Hybrid Wavelet Model for Time Series Prediction

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Abstract

To improve time series forecasts the wavelet decomposition has been applied. The combination of forecasting methods as the Autoregressive Integrated Moving Average (ARIMA) and Artificial Neural Networks have been used to achieve a higher quality time series forecasting than. This paper proposed a hybrid model composed of wavelet decomposition, ARIMA and neural network Multilayer Perceptron. These models are combined linearly then yielding the time series forecasting. The series studied are the Wolf's sunspots and the British pound/US dollar exchange rate data. The comparison of the proposed model in this paper with literature indicated an effective way to improve forecasting.
**Keywords:** Wavelet Decomposition, ARIMA, Multilayer Perceptron, Combined Forecasts

1 Introduction

Time series prediction has an important role in the area of decision-making. Analyses are made from past observations to model mathematically the time series, to try to get future values, with an acceptable margin of error by the decision maker. The time series prediction models are applied, when a little knowledge about the process of series generation is available [15]. Examples of usual methods of forecasting are Box and Jenkins and Artificial Neural Network (ANN), the first is able to model data which have linear and stationary characteristics, the second is applied for complex data relation [1, 4, 3]

To overcome limitations of the usual forecasting models and to increase the accuracy of the predictions, hybrid models have been used. These types of forecasters are composed of various models which can express linear and nonlinear pattern, and become capable of modeling any type of data. Whereas the real data sometimes has both characteristics, and this makes using the usual methods unsatisfactory outcome.

Researchers such as Wedding e Cios [14], Zhang [15], Khashei and Bijari [6], show the effectiveness of this hybrid methodology in relation to single model. The application of various methods can complement each other and produce a weighty result for the time series prediction [11]. The manner of combine different models is generally additive; this feature can influence the performance of forecasting [5, 7, 13].

In some cases, the time series is preprocessing, so that the adjustment obtain better results, this is shown in Zhang [15], in this paper it's applied for example logarithm in some series of study. The wavelet decomposition (WD) of the time series is applied for Teixeira, Teixeira Jr and Siqueira [10], the series is turn into parts, allowing the data analysis in various resolution levels. This method makes the time series more appropriate to model [12, 2].

This study proposes a hybrid methodology consists on WD series and this modeling using ARIMA and Artificial Neural Network Multilayer Perceptron (MLP). To investigate the methodology accuracy is used the series Canadian lynxes, Wolf's sunspot and exchange rate [11].

This paper is organized as follows. In the next sections 2, 3, 4 have the methods used in the hybrid model respectively the wavelet, ARIMA and ANN and forecasting combination, in section 5 the materials and methods, section 6 the results and the last section the conclusions.

2 Wavelet

According to Kubrusly and Levan [8], the wavelet decomposition of a state of the time series $y_t (t = 1, ..., T)$ is given by Equation (1):
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\[ y_t = \sum_{m=n}^{\infty} a_{m,n} \phi_{m,n}(t) + \sum_{m=n}^{\infty} \sum_{n=\infty}^{\infty} d_{m,n} \omega_{m,n}(t) \]  

where: \( a_{m,n} \) are the approximation coefficients and \( d_{m,n} \) the detail coefficients; \( \phi_{m,n}(t) \) defines the function wavelet scale levels \( m \) and \( n \); \( \omega_{m,n}(t) \) represents the wavelet function level \( m \) and \( n \), wherein \( m \) is the approach parameter and \( n \) detail parameter. The component approach such as detail is calculated by the inner product of the wavelet scale function and wavelet function by \( y_t \) data.

3 ARIMA Method

The ARIMA method is one of the most established linear models for time series forecasting. This method is widely adopted in hybrid models, to deal with the linear underlying process of the time series, causing the increase of predictive capacity [14, 12, 9]. The ARIMA model does the differentiation of the time series, this action is necessary to make it stationary, thus enabling the application of Box and Jenkins method [1].

The process that generates the time series, with mean \( \mu \) has the form:

\[ y_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} \]  

where \( p \) the order autoregressive part, \( q \) the order of the moving average part, \( \varepsilon_t \) the random errors [9].

4 Artificial Neural Network

Some authors list the benefits of RNA such as: ability to approach any measurable continues function with an arbitrary precision; imposing few assumptions for their implementation; generalizability even being applied to non-stationary means and finally using fewer parameters compared to other methods [15, 12, 2]. Its main advantage is the universal approximation property; its parameters are iteratively adjusted and optimized through learning historical standards [13].

The Multilayer Perceptron (MLP) is one of the RNAs used for modeling and time series forecasting. The MLP three layers produce effective time series forecasting. It consists of a layer composed of the input pattern, represented by the set \( \{ y_{t,H} \}_{t=1}^{T} \), the hidden layer of neurons, processing the information from the synaptic weights and the output layer, providing the network response. The sum of the weights is used in \( g \) activation function according to equation (3) to activate the information provided the network then generating the output of the network [4].

\[ y_t = w_0 + \sum_{j=1}^{a} w_{j} g \left( w_{0j} + \sum_{i=1}^{p} w_{i,j} y_{t-i} \right) \]
where \( w_0 \) and \( w_0j \) are the bias of RNA, \( w_j\) and \( w_{ij} \) are the synaptic weights; \( q \) is the number of input nodes, \( p \) is the number of neurons in the hidden layer.

5 Materials and Methods

The time series used in this study are: Wolf's sunspots\(^1\) and the British pound/US dollar exchange rate \(^2\). These series have different statistical characteristics; they are studied both statistically and in neural networks.

Wolf's sunspot series is considered nonlinear and non-Gaussian, used for verification of nonlinear models. It is composed of 288 observations of the number of annual sunspots from 1700 to 1987. The time horizon considered is 67 years.

The weekly British pound/US dollar exchange rate exchange rate is the last series to be used. Weekly observations from the year 1980 to 1993, the logarithm is used to transformed data and considering a time horizon of 12 months.

Time series horizon has been defined according the papers of Khashei and Bijari \([6]\) and Zhang \([15]\) so that it can be compared to the results obtained by these papers. The forecast of these series is made one step forward. The Mean Absolute Error (MAE) and Mean Squared Error (MSE) are used to compare the forecasting accuracy.

The time series was decomposed using a Haar wavelet, with a decomposition level equal to two. Consequently, it was decomposed into an approximation component (\( A_2 \)) and two details (\( D_1 \) and \( D_2 \)). Each decomposition was modeled by ARIMA method and linearly combined as shown in Equation (4).

\[
\hat{y}_{1,t} = \alpha_2 A_2 + \alpha_1 D_1 + \alpha_2 D_2
\]

(4)

where \( \hat{A}_2 \), \( \hat{D}_1 \) and \( \hat{D}_2 \), are the adjusted values of the time series. The adjustment of this phase is represented by \( \hat{y}_{1,t} \). The weights of the Equation (4) are calculated from a nonlinear programming problem as in Equation (5), whose objective function is to minimize the MSE, subject to the unrestricted variables.

\[
\text{Objective function : } \min MSE = \frac{1}{m} \sum_{t=1}^{m} ||y_t - \hat{y}_t||^2
\]

(5)

\( \alpha_2; \alpha_0; \alpha_0; \) unrestricted

The residue of this adjustment was again decomposed by wavelet but each component was used as a standard MLP network input, generating adjusting the residue \( \hat{e}_t \). The final adjustment of the series is a linear combination of ARIMA and MLP model defined as Equation (6).

\[
\hat{y}_t = \beta \hat{y}_{1,t} + \delta \hat{e}_t
\]

(6)

whereas \( \beta \) and \( \delta \) are calculated similarly to the parameters equation (5).

For the development of this methodology were used the software E-views and

\(^1\) http://www.sidc.be/silso/datafiles
\(^2\) https://research.stlouisfed.org/fred2/series/DEXUSUK/downloaddata
Matlab. The time series forecasting is divided into training and testing sample. The training set is used to determine the parameters of the methods, the testing set to evaluate the model proposed.

6 Results

As an example, it presents the methodology step for the series of sunspot. The series undergoes wavelet decompositions Haar and with level two, generating one approximation component \( A_2 \) and the detail components \( D_1 \) and \( D_2 \). This configuration is used for all series.

The ARIMA modeling is performed for the training wavelet components. The weights of linear combination are determined from the nonlinear programming problem as defined in Equation (5). This phase aims to fit the linear characteristics of the series. The residue of this combination is determined, decomposed by wavelet and applied to RNA MLP, to retain the nonlinear characteristics of the series.

The residues are decomposed also by Haar wavelet decomposition with level two. The components of this stage serve as input patterns neural network. The parameters to be investigated are the size of the time window and the number of neurons in the hidden layer. The residues are also separated into training and testing set. The best result for the ANN MLP was obtained to time window with a size of five and seven neurons in the hidden layer.

The final adjustment has been made by the combination of ARIMA modeling with residues as Equation (6) and the weights defined similarly by Equation (5). The result of the methodology in respect of test data can be ascertained in Figure 1, with the time horizon equal to 67 years, one step forward.

![Figure 1](image-url)
The hybrid methodology developed here is compared to hybrid models of Khashei and Bijari [6], Zhang [15]. The comparison with the models ARIMA and ANN MLP will not be performed, since this work obtained a lower performance than aforementioned. The table 1 shows the results.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sunspot MSE</th>
<th>Sunspot MAE</th>
<th>Exchange rate MSE</th>
<th>Exchange rate MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang</td>
<td>280.15956</td>
<td>12.780186</td>
<td>4.359x10^-5</td>
<td>0.0051212</td>
</tr>
<tr>
<td>Khashei e Bijari</td>
<td>218.642153</td>
<td>11.446981</td>
<td>3.648x10^-5</td>
<td>0.0049691</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>69.28095</td>
<td>3.903514</td>
<td>4.23x10^-6</td>
<td>9.07x10^-3</td>
</tr>
</tbody>
</table>

7 Conclusions

Increased accuracy of time series forecasting is of extremely important in relation to decision making. This is achieved in some cases using hybrid models forecasting. The single models cannot always capture the characteristics of time series, since most of them have linear and nonlinear relationships among its components. For this reason the combination of forecasting methods has been increasingly studied.

The method proposed in this paper is a hybrid model consisting of the wavelet decomposition of the initial series, modeling then by ARIMA. The ARIMA model for each component is combined through a linear combination. The weights are defined using a nonlinear programming problem; this stage of modeling has a different configuration of the papers presented in this article. The residue from the combined model was again decomposed, its components have been used as patterns in the input RNA MLP; wavelet decomposition was also not been used in other methods hybrid at this stage. The linear combination was used again to determine final prediction. All of these steps constitute a modeling that differs from the others presented here.

Based on the series of sunspots, Canadian lynx and the exchange rate, it can be seen that the results of the proposed method were promising. This can be seen from the comparison of this model with the methods proposed by Khashei and Bijari and Zhang.

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References

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