A Hybrid Optimization Model: An Approach for the Humanitarian Aid Distribution Problem

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Abstract

This paper models a humanitarian aid distribution problem, which aims to satisfy the basic needs of a population in the aftermath of a natural or man-made disaster. The model considers dispatching goods from local distribution centers to distribu-
tion points during a given time horizon covering several periods (days). It takes into account practical constraints related to site’s storage capacity, truck’s capacity, and restrictions on the drivers’ maximum working time. To tackle this difficult problem, a hybrid optimization algorithm combining integer linear programming and a simulated annealing heuristic is proposed. Numerical results produced for a set of academic instances show that the hybrid optimization algorithm is very efficient, providing excellent results in short computational time.

**Keywords:** Humanitarian logistics, hybrid optimization, routing and distribution, transportation

### 3.1 Introduction

Improvements on the state of the art and practice of humanitarian logistics have significant economic and social implications. The human and economic impacts of natural disasters are increasing (Centre of Research for the Epidemiology of Disasters, 2009). According to the Office of U.S. Foreign Disaster Assistance and the Center for Research on the Epidemiology of Disasters, more than 297,000 people were killed and over 217 million were affected by natural disasters during 2010, and the economic damage has been estimated at over US$123.9 billion (Guha-Sapir et al., 2011 [12]). However, as several authors state, research on humanitarian logistics is not commensurate with its crucial role.

In a natural disaster situation, infrastructures have been partially or totally destroyed and supply chains are inoperative [16]. Humanitarian aid distribution (HAD) is the activity related with transportation of resources and items that provide humanitarian aid from the Local Distribution Centers (LDC) to the Distribution Points (DP), through a fleet of heterogeneous vehicles. From the distribution points, humanitarian aid is delivered to the affected population. Balcik et al. [3] establish that a poor HAD performance produces suffering to the affected population due to privation of their basic needs. Kovacs and Spens [15] indicate that the delay in supplying humanitarian aid may result in casualties. As a consequence, a good HAD plan is absolutely necessary in order to optimally distribute the available humanitarian aid; this is called the Humanitarian Aid Distribution Problem (HADP).

Huang et al. [13] and Perez et al. [18] point out that the HAD plan is frequently made in an ad-hoc manner, leading to an inefficient resource usage, a slow response and an unequal aid distribution. Similarly, Ergun et al. [11] affirm that people responsible for the administration of the supply chain and humanitarian logistics in most of the organizations generally, are not specialists and do not have adequate expertise to solve the problems related to the HAD. Since a successful response to a disaster is not improvised, Van Wassenhove [25] recommends being prepared to establish the most effective response. But the fulfill-
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...ment of the HAD plan in an optimal way is a very difficult task, so operations research techniques may be of great help to decision makers [3, 11].

Humanitarian aid distribution is different from commercial distribution since its objective is to provide humanitarian assistance subject to resource limitation, represented by the available human aid and transportation capacity [3, 11, 15, 23]. On the other hand, the main objective of commercial logistics is to minimize (maximize) the costs (profit) involved in the process [19, 22]. The operations, features and dynamics of commercial logistics are well known, such that researchers have been able to develop highly sophisticated analytical models to optimize the various components of modern supply chains. On the contrary, the characteristics, dynamics, and features of HAD are significantly different from commercial logistics.

This paper presents a research that seeks an adequate distribution of goods to population affected by a disaster, represented as a HADP. To this end, this paper presents a mathematical formulation to model the HADP considering all of the routing and allocation decisions to send humanitarian aid from LDCs to affected people through delivery of goods to DPs. The model is formulated in multiple periods in order to be able to consider adequately a balanced distribution, the satisfaction of demand with delays and the inventory in the DPs. We also propose a hybrid optimization metaheuristic algorithm to solve the HADP. The algorithm is based on a simulated annealing algorithm that includes the resolution of integer programming subproblems to efficiently explore a vast neighborhood. The algorithm’s performance is validated on academic instances and the results produced encourage application to bigger and more complex problems.

The paper is organized as follows. Section 2 outlines an exhaustive revision of the related literature. Section 3 presents the optimization model, as well as the hybrid optimization algorithm, including the simulated annealing metaheuristic with a sophisticated perturbation function that requires the solution of an integer programming subproblem. Section 4 outlines the implementation of the algorithm and details experimental results on a group of academic instances. Conclusions are presented in Section 5.

2 State of the Art

Commercial logistics are primarily concerned with the optimization of different stages of manufacturing, distribution and waste retrieval. They encompass a wide range of activities which require specific analytical models. Van Wassenhove [25] states that commercial logistics possess the following characteristics: (1) the main objective is to either minimize the cost of transportation or logistics (i.e., the summation of inventory and transportation costs); (2) the goods to be transported originate from the company’s suppliers and flows to the company’s customers; (3) not only are the origin and destination of
the freight known, but so is the demand, enabling the company to optimize its actions; (4) there are established decision-making procedures, involving a relatively small set of decision makers; (5) the logistic system transports large volumes of freight on a routine basis, which enables it to optimize operations; (6) the social networks in charge of logistical operations are intact and able to function at their maximum capacity; and (7) all these activities take place in conditions in which the supporting systems, e.g., transportation, are relatively stable and functional.

On the contrary, humanitarian logistics cover a wide range of activities that occur at any one of the phases of emergency management, i.e., mitigation, preparedness, response and recovery (National Governors’ Association Center for Policy Research, 1979; FEMA IS-1, 2010). Mitigation and preparedness activities are performed before the disaster to enhance safety and reduce the potential impact on people and infrastructure. Response-related humanitarian logistic activities include the transportation of supplies and equipment for search and rescue, and of equipment and material for emergency repairs to the infrastructure (FEMA IS-1, 2010). Finally, the recovery process is characterized by two sub-phases (Natural Hazards Research and Applications Information Center, 2005, Section 2-4). Short-term recovery is the transitional stage between response and long-term recovery, where activities such as managing donations and volunteers, conducting damage assessments, securing temporary housing, restoring lifelines and clearing debris take place. The work of Altay and Green [1] provides an important review of the literature in humanitarian logistics management. They conclude that very little information about transport planning and human aid distribution exists. However, more recent reviews devoted to the logistics in the humanitarian context (Caunhye et al. [7]; de la Torre et al. [9]; Anaya-Arenas et al. [2]) show the rapid and unstoppable development of this research field.

Disasters can be localized (a small tornado) or widespread (floods that impact a large portion of a country). The magnitude of the logistical challenge in terms of the volume of help to be transported after a disaster is significant. Also, humanitarian logistics imply an inbound flow of relief towards the affected population, but also an outbound flow aimed at evacuating people or materials towards safer areas located either inside or outside the affected zone. Despite the importance of such outbound flows, this work focuses on the inbound part. In addition to supplies for survivors, humanitarian logistics must deliver supplies to meet the needs of the response process. These include: (1) food and water for the responders; (2) equipment and supplies for medical teams, search and rescue, security forces; and (3) transportation, construction equipment and fuel. All of these conditions create a vast difference between the decision-making process involved in commercial logistics and the variants of humanitarian logistics. Commercial logistics are managed by a small number of decision makers using formal decision-making structures and standard procedures, with clearly defined roles and responsibilities for all players. Each participant knows what they are sup-
posed to do and how to interact with others. Established protocols enforce compliance with rules and regulations. It is a mature and highly structured market, with common information systems, standardization, transparency and visibility, accountability, clear objectives and well aligned incentives.

In the particular case of the HADP, its resolution through exact methods can only be made for small instances in a reasonable timeframe [3, 26]. De Angelis et al. [8] formulate the HADP as a Mixed Integer Linear Programming model and solve it by traditional operations research techniques. Due to the complexity of the problem, several researches have proposed to solve the problem in two stages [3, 4, 16, 26], however it is possible to find works that solve the problem in one stage [8, 17]. Barbarosoglu et al. [4] present a heuristic interactive method for the coordination of both subproblems with a hierarchical system of multiple criteria. Yi and Ozdamar [26] and Perez et al. [18] use a method with two stages: The first one defines a network flow problem, in which the vehicles are represented as products, being their flow an integer variable. The second stage defines the routes and the cargo for each vehicle. Balcik et al. [3] present a system with two phases, the first one generates all the non-dominated routes and the second one, through mixed integer linear programming, assigns routes to the vehicles, determines their loads and the quantities to be distributed to the DPs. Lin et al. [16] present two heuristics. The first one reduces the number of routes when considering a sub-group; the routes are then filtered through a genetic algorithm, the second one separates the initial problem into several subproblems, which contain a partial number of vehicles and DPs.

Nolz et al. [17] present a mathematical, non-linear formulation based on the Non-dominated Sorted Genetic Algorithm (NSGA-II). Berkoune et al. [5] also propose a Genetic Algorithm based algorithm to solve a relief distribution situation which is modeled as a transportation problem. Yi and Ozdamar [26] aim at minimizing the unsatisfied demand for relief, and Pérez et al. [18] use the concept of “social cost”, establishing the objective as minimizing the sum of the operational costs and social costs. Huang et al. [13] present a model of the HAD for one single period without limitations, in which the objective is the minimization of the balance in the distribution time to the DPs.

3 Problem description and model formulation

The HADP may be formally defined as follows: let $G = (V, A)$ a directed graph, $V = \{0, \ldots, n\}$ a group of nodes and $A = \{(i, j) : i, j \in V, i \neq j\}$ the group of arcs. Node 0 corresponds to the LDC and the rest of the nodes $\{1, \ldots, n\}$ represent the DPs. A non-negative travel time $c_{ij}$ is associated to each arc $(i, j) \in A$ and represents the required time to travel from node $i$ to node $j$. In this work, it is considered that travel times are symmetrical, in other words, $c_{ij} = c_{ji}$ for every arc $(i, j) \in A$. A heterogeneous fleet is available, each vehicle represented by $k$, and its load capacity by $C_k$. The maximum work time
available for each vehicle is denoted by \( t \). Each \( DP_i \) \( (i = 1, \ldots, n) \) is associated to a known non-negative demand \( d_t \) for each period \( t \) of the planning’s horizon, the LDC is assigned with demand equal to zero. The available aid in the LDC is known and non-negative for each period \( t \) of the planning’s horizon and inferior to the total demand in each period.

The model also considers: i) the capacity of humanitarian aid storage in the DPs to satisfy the next period’s demand, ii) vehicles are able to travel through several routes in each period, iii) various vehicles can visit the same DP within a single period, iv) it is possible to partially satisfy the DP’s demands and v) vehicles are able to complete different routes without surpassing the maximum work time.

The HADP’s main objective is to minimize penalties due to delivery delay and therefore non-satisfied demand. The penalty factors are based on the number of periods of delay: i) if the demand is satisfied in the same period, there is no penalization, ii) if the demand is satisfied with a delay of 1 or 2 periods, the penalization factors \( \beta_1 \) and \( \beta_2 \) (\( \beta_1 << \beta_2 \)) are defined, iii) if the demand is not satisfied, it is associated to a \( \beta_3 \) (\( \beta_1 << \beta_2 << \beta_3 \)) factor. The objective function includes a term which represents the distribution balance between the DPs to achieve equity in the service received by the DPs. Needless to say, complete equity would imply that everyone gets the exact same service, which cannot be achieved due to the restrictions on the available resources.

The service received by each DP is measured by calculating the demand that is not satisfied as a percentage. The service’s balance is incorporated to the objective at the moment the percentage of the unsatisfied demand for each DP is included through a convex, piecewise linear function, that relates the percentage of the unsatisfied demand with a particular weight value. This approach was proposed by Huang et al. [13], who declare that this form of establishing the balance has two advantages ahead of minimizing the maximum difference of the service received by the DPs: the first one is that the model maintains itself as linear. The second one is that better results are obtained in the other objectives.

The solution to the HADP establishes: The DPs that receive humanitarian aid in each period, the amount of humanitarian aid each DP receives during each period, the routes of the vehicles and the assignation of the routes to the vehicles.

### 3.2 Formulation of the problem

The following nomenclature is used to formulate the HADP:

- \( I, J \): Group of nodes
- \( T \): Group of time periods, size of the planning horizon
- \( K \): Group of vehicles
- \( R \): Group of routes
**B**: Segments of the convex linear function  

**C_k**: Capacity of vehicle **k** (product units)  

**τ**: Maximum work time of vehicle **k**  

**β_1, β_2 and β_3**: penalty factors due to delays  

**d_i**: DP’s demand within the **t** period  

**H_i**: Total available aid to be split in the **t** period  

**c_{ij}**: Travel time from node **i** to node **j**  

**M**: A very big value  

**xval_b, yval_b**: values of the intersections in the convex linear function by segments. \( f(xval_b) = yval_b \)  

**x_{ijrkt}**: 1 if vehicle **k** travels from node **i** to node **j** in the **t** period in the **r** route, otherwise 0.  

**y_{jrkt}**: 1 if vehicle **k** visits the DP **j** in the **t** period in the **r** route, otherwise 0.  

**q_{irk}**: Quantity of aid to be given by vehicle **k** in the route **r** in the **t** period in the DP **i** for **t**’s demand.  

**s_{kr}**: Quantity of aid that vehicle **k** will give in the route **r** in the **t + 1** period.  

**v_{irk1}**: Amount of aid that vehicle **k** will give in the route **r** in the **t** period to the DP **i** for the demand of the **t − 1** period.  

**v_{irk2}**: Amount provided by vehicle **k** on route **r** for period **t** to PD **i** for demand of **t − 2** period.  

**l_{it}**: Amount of unsatisfied demand of the DP **i** in the **t** period.  

**p_{i}**: Percentage of the unsatisfied demand of the DP **i**.  

**w_{i}**: Weight of the DP **i**’s unsatisfied demand.  

**m_{ib}**: Variable used to establish the relationship between **p_{i}** and **w_{i}** with the convex linear function by segments.  

The HADP model:  

\[
\text{Min} \sum_{i \in V} \sum_{r \in R} \left( \sum_{k \in K} \sum_{r \in R} \beta_1 v_{irk1}^1 + \beta_2 v_{irk2}^2 \right) + \beta_3 l_{it} + \sum_{i \in V} w_{i} \quad (1)
\]

Subject to:  

\[
\sum_{j \in V} x_{ijrkt} = \sum_{j \in V} x_{jirkt} = y_{irk} \quad \forall i, r, k, t \quad (2)
\]

\[
\sum_{i \in V} \sum_{j \in V} \sum_{r \in R} c_{ij} x_{ijrkt} \leq \tau \quad \forall k, t \quad (3)
\]
\[
\sum_{i \in V} v_{irkt}^2 + v_{irkt}^1 + q_{irkt} + s_{irkt} \leq C_k \quad \forall \ r, k, t \tag{4}
\]

\[
\sum_{i \in V} \sum_{r \in R} \sum_{k \in K} v_{irkt}^2 + v_{irkt}^1 + q_{irkt} + s_{irkt} \leq H_t \quad \forall \ t \tag{5}
\]

\[
v_{irkt}^2 + v_{irkt}^1 + q_{irkt} + s_{irkt} \leq M y_{irkt} \quad \forall \ i, r, k, t \tag{6}
\]

\[
\sum_{k \in K} \sum_{r \in R} s_{irkt} \leq d_{i,t+1} \quad \forall i; \ t = 1, \ldots, T - 1 \tag{7}
\]

\[
s_{irkt} = 0 \quad \forall \ i, r, k; \ t = T \tag{8}
\]

\[
d_{it} \leq \sum_{k \in K} \sum_{r \in R} q_{irkt} + v_{irk,t+1}^1 + v_{irk,t+2}^2 + l_{it} \quad \forall i; \ t = 1 \tag{9}
\]

\[
d_{it} \leq \sum_{k \in K} \sum_{r \in R} s_{irk,t-1} + q_{irkt} + v_{irk,t+1}^1 + v_{irk,t+2}^2 + l_{it} \quad \forall i; \ t = 2, \ldots, T - 2 \tag{10}
\]

\[
d_{it} \leq \sum_{k \in K} \sum_{r \in R} s_{irk,t-1} + q_{irkt} + v_{irk,t+1}^2 + l_{it} \quad \forall i; \ t = T - 1 \tag{11}
\]

\[
d_{it} \leq \sum_{k \in K} \sum_{r \in R} s_{irk,t-1} + q_{irkt} + l_{it} \quad \forall i; \ t = T \tag{12}
\]

\[
u_{ikrt} - u_{jkr} + C_k x_{ijrkt} \leq C_k - d_{jt} \quad \forall \ i, j, k, r, t \tag{13}
\]

\[
d_{it} \leq u_{irkt} \leq C_k \quad \forall \ i, r, k, t \tag{14}
\]

\[
p_i = \frac{\sum_{t \in T} l_{it}}{\sum_{t \in T} d_{it}} \quad \forall \ i \tag{15}
\]

\[
p_i = \sum_{b \in B} m_{ib} x_{ival_b} \quad \forall \ i \tag{16}
\]

\[
w_i = \sum_{b \in B} m_{ib} y_{ival_b} \quad \forall \ i \tag{17}
\]
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\[ \sum_{b \in B} m_{ib} = 1 \quad \forall i \]  

(18)

\[ s_{irkt}, u_{irkt}, v^{1}_{irkt}, v^{2}_{irkt}, s_{irkt} \geq 0 \quad \forall i, r, k, t \]  

(19)

\[ l_{it} \geq 0 \quad \forall i, t \]  

(20)

\[ p_i, w_i \in [0, 1] \quad \forall i \]  

(21)

The objective function (1) seeks to minimize the penalty due to delays and at the same time the minimization of unbalanced distribution. Equation (2) establishes flow conservation and the relationship vehicle – DP – period – route. Equation (3) imposes a restriction on the maximum working time. In a similar fashion, the restriction (4) considers the vehicle’s load capacity. Equation (5) considers the available humanitarian aid. Equation (6) evaluates the amounts that are delivered if the DP is visited. Equations (7 and 8) limit the inventory’s maximum values. Equations (9, 10, 11, and 12) relate the demand with the delivered humanitarian aid and the unsatisfied demand. Equations (13 and 14) allow eliminating sub-tours. Equation (15) calculates the value of the percentage of unsatisfied demand. Equation (16) relates the percentage of the unsatisfied demand through the convex linear function by segments to establish the value of \( p_i \). Restriction (17) calculates the service’s weight by unsatisfied demand \( w_i \) through the linear convex function by segments and the value of \( m_{ib} \). Restriction (18) establishes that \( m_{ib} \) has a value of 1, in order to adequately establish the value of \( w_i \) from \( p_i \). Equations (19 – 21) limit the variables in a general way. Analyzing the HADP’s formulation it is observed that it is the union of two problems: the humanitarian aid allocation problem and the routing problem.

Unfortunately, the HADP formulation can’t be solved efficiently by commercial branch-and-bound software. Therefore we turned to metaheuristics methods, which have been proven their ability to solve difficult combinatorial problems, to tackle the HADP instances. We designed a simulated annealing (SA) metaheuristic aimed at solving the HADP. However, since preliminary results were not very promising, we decided to use the natural decomposition of the HADP’s to conceive a hybrid solving algorithm.

3.3 Hybrid optimization algorithm

Raidl and Puchinger [21] recognize the strengths and the advantages of hybrid methods. Since exact methods have their own advantages and disadvantages, it
seems natural to combine the ideas of both methods. Hybrid algorithms are the combination of exact algorithms and metaheuristics. In the scientific community, the term “metaheuristic” refer to general purpose approximated optimization methods, such as tabu search, evolutionary computation, and simulated annealing, among others. Puchinger and Raidl [20] present a general classification of metaheuristic algorithms grouped into two categories:

- **Collaborative combinations**: In an environment of collaboration, the algorithms exchange information, but are independent. The exact and metaheuristic algorithms may be executed sequentially, in parallel or intertwined.

- **Integrated combinations**: In integrated methods, an algorithm is a subordinated component of another algorithm.

In the integrated combinations category, Caserta and Vob [6] and Raidl and Puchinger [21] identify two subcategories (i) the metaheuristic algorithm is the master and controls the calls to the exact algorithm and (ii) the exact algorithm is the master and calls the metaheuristic algorithm.

This paper presents a hybrid optimization algorithm merging a simulated annealing metaheuristic with a sophisticated perturbation function around an integer programming subproblem. The pseudocode of the hybrid optimization algorithm is shown in Fig 2. The algorithm starts with the definition of the algorithm’s parameters. An initial solution is generated randomly. Step 2 initializes the simulated annealing algorithm. Step 3 generates a new solution from the current solution. To this end, it executes the following stages: i) the current solution ($s$) is split in two parts, the partial-solution-routes ($sp_{routes}$) and the partial-solution-assignation ($sp_{asig}$). ii) the partial-solution-routes ($sp_{routes}$) are modified when the vehicle’s route change is applied several times, defining the new partial-solution-routes ($sp_{newroutes}$). iii) a humanitarian aid allocation problem is defined. This problem is related to the new partial solution routes ($P(sp_{newroutes})$). iv) the problem is solved to optimality, generating the new partial solution assignation ($sp_{newasig}$). v) in this stage, the two new partial solutions are joined, forming a new solution ($s_{new}$). vi) the new solution ($s_{new}$) is generated, eliminating the DPs that don’t receive humanitarian aid from the routes. Objective values of new solution ($s_{new}$) and the current solution ($s$) are compared. $s_{new}$ becomes the current solution if its objective value is greater than the one of solution $s$ or if it is within a random distance related to the algorithm evolution (the temperature). The algorithm ends when the temperature reaches the stop criteria.
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3.4 Hybrid algorithm, solution codification structure

An important element for the use of any metaheuristic algorithm is the structure that represents the solutions to the problem, normally using matrices or vectors, where each element represents a component of the solution. This structure or codification has to be able to represent any solution of the problem and, simultaneously, must be suitable to efficiently apply the algorithm. Because the HADP is the combination of a routing problem and an allocation problem, the structure of the solution is formed by two parts. Figure 1 shows an example of the structure of the solution defined for the HADP. The first part, route group, defines the routes followed by the vehicles. Each element of the route group is the route taken by a vehicle ($k$), in a period ($t$) and in a route ($r$). The first value indicates the total number of DPs to be visited, the rest indicates the DPs to visit and their order indicates the order in which they will be visited. The LDC is not included (node 0) because the route starts and ends in the LDC. The second part is the allocation matrix that establishes the amount of humanitarian aid delivered to each DP. Each row of the assignation matrix is related to a route and each column represents a DP, the value of the matrix indicates the amount of humanitarian aid to be delivered in the DP.

Step 1: Initialization

- $s =$ initial solution
- $\text{temp} = T_{ini}$ (Initial temperature)
- $\alpha =$ reduction factor
- $\text{iter} =$ number of iterations
- $T_{fin} =$ Final temperature (stop criteria)

Step 2: Search

while Temp > $T_{fin}$
    from $x=1$ to iter

Step 3: New solution

(i) $s_{\text{routes}}$ = establish the partial solution route ($s = s_{\text{routes}} + s_{\text{assign}}$)
(ii) $s_{\text{assign}}$ = establish the partial solution assignation
(iii) $P(s_{\text{routes}})$ = Disturb ($s_{\text{routes}}$)
(iv) $s_{\text{assign}}$ = Optimize $P(s_{\text{routes}})$
(v) $s_{\text{new}} = s_{\text{routes}} + s_{\text{assign}}$
(vi) Purify ($s_{\text{new}}$)

Step 4: Comparison

$\Delta E = c(s_{\text{new}}) - c(s)$
if $\Delta E > 0$ then
    $s = s_{\text{new}}$
else if
    if random [0,1] < exp($\Delta E$/temp) then
        $s = s_{\text{new}}$
    end if
end if
end while

Step 5: Final solution $s$

Fig. 2. Pseudocode for hybrid optimization algorithm
A set of academic instances were generated and split into two categories, namely small and large instances, containing 10 and 20 DPs, respectively. DPs’ locations were randomly generated from a uniform distribution within 100 X 100 distance units square. LDC is always located in the square’s center with coordinates [50, 50]. The travel times between nodes are computed as the downward rounded Euclidean distance in units of time. The demand of each DP in each period was generated from a uniform distribution $\text{U}[1000, 6000]$. After defining the demand of each DP for each period, the available supply for each period was set using a random uniform distribution to satisfy between 80 and 95% of the period’s demand. It is indicated that the transportation capacity is defined by four elements: number of vehicles, load capacity, maximum working time (MWT) and the number of routes. Small instances have 2 vehicles and a load capacity of 8000 and 6000 units. In the case of large instances, 2 vehicles with load capacity of 16000 and 12000 units were allowed. The maximum number of routes that a vehicle can do by period is limited to 2. Consequently, four types of problems were defined according to the number of DPs and the number of vehicles. For each type of problem, 6 instances were generated based on their location data, travel times, demand and supply. The total number of instances is 24.

The weights of the objective function were set \textit{a priori} and kept unchanged for all the instances. Their values are: P3: 100, P2: 40 and P1: 10. The penalty convex piecewise linear function used for the objective of balance in the unsatisfied demand is presented in figure 3 and contains 10 segments.
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In order to solve the proposed instances, two methods were used. The first method solves the mathematical formulation with the branch-and-bound algorithm of Cplex 11.0 software. A limit of 7200 seconds (2hrs) computation time was imposed. The second method is the hybrid optimization method. It was implemented in Python 2.7.2 and the integer programming problem is solved using the software Gurobi 4.6. The algorithm’s parameters were set, after a run of preliminary experiments, to: $T_{\text{ini}}= 10000$, $T_{\text{fin}}= 1000$, $\alpha$ factor$= 0.95$ and iter$= 20$. Experiments were run on a computer with a 1.80 GHz AMD Athlon 64 processor with 2.00 GB of RAM. Process times are reported in seconds.

Numerical results are shown in Table 1. The left part of Table 1 identifies the instance according to the number of DPs (column DP) and Working time (column MWT). For the exact method, column Value Obj Funct reports the best solution produced within the time limit, the best lower bound (column Lower Bound), the optimality gap in percentage (column GAP), and the computational time in seconds. Gap (in percentage) is computed as the difference between the best value produced by the concerned method and the best lower bound produced by Cplex, divided by the lower bound. Since the hybrid method is affected by randomness, we executed the algorithm four times per instance and the average results were computed. Hence, each line under header Value Obj Funct reports the average objective value produced by 4 executions of the hybrid algorithm. Column GAP gives the average over the four executions with respect to the best lower bound produced by the exact method. The bottom line in Table 1 reports the total average over the 24 instances.

![Figure 3. Penalty convex piecewise linear function of the unsatisfied demand](image-url)
The results show that, unlike the exact method, the hybrid optimization algorithm has a remarkable performance. In all the cases the hybrid method is able to produce better, or at least the same results than the exact method. Moreover, it produces an average gap of only 0.35%, while the exact method’s one raises up to 208%. Finally, the hybrid optimization only requires an average execution time of 194 seconds, which is negligible when compared to the 6747 seconds allotted to the exact method.

### Table 1. Numerical results

<table>
<thead>
<tr>
<th>Instance</th>
<th>PD</th>
<th>MWT</th>
<th>Value Obj Funct</th>
<th>Lower Obj Funct</th>
<th>Lower Bound</th>
<th>GAP</th>
<th>Time</th>
<th>Value Obj Funct</th>
<th>GAP</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>300</td>
<td>5,121,408</td>
<td>1,916,588</td>
<td>167.2%</td>
<td>7,200</td>
<td>1,918,003</td>
<td>0.07%</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>300</td>
<td>11,226,810</td>
<td>3,820,891</td>
<td>193.8%</td>
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<td>3,828,887</td>
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5 Conclusions

This work proposes a mathematical formulation and an efficient hybrid heuristic solving approach for a Humanitarian Aid Distribution Problem (HADP). The problem seeks to make all the routing and allocation decisions to send humanitarian aid from Local Distribution Centers to the affected people through delivery points called DPs. It considers multiple which capture the dynamics of demand as well as partial satisfaction of the demand at some DPs due to the resources’ limitation. The model also considers delays in demand satisfaction and the creation of multiple distribution routes. The objective of the model is to mini-
A hybrid optimization model: an approach for the humanitarian aid

mize penalties caused by unsatisfied demand or satisfying the demand out of schedule. It also includes a secondary objective, which is the distribution balance between the DPs. The balance’s goal aims to achieve a degree of equity in the service received by the DPs. The service received by each DP is measured by calculating the demand that is not satisfied as a percentage.

A hybrid optimization algorithm is also proposed to solve such a difficult problem. The hybrid algorithm’s performance, both in terms of objective value and computational time, is remarkable, and clearly outperforms the results produced by the exact method on a random generated set of 24 academic instances.

The insights provided by the model allow was to consider the representation of un-satisfied demand in a quantitative decision support system. The model represents quasi-real-live conditions and the efficiency of the hybrid algorithm lets us be confident with respect to its implementation within a decision support system covering a wide range of applications within several decision levels (operational, tactical and strategic) and it can be used for all stakeholders perspective based upon de crisis characteristic.

Nonetheless, further research is required to adequately support decision makers facing humanitarian crisis. In particular, aid distribution models should be linked to early-warning systems, or advanced prediction models (i.e. ground movements prediction models or whether forecasting systems) to better plan response to a larger and more likely set of disaster scenarios. Also, although this work concerns exclusively the transportation of relief to affected people, it might be easily adapted to tackle situations where a large part of the affected population has been evacuated to shelters. As a matter of fact, shelters would be required to be supplied within a variety of relief operations, becoming the HADP’s PODs.

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References


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