Domination, Co-Domination Numbers of Cube Connected Cycles

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Abstract

In this paper, the results on some domination parameters i.e., domination and co-domination numbers $\gamma$ and $\overline{\gamma}$ in cube connected cycles is analysed. Also, the relationship between diameter, domination and co-domination numbers of cube connected cycles is given.

Keywords: cube connected cycles, diameter, domination and co-domination numbers

1 Introduction

In mathematics as well as in computer science, graph theory is nothing but study of graphs. Nowadays, research in graph theory is being increased because of its numerous applications. It includes, biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). The powerful combinatorial methods found in graph theory have also been used to prove significant and well-known results in a variety of areas in mathematics itself and graph theory ideas are used in data mining, image segmentation, clustering, image capturing, networking etc., Similarly modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph colouring is utilized in resource allocation, scheduling. This leads to the development of new algorithms and new theorems that can be used in tremendous applications. [1]
An interconnection network connects the processors of a parallel and distributed system. The topology of an interconnection network for a parallel and distributed system can always be represented by a graph, where each vertex represents a processor and each edge represents a vertex-to-vertex communication link. Communication is a critical issue in the design of a parallel and distributed system [3]. The interconnection network plays a central role in determining the overall performance of a multicomputer system. If the network cannot provide adequate performance, for a particular application, nodes will frequently be forced to wait for data to arrive. Some of the more important networks include Mesh, Rings, Hypercube, Butterfly, Benes and Cube Connected Cycles etc., [4].

2 Basic Concepts

In this paper, graphs are finite and simple, that is, they have no loops or multiple edges. Let \( G = (V, E) \) be a graph where \( V \) is the vertex set and \( E \) is the edge set. A subset \( D \) of \( V \) (\( G \)) is called a dominating set if every vertex in \( V - D \) is adjacent to at least one vertex in \( D \). A subset \( S \) of \( V \) is said to be a minimal dominating set if \( S - \{u\} \) is not a dominating set for any \( u \in S \). The domination number \( \gamma (G) \) of \( G \) is the smallest size of a dominating set of \( G \). The domination number of its complement \( \bar{G} \) is called the co-domination number of \( G \) and is denoted by \( \gamma (\bar{G}) \) or simply \( \bar{\gamma} \). A shortest u-v path of a connected graph \( G \) is often called a geodesic. The diameter denoted by \( \text{diam} (G) \) is the length of any longest geodesic. A vertex \( v \) of degree zero in \( G \) is called an isolated vertex of \( G \). A hypercube of order \( n \), denoted by \( Q_n \), is a graph constructed of two copies of the graph \( Q_{n-1} \), where the corresponding nodes of each sub graph are joined. Equivalently, the vertex set \( V \) of \( Q_n \) consists of all binary sequences of length \( n \) on the set \( \{0, 1\} \). In other words \( V = \{x_1x_2 \ldots x_n : x_i \in \{0, 1\}, i = 1, 2 \ldots n\} \). Two vertices \( x = x_1x_2 \ldots x_n \) and \( y = y_1y_2 \ldots y_n \) are linked by an edge if and only if \( x \) and \( y \) differ exactly in one coordinate, i.e. \( \sum_{i=1}^{n} |x_i - y_i| = 1 \). Another definition of \( Q_n \) is the Cartesian product of \( n \) two-vertex complete graphs \( K_2 \) [2].

3 Cube Connected Cycles

A cube connected cycles network is a \( d \)-dimensional hypercube in which each of the \( 2^d \) vertices has been replaced by a cycle of length \( d \) in such a way that the edge of the \( i^{th} \) dimension originally incident with the hypercube vertex is now made incident with the \( i^{th} \) vertex of the cycle. Every vertex is of degree 3. For the cube connected cycles network, \( n = d2^d \). Figure 3 shows the cube connected cycles network for \( d = 3 \).
Example 1

![3-Dimensional Cube Connected Cycle Network](image)

Figure 1: 3-Dimensional Cube Connected Cycle Network

### 3.1 Theorem

Let $G$ be a cube connected cycles of dimension $d$, $d \geq 4$, $d$ is even. $\text{diam} (G) \geq 3$ if and only if $\gamma (\bar{G}) = 2$.

**Proof:**
Let $G$ be a cube connected cycles of dimension $d$, $d \geq 4$, $d$ is even.
Assume, $\text{diam} (G) \geq 3$.
Then there exists at least two vertices $u, v$ in $G$ such that $\text{diam} (u, v) \geq 3$.
Hence, there does not exist a vertex in $G$ which is adjacent to both $u$ and $v$.
$\Rightarrow$ All the vertices in $\bar{G}$ are either adjacent to $u$ or $v$.
$\Rightarrow \{u, v\}$ is a dominating set of $\bar{G}$.
Since $G$ has no isolated vertex, $\gamma (\bar{G}) \neq 1$.
$\Rightarrow \gamma (\bar{G}) = 2$.
Conversely assume that $\gamma (\bar{G}) = 2$.
Let $D = \{u, v\}$ be a minimum dominating set of $\bar{G}$.
$\Rightarrow$ All the vertices in $\bar{G}$ are either adjacent to $u$ or $v$.
Then there does not exist a vertex in $G$ which is adjacent to both $u$ and $v$.
$\Rightarrow \text{diam} (u, v) \geq 3$.
$\Rightarrow \text{diam} (G) \geq 3$.
Hence the proof.

### 3.2 Theorem

Let $G$ be a cube connected cycles of dimension $d$, $d \geq 4$, $d$ is even. $\gamma (G) > 1$ if and only if $\text{diam} (\bar{G}) \leq 3$.

**Proof:**
Let $G$ be a bipartite graph with no isolated vertex which is Super Strongly Perfect.
Assume $\text{diam} (\bar{G}) \leq 3$. 
To prove \( \gamma (G) > 1 \).
Suppose \( \gamma (G) = 1 \),
\[ \Rightarrow \] There exists a vertex \( v \in G \) which is adjacent to all the remaining vertices in \( G \).
\[ \Rightarrow \] \( v \) is an isolated vertex in \( \bar{G} \).
\[ \Rightarrow \] \( \text{diam} (\bar{G}) \) cannot be defined, which is a contradiction to the assumption.
Conversely assume that \( \gamma (G) > 1 \),
To prove \( \text{diam} (\bar{G}) \leq 3 \)
Suppose \( \text{diam} (\bar{G}) > 3 \).
Then there exists at least two vertices \( u, v \) in \( \bar{G} \) with \( d(u,v) > 3 \) in \( \bar{G} \).
\[ \Rightarrow \] \( \bar{G} \) has no vertex which is adjacent to both \( u \) and \( v \).
All the vertices are either adjacent to \( u \) or \( v \) in \( \bar{G} \).
\[ \Rightarrow \{u, v\} \) is a dominating set in \( G \)
\[ \Rightarrow \gamma (G) < 2 \), which is a contradiction to the assumption.
Hence \( \gamma (G) > 1 \).

4 Conclusion

We have investigated the relationship between diameter, domination and co-domination numbers of cube connected cycles network. In future, these investigations will be extended to the remaining well known architectures.

References


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