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Image Reconstruction Using Rational Ball Interpolant and Genetic Algorithm

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Abstract

In this paper we employed rational Ball interpolant for the reconstruction of different images. The unknown parameters involved are optimized with the help of genetic algorithm. The error tables of three chosen images are displayed.

Keywords: Boundary Extraction, Corner Detection, Genetic Algorithm, Image Reconstruction

1 Introduction

Curve reconstruction is significant field in computer graphics particularly for the curves which are not easily computed. There are various aspect of curve reconstruction such as font designing, data visualization, capturing hand-drawn images on computer screens, and computer-supported animations. The literature is rich for such related studies, see [1-15]. The smoothness and accurateness of the curve is the main challenge in curve reconstruction technique.
We have developed a rational cubic interpolant with two free parameters based on Ball bases. The Ball bases have the following properties: barycentric, satisfy convex hull property and variation diminishing property and they are symmetric.
One of the most important properties of Ball bases is it forms a curve which is far from the middle control polygon and it helps in curve fitting.

For reconstruction of three different $2D$ images we have utilized the mentioned interpolant. Control points and free parameters are used to control rational Ball interpolant. The curve is first segmented in corner detection process, then the curve control points are calculated based on the corner points. For optimization of free parameters we use genetic algorithm.
Then, we use mathematical morphology to obtain the boundary of the image.

2 Rational cubic Ball interpolant

Cubic Ball polynomial bases (see Figure 1) proposed by Ball in his paper, Generalized Ball curve and its recursive algorithm, are

$$S_0(\theta) = (1-\theta)^2,$$

$$S_1(\theta) = 2\theta(1-\theta)^2,$$

$$S_2(\theta) = 2\theta^2(1-\theta),$$

$$S_3(\theta) = \theta^2,$$

where

$$\sum_{i=0}^{3} S_i(\theta) = 1.$$

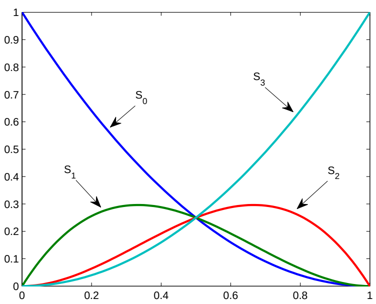


Figure: 1 **Ball Basis Functions**

These were generalized by [**?**] to curves of arbitrary degree. We will use cubic forms for both the denominator and numerator of

$$s_i(\theta) = \frac{P_i(\theta)}{Q_i(\theta)}, i = 1, 2, 3, ...n-1 \, and \, 0 \le \theta \le 1, \tag{1}$$

where

$$P_i(\theta) = \alpha_i f_i(1-\theta)^2 + V_i(1-\theta)^2\theta + W_i(1-\theta)\theta^2 + \beta_i f_{i+1}\theta^2,$$
$$Q_i(\theta) = \alpha_i(1-\theta)^2 + a_i(1-\theta)^2\theta + b_i(1-\theta)\theta^2 + \beta_i\theta^2,$$

where

$$V_i = a_i f_i + h_i d_i,$$
$$W_i = b_i f_{i+1} - h_i d_{i+1},$$

and α_i, β_i, a_i, b_i are free parameters, D_i are the data points of each segment and f_i, f_{i+1} are the control points of each segment, and d_i, d_{i+1} are the derivatives at f_i and f_{i+1}, respectively. [?] established a relation between rational Ball and Bezier representations, where the first and last weights remain unchanged when transforming from one form to the other. Taking this into account, and also noting that the first and last weights in the rational Bezier form can both be set to 1 [?], we will assume that $\alpha_i = \beta_i = 1$, so that (1) can be written as

$$s_i(\theta) = \frac{f_i(1-\theta)^2 + V_i(1-\theta)^2\theta + W_i(1-\theta)\theta^2 + f_{i+1}\theta^2}{(1-\theta)^2 + a_i(1-\theta)^2\theta + b_i(1-\theta)\theta^2 + \theta^2}, \tag{2}$$

$$0 = \theta_0 < \theta_1 < \theta_2 < \theta_3.... < \theta_n = 1,$$

and (2) satisfies the following conditions

$$\begin{cases} s(\theta_0) = f_i, s(\theta_n) = f_{i+1}, \\ s'(\theta_0) = d_i, s'(\theta_n) = d_{i+1}. \end{cases} \tag{3}$$

3 Boundary extraction and corner detection

The boundary of the original image is obtained using mathematical morphology defined by $\beta(A) = A - (A\Theta B)$ where A is the set of all black pixel, B is the 3×3 structuring element and $\beta(A)$ is the boundary of set A. Θ and $-$ represent the operation of erosion and difference, respectively.
Corner points divide the boundary into smaller segments. The method used in this paper to find corner points is by [?].

3.1 Parameterization

In this paper, chord length parameterization is used to evaluate the values of θ_i related to points D_i, where D_i represent the data points of segments

$$\begin{cases} \theta_0 = 0, \\ \theta_k = \frac{\sum_{i=1}^{k}|D_i - D_{i+1}|}{\sum_{i=1}^{n}|D_i - D_{i+1}|} \quad 1 \le k \le n-1, \\ \theta_n = 1, \end{cases} \tag{4}$$

It can be observed that θ_i is a normalized form and varies from 0 to 1

$$h_i = \theta_{i+1} - \theta_i,$$

so h is always one, where h is

$$h = \sum_{i=1}^{n-1} h_i.$$

3.2 Tangent vectors

Tangent vectors d_i at f_i are defined as follows:

$$\begin{cases} d_0 = 2(f_1 - f_0) - (f_2 - f_0)/2, \\ d_n = 2(f_n - f_{n-1}) - (f_n - f_{n-2})/2, \\ d_i = a_i(f_i - f_{i-1}) - (1 - a_i)(f_{i+1} - f_i), \\ i = 1, 2, 3, 4, ..., n - 1. \end{cases}$$

For open curve, and

$$\begin{cases} f_{-1} = f_{n-1}, f_{n+1} = f_1, \\ d_i = a_i(f_i - f_{i-1}) - (1 - a_i)(f_{i+1} - f_i), \\ i = 0, 1, 2, 3, 4, ..., n. \end{cases}$$

For closed curve with

$$a_i = \frac{|f_{i+1} - f_i|}{|f_{i+1} - f_i| + |f_i - f_{i-1}|}, i = 0, 1, 2, 3, ...n.$$

4 Genetic algorithm

In this paper, we use the continuous genetic algorithm (GA) to optimize free parameters in our proposed interpolant. This GA is very similar to binary GA. The primary difference is variables are no longer represented by bits of zeros and ones. We start the process of GA by defining a chromosome as an array of variable values to be optimized. Since there are two variables for our interpolant, the chromosome is written as an array of 1×2 elements i.e $chromosome = [a_i, b_i], i = 1, 2, 3...n$.
GA as a search technique must be limited to explore a reasonable region of variable space. Sometimes this is done by imposing a constraint on the interpoalnt. If an initial search region is unknown, then we define a suitable search space to start the GA. After this, we define an initial population of

chromosome; mostly the initial population varies from $2k$ to $4k$, where k is the number of chromosomes. In our case, initial population size is 8. Then we evaluate the cost of interpolant.
In the next step, we select chromosomes in the initial population fit enough to survive and possibly reproduce offspring in the next generation. For this, cost and associated chromosomes are ranked from lowest to highest. Only the top 50% of population with lowest cost will be selected for crossover, i.e parents. This process of natural selection must occur at each iteration of the algorithm. The simplest method for crossover is to choose one or more points in the selected chromosomes to be crossover points. Then, the variables of these points are merely swapped between the two parents.
Mutation is the final step where we change some chromosome values randomly from search space; usually the mutation rate is 20%. This process continues iteratively for a given number of generations or until a result obtained is less than a defined value.

5 Normalized mean squares error

In this paper, normalized mean squares error is used to find the error and free parameters of rational Ball interpolant. Normalized mean square error is defined as

$$E^2 = \frac{\sum |s(\theta) - (x,y)|^2}{\sum |(x,y)|^2},$$

where (x, y) represents the pixel values of the segments and θ is parameterized by chord length. We want to find the values of unknown parameters a_i and b_i so that normalized mean squares error is minimized. A genetic algorithm is used to optimize the value of a_i and b_i for image reconstruction.

6 Adopted procedure

Our adopted procedure for image reconstruction is described as follows:

6.1 Initialization

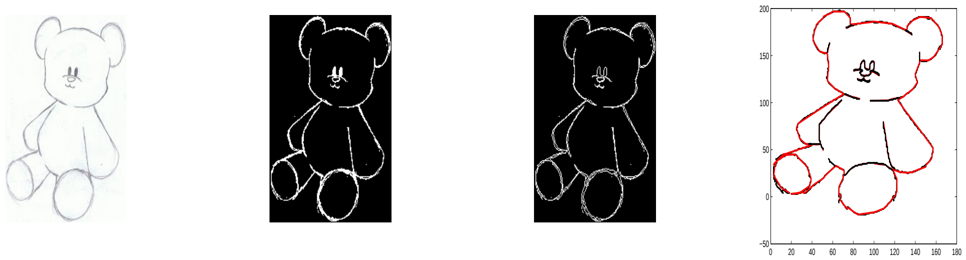
After getting the original image for image reconstruction as in figure $2(a)$, we will find the boundary and corners of the image using the method defined above.

6.2 Image reconstruction

Each segment will be approximated by using a rational Ball interpolant. The unknown parameters a_i and b_i, in (2) will be optimized using genetic algorithm. Errors are calculated using the normalized means square method discussed in above section.

7 Numerical examples

We construct three images of a bear, a vase and an arabic letter '*Dal*' and calculate normalized mean squares error of these images. Fig. 2 represent the various images of the bear sketch, in table 1 and figure 3 we display its normalized mean squares error. Table 2 and Figs. 4, 5, and 6 are related with the normalized mean squares error of three segments 60, 36, and 15.



(a) Original Sketch (b) Binary Image (c) Boundary (d) Constructed Image using Rational Ball Interpolant

Figure: 2 **Reconstruction of a Bear Sketch**

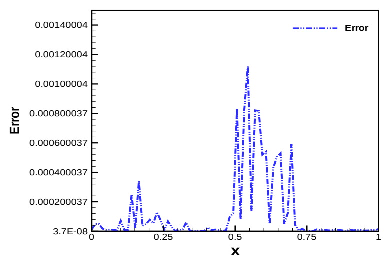


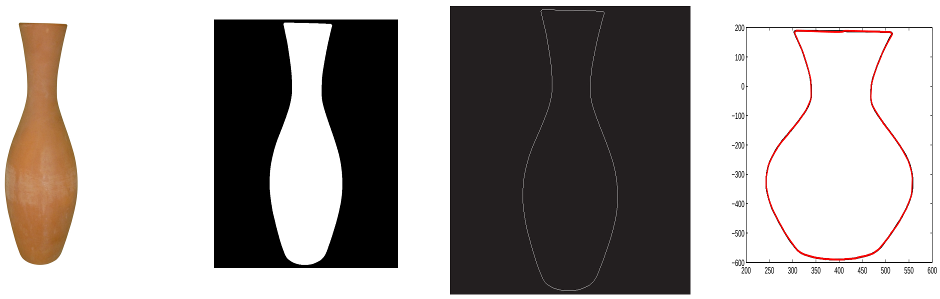
Figure: 3 **Graph of the Normalized Mean Squares Error of a Bear Sketch in Table 1**

Table: 1 **Bear Sketch errors from left to right**

1.20e-05	4.97e-05	4.99e-05	1.68e-05	9.504e-06	1.155e-05	9.31e-06	9.4e-06
7.15e-05	8.53e-06	9.23e-06	0.000250	1.8e-05	0.000342	5.02e-05	4.94e-05
7.84e-05	5.52e-05	0.000129	7.3e-05	8.26e-06	6.74e-05	2.92e-05	2.55e-06
1.15e-05	9.523e-06	5.98e-05	4.91e-06	3.51e-06	1.356e-07	2.7e-06	2.166e-06
2.5e-05	6.71e-06	1.21e-05	2.73e-06	2.369e-06	7.25e-06	9.673e-05	0.123e-05
0.000834	8.2e-05	6.78e-05	0.00112	1.14e-05	0.000821	0.000817	0.00052
1.540e-06	4.97e-05	0.00043	7.504e-06	0.00053	5.02e-05	0.000123	0.00059
4.3e-05	4.85e-06	1.667e-07	1.84e-06	3.65e-06	2.366e-06	1.284e-05	8.44e-06
1.050e-05	6.534e-06	5.37e-06	1.14e-05	9.00e-06	6.33e-06	3.853e-08	1.122e-05
6.183e-06	8.45e-06	9.6030e-06	4.88e-06	5.38e-06	8.1330e-06	7.213e-06	1.553e-05

Normalized Mean Squares Error of Bear Sketch.

Figure 7 shows the various images of a vase, Table 3 and Figure 8 shows the normalized mean squares error of this image.



(a) Original Image (b) Binary Image (c) Boundary (d) Constructed image using Rational Ball Interpolant

Figure: 4 **Reconstruction of a Vase**

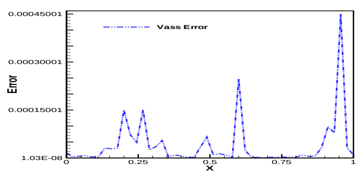


Figure: 5 **Graph of the Normalized Mean Squares Error of a Vase in figure 7**

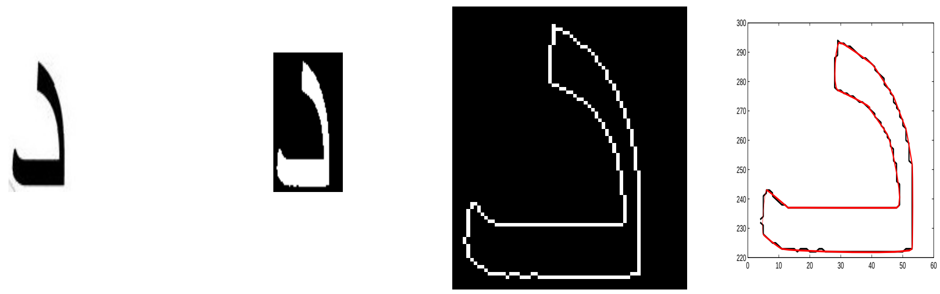
Figure 9 shows the various images of the arabic letter '*Dal*', Table 4 and

Table: 2 **Vase errors from left to right**

1.606e-05	3.0997e-06	5.639e-05	7.696e-06
3.315e-06	3.99e-06	4.89e-06	3.106e-05
2.87e-05	3.13e-05	1.9e-04	7.286e-06
4.920e-05	1.523e-04	2.56e-05	5.45e-05
3.39e-06	9.64e-06	6.08e-06	4.12e-06
2.605e-06	3.39e-06	6.55e-05	1.03e-05
1.323e-05	1.97e-06	1.665e-06	2.473e-05
2.45e-05	2.31e-06	2.07e-06	1.44e-06
2.34e-06	5.85e-06	4.932e-06	4.514e-06
3.2e-06	8.937e-06	5.2390e-06	6.486e-06

Normalized Mean Squares Error of Vase.

Figure 10 shows the normalized mean squares error of this image.



(a) Original Image (b) Binary Image (c) Boundary (d) Constructed Image using Rational Ball Interpolant

Figure: 6 **Reconstruction of the Arabic Letter** '*Dal*'

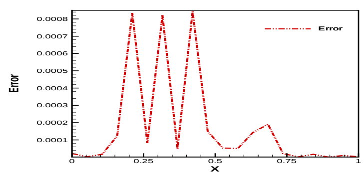


Figure: 7 **Graph of the Normalized Mean Squares Error of** '*Dal*'

Table: 3 **The Arabic Letter** '*Dal*'

2.120e-05	2.510e-06
1.6730e-05	0.000123
2.80e-6	8.20e-05
0.000821	4.97e-05
3.843e-5	0.00015
5.3e-05	5.02e-05
0.000143	0.00019
2.113e-05	1.85e-06
1.667e-05	8.723e-07
9.85e-06	2.366e-06

Normalized Mean Squares Error of The Arabic Letter '*Dal*'.

8 Conclusion

In this paper, we construct various images using rational cubic interpolant Ball, and free parameters are optimized using genetic algorithm. The error tables and graphs represent the accuracy of our proposed method.

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