has a hypothetical required rate of return namely $r_K > 0$. The bank is a risk neutral agent and would agree to finance project if the return is at least r_K , hence

$$\alpha_H = \frac{(1 + r_K)K}{\theta_H R_H}, \quad \alpha_L = \frac{(1 + r_K)K}{\theta_L R_L}.$$

Assumption $\theta_H R_H > \theta_L R_L$ provides $\alpha_L > \alpha_H$, which means that project with lower expected return is required to surrender a larger proportion of cash flow.

4 Concluding Remarks

Findings elaborated in this paper have important replication for practical purposes. In fact, PLS scheme can naturally drive business practices into a less risky activity. To see this possibility, lets examine three classes of borrower:

1. The first class is when the population of businessman is dominated by risk neutral persons. In this group, the expected return is just the same, regardless the technology they applied. Each type of technology will be randomly picked-out by the borrower. In other words, the proportion of technology being selected will be equally distributed.
2. The second group is dominated by the risk lover population. In this group, all borrowers will implement type X technology, but this does not affect the expected return of the bank provided the size of population is large enough. Even if all borrowers are type X , the expected return will be just equal to the case where all borrowers are type Y .

3. The third group which is more akin to the real life is the case of risk averse borrowers. Certainly they will prefer type Y technology. But again, the expected return of the bank is not influenced by this choice.

One important lesson tells that the income of the bank is independent of the choice over technology and PLS scheme does not provoke any particular class of borrowers to engage in a riskier technology. The choice of technology fully represents the risk appetite of the borrowers. Since risk aversion is more common to be observed in real life, PLS scheme retains the borrowers to be consistent with their own nature. In other words, PLS scheme promotes honesty. This prodigy cannot happen to an interest based credit scheme. Since, it elicits borrower to engage in a riskier project. Type X borrowers have incentives to declare and therefore attempt to prove that they are type Y to rip off extra profit, then this credit scheme promotes an untruthfulness.

In the case of moral hazard problem, the incentive compatible mechanism offered by PLS arrangement drives operator to choose less risky and high return project. When operator is rational, this scheme essentially promotes business activities that are more valuable and less risky. But greed may push an operator to employ project L instead of H . Realizing that $\alpha_L > \alpha_H$, all operators will sign a PLS financing contract for project H but then actually carry out project L . Such a decision will only be feasible if such immoral action is more profitable than doing H . Since $\theta_H R_H > \theta_L R_L$, it is not rational if operator chooses type L project after signing financial contract for type H project. It is always rational for operator to stick on the contract and thus, there is no such thing as moral hazard in the PLS mechanism.

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