

Use the Refuge to Protect the Food Chain when the Area Ecosystem is Reduced

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Abstract

In this work deterministic continuous-time predator-prey models are analyzed considering the use of refuge by a part of prey population, when the area of where inhabit the preys and predators decreases.

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1 Introduction

In earlier works it has been claimed that the prey refuge use exerts a stabilizing effect in the dynamics of the interacting populations. We show that the above statement it is true assuming that the quantity, since we demonstrate that when we include a refuge to the preys we can maintain the equilibrium points and to maintain the stability. The function refuge given by Almanza-Vásquez presents the best alternative to preserve the species, relates the size refuge the capacity and the fraction of land that can trim and stability conditions to maintain original. Population ecology has given emphasis on the introduction of natural complexity and realism into the basic Lotka-Volterra framework [1]. The initial steps were to include density-dependent effects on the endogenous dynamic of predators and prey, and to develop non-linear functions for consumption of prey by predators, the so-called functional response [8, 11]. So, the predator behavior was explicitly considered in predator-prey models.

More recently, the different behavior of prey and its consequences at the population level has been worked out and incorporated into the predation theory or in the growth prey function due the Allee effect [4]. In this context, a more relevant behavioral trait that affect the dynamics of predator-prey systems is the use of spatial refuges or covers by the prey.

Spatial refuges are found where environmental heterogeneity provides less accessible sites for predators in which a number of prey can stay, at least temporarily. In this way, some fraction of the prey population is partially protected against predators and we assume that the refuge is a physical location in which prey either live or temporally hide [7].

In the ecological literature have studied three types of refuge:

Two refuge types proposed by Maynard-Smith [9]; $X_r = \sigma$. The quantity of hidden prey is a constant number, $X_r = \alpha$. The quantity of hidden prey is proportional to the prey population at instant t , $X_r = \beta X$.

One proposed by Almanza-Vasquez [2] where analyze the population consequences of refuge use in the Lotka-Volterra model with self-limitation, assuming that the amount of prey in refuge using a saturated function that it is growing monotonously in the way $X_r = \frac{\alpha X}{X + \beta}$

Here α represents the maximum physical capacity of refuge and where the population's fraction in cover is falling in the way. β is the quantity of necessary preys to reach half of the maximum capacity α .

Where $X = X(t)$ represents prey population size for $t \geq 0$ and $X_r = X_r(t)$ belongs to the fraction of the covered population.

According to Taylor [10] the different kinds of refuges can be arranged into three types:

a) Those which provide permanent spatial protection for small subsets of the prey population,

- b) Those which provide temporary spatial protection, and
- c) Those which provide a temporal refuge in numbers, i.e., the risk of predation decreases by increasing the abundance of vulnerable prey.

Frequently, in Population Dynamics it has been claimed that prey refuge use has a stabilizing effect in interaction [9] and damp predator-prey oscillations: this affirmation is based on theoretical studies of very simple deterministic continuous-time predator-prey models [5].

We denote for $X(t) = X$ and $Y(t) = Y$ the population sizes of preys and predators, respectively for $t > 0$, considered as continuous variables that can represent density, biomass or quantity of each population's individuals; and $X_r(t) = X_r$ the quantity of prey population that occupies a refuge (heterogeneity of the means), the quantity of preys in refuge, then the quantity of preys that interact with the predators is $X - X_r$. According to [3] we modified the functional response of the Lotka-Volterra model when prey refuge is expressed by the system of self-dimensional differential equations:

$$\begin{aligned}\frac{dX}{dt} &= r\left(1 - \frac{X}{K}\right)X - q(X - X_r)Y \\ \frac{dY}{dt} &= p(X - X_r)Y - cY\end{aligned}\tag{1}$$

which is a Kolmogorov type system, where the functional response is linear.

The parameters have the following biological meanings:

r : is the intrinsic per capita prey growth rate;

K : is the prey environmental carrying capacity;

q : is the maximal per capita predator consumption rate;

p : is the efficiency with which predators convert consumed prey into new predators;

c : is the natural per capita death predator rate.

If assuming that $X_r = \alpha$, the system ceases to be a Kolmogorov type, but the positive equilibrium point is globally asymptotically stable [10]

If the prey quantity in refuge $X_r = \beta X$, then the system is topologically equivalent to the original [5], changing only the coordinates of the positive equilibrium point [6].

Assuming that the amount of prey in refuge is given by $X_r = \frac{\alpha X}{X + \beta}$. The positive equilibrium point can change the stability depending on the parameters.

2 Effect of the refuges on the food chain when you intervene the area and stay the carrying capacity. The Models

The behavior of the species can be affected by the effect of ecological variables (readiness of refuges, formation of defence groups, difficulty of mating, appear-

ance of other strategies antipredator etc). It is shown the influence that has the refuge when the area have to be intervened by a civil work, comparing the stability and values of the equilibrium points of the systems generated when analyzing all the refuges functions presented in the literature. Maintaining the carrying capacity.

As $q = \sigma \frac{S_y}{S}$ where σ is the fraction of encounters prey-predator where the prey dies. S_y is the area where each predator looks for the preys and S is the region where the preys are distributed. And $p = \xi q$ where ξ is the quantity of new predators taken place by each consumed prey.

If we diminish the area, let us say that it exists $0 < \phi < 1$ such that the not intervened area is $\phi S < S$, the consumption average changes to $\frac{1}{\phi}q > q$, then the rate of conversion of preys in new predators rate changes to $\frac{1}{\phi}p > p$.

Now it is shown the influences that have the refuges functions X_r on the variation of the preys and predators species, in the system of Lotka-Volterra. The refuges functions increase the variation of the preys, positively and negatively to the predators.

It is considered that $a = \frac{1}{\phi}$. And the refuge function in the system (1)

- (a) The system of linear equations when the area is modified from S to ϕS , stays the carrying capacity and the $X_r = \alpha$ refuge function proposed by Maynard-Smith is included [9], it is represented by:

$$\begin{aligned}\frac{dX}{dt} &= r\left(1 - \frac{X}{K}\right)X - qXY - (a-1)qXY + \alpha a qY \\ \frac{dY}{dt} &= pXY - cY + (a-1)pXY - \alpha a pY\end{aligned}\tag{2}$$

where the equilibrium points are $(0, 0)$, $(K, 0)$ and

$$(X_2^*, Y_2^*) = (\phi X_o^* + \alpha, (\phi c + \alpha p)(Y_o^* + \frac{r(1-\phi)}{q} + \frac{\alpha r}{qK}))$$

where the equilibrium point is into the first quadrant Lotka-Volterra model.

Theorem 2.1. *The nature of the equilibrium points of the system (2). For all parameter values it has*

- (a) *The singularity $(0, 0)$ is saddle point.*
 (b) *$(K, 0)$ saddle point, if and only if, $K > \phi_p^c + \alpha$; an attractor point, if and only if, $K < \phi_p^c + \alpha$ and an saddle-node attractor, if and only if, $K \leq \phi_p^c + \alpha$*
 (c) *If $K > \phi_p^c + \alpha$, the singularity (X_2^*, Y_2^*) is a locally asymptotically stable equilibrium point*

Proof. The Jacobian matrix of system (2) is

$$J(X; Y) = \begin{bmatrix} r(1 - \frac{2X}{K}) - aqY & -aq(X - \alpha) \\ apY & ap(X - \alpha) - c \end{bmatrix}$$

(a) Evaluating the Jacobian matrix at $(0, 0)$ we have that

$$J(0, 0) = \begin{bmatrix} r & aq\alpha \\ 0 & -(ap\alpha + c) \end{bmatrix}.$$

The eigenvalues are: $\lambda_1 = r > 0$ and $\lambda_2 = -(ap\alpha + c) < 0$, this way $(X^* = 0, Y^* = 0)$ is hyperbolic saddle

(b) $J(K, 0) = \begin{bmatrix} -r & -aq(K - \alpha) \\ 0 & ap(K - \alpha) - c \end{bmatrix}$. The eigenvalues are: $\lambda_1 = -r < 0$ and $\lambda_2 = ap(K - \alpha) - c$, then the sign of λ_2 depends on the sign of $ap(K - \alpha) - c$.

i.e.: hyperbolic saddle for $ap(K - \alpha) - c > 0$; an attractor point, for $ap(K - \alpha) - c < 0$

(c) For the unique equilibrium point at the first quadrant we get:

$$J(X_2^*, Y_2^*) = \begin{bmatrix} -(\frac{r(ap\alpha + c)}{apK} + aq(\frac{c}{a} + \alpha p)(Y_o^* + \frac{r(a-1)}{aq} + \frac{\alpha r}{qK})) & -qX_o^* \\ ap(\frac{c}{a} + \alpha p)(Y_o^* + \frac{r(a-1)}{aq} + \frac{\alpha r}{qK}) & 0 \end{bmatrix}.$$

The $Trace J(X_2^*, Y_2^*) = -(\frac{r(ap\alpha + c)}{apK} + aq(\frac{c}{a} + \alpha p)(Y_o^* + \frac{r(a-1)}{aq} + \frac{\alpha r}{qK})) < 0$ and $Det J(X_2^*, Y_2^*) = apqX_o^*(\frac{c}{a} + \alpha p)(Y_o^* + \frac{r(a-1)}{aq} + \frac{\alpha r}{qK}) > 0$.

Then (X_2^*, Y_2^*) node attractor.

□

We can find the relationship between the area that is clipped and the refuge functions so that it does not change the quantity of preys in the ecosystem $\frac{1}{a}X_o^* + \alpha = X_o^*$, therefore $\alpha = (1 - \phi)\frac{c}{p}$

(b) The system of linear equations when the area is modified from S to ϕS , stays the carrying capacity and the $X_r = \beta X$. The system that incorporates refuge function uses in proportion to the prey size is given by:

$$\begin{aligned} \frac{dX}{dt} &= r(1 - \frac{X}{K})X - qXY - (a-1)qXY + \beta aqY \\ \frac{dY}{dt} &= pXY - cY + (a-1)pXY - \beta apY \end{aligned} \quad (3)$$

The equilibrium points:

$$(0, 0), (K, 0) \text{ and } (X_3^*, Y_3^*) = (\frac{1}{a}X_o^* + \frac{\beta}{ap(1-\beta)}, \frac{1}{(1-\beta)^2}(\frac{1}{a}Y_o^* + \frac{r(a-1)(1-\beta)}{a^2q} - \frac{\beta r}{aq(1-\beta)^2}))$$

Theorem 2.2. *For the singularities of the system (3) one has:*

- (a) *The singularity $(0, 0)$ is saddle point for all parameter values.*
- (b) *The equilibrium point $(K, 0)$ is a saddle-node attractor (a nonhyperbolic equilibrium point) $ap(1 - \beta)K - c > 0$; globally asymptotically stable if and only if, $ap(1 - \beta)K - c < 0$; in this case it does not exist an equilibrium point at interior of the first quadrant.*
- (c) *The unique positive equilibrium point (X_3^*, Y_3^*) is globally asymptotically stable if and only if, $ap(1 - \beta)K - c > 0$; in this case the equilibrium $(K, 0)$ is saddle point.*

Proof. The Jacobian matrix of system (3) is

$$J(X; Y) = \begin{bmatrix} r(1 - \frac{2X}{K}) - aq(1 - \beta)Y & -aq(1 - \beta)X \\ ap(1 - \beta)Y & ap(1 - \beta)X - c \end{bmatrix}$$

- (a) Evaluating the Jacobian matrix at $(0, 0)$ we have that $J(0, 0) = \begin{bmatrix} r & 0 \\ 0 & -c \end{bmatrix}$. As $\text{Det}J(0, 0) = -rc < 0$, then $(0, 0)$ is saddle point

- (b) $J(K, 0) = \begin{bmatrix} -r & -aq(1 - \beta)K \\ 0 & ap(1 - \beta)K - c \end{bmatrix}$.

The eigenvalues are: $\lambda_1 = -r < 0$ and $\lambda_2 = ap(1 - \beta)K - c$, then the sign of λ_2 depends on the sign of $ap(1 - \beta)K - c$.

i.e.: hyperbolic saddle for $ap(1 - \beta)K - c > 0$ an attractor point for, $ap(1 - \beta)K - c < 0$

- (c) The Jacobian matrix at (X_3^*, Y_3^*)

$$J(X_3^*, Y_3^*) = \begin{bmatrix} -r & -aq(1 - \beta)X_3^* \\ ap(1 - \beta)Y_3^* & 0 \end{bmatrix}.$$

The $\text{Trace}J(X_3^*, Y_3^*) = -r < 0$ and $\text{Det}J(X_3^*, Y_3^*) = a^2qp(1 - \beta)^2X_3^*Y_3^* > 0$.

Then $(\frac{c}{ap}, r(\frac{a^2pK-c}{a^3pqK}))$ an attractor point.

□

We can find the relationship between the clipped area and the refuge function so that it doesn't change the quantity of preys in the equilibrium point in the quadrant with biological sense.

$\frac{1}{a}X_o^* + \frac{\beta}{ap(1-\beta)} = X_o^*$. Therefore $\beta = \frac{(a-1)c}{(a-1)c+1}$ With the same conditions it can be a relationship between α of $X_r = \alpha$ and β of $X_r = \beta X$. Obtaining that $\alpha = \frac{\phi\beta}{p(1-\beta)}$

- (c) The system of linear equations when the area is modified from S to ϕS , stays the carrying capacity and considering the refuge function $X_r = \frac{\alpha X}{X+\beta}$

The system is expressed by:

$$\begin{aligned}\frac{dX}{dt} &= r(1 - \frac{X}{K}) X - qXY - (a-1)qXY + \alpha a q \frac{XY}{X+\beta} \\ \frac{dY}{dt} &= pXY - cY + (a-1)pXY - \alpha ap \frac{XY}{X+\beta}\end{aligned}\quad (4)$$

The equilibrium points:

$$(0,0), (K,0) \text{ and } (X_4^*, Y_4^*) = (\frac{r}{2apK}(H + \sqrt{H^2 + 4\frac{acp\beta}{r^2}}); \frac{r}{2c}H - \frac{r^2}{2acpK}(H^2 + H\sqrt{H^2 + 4\frac{acp\beta}{r^2}} + 2\frac{acp\beta}{r^2}))$$

where

$$H = \frac{(\alpha-\beta)ap+c}{2apK} = \frac{(\alpha-\beta)+\phi X_0^*}{2K} \text{ and } \alpha - \beta > 0.$$

The following theorem considers that if the reason between the size of the refuge and the carrying capacity of the system spreads to $(1 + \frac{\beta}{K})(\frac{Kap-c}{r})$ (observes that the area portion that is when clipping the ecosystem appears), the equilibrium point inside the first quadrant can change its stability

Theorem 2.3. *The nature of the equilibrium points of the system (4)*

For all parameter values it has

- (a) *The singularity $(0,0)$ is saddle point.*
- (b) *$(K,0)$ saddle point, if and only if, $(1 + \frac{\beta}{K})(\frac{Kap-c}{r}) - \frac{\alpha}{K} > 0$; an attractor point, if and only if, $(1 + \frac{\beta}{K})(\frac{Kap-c}{r}) - \frac{\alpha}{K} < 0$ and an saddle-node attractor, if and only if, $(1 + \frac{\beta}{K})(\frac{Kap-c}{r}) - \frac{\alpha}{K} \leq 0$*
- (c) *For $(1 + \frac{\beta}{K})(\frac{Kap-c}{r}) - \frac{\alpha}{K} > 0$, the singularity (X_4^*, Y_4^*) one has*
 - (i) *If $\frac{\alpha}{K} = \frac{(X_4^* + \frac{\beta}{K})^2}{2X_4^* + \frac{\beta}{K} - 1}$, the system (4) has a unique limit cycle, surrounding the (X_4^*, Y_4^*) , unique equilibrium point at the first quadrant.*
 - (ii) *Unstable focus, if $\frac{\alpha}{K} > \frac{(X_4^* + \frac{\beta}{K})^2}{2X_4^* + \frac{\beta}{K} - 1}$*
 - (iii) *If $\frac{\alpha}{K} < \frac{(X_4^* + \frac{\beta}{K})^2}{2X_4^* + \frac{\beta}{K} - 1}$, the singularity (X_4^*, Y_4^*) of system (4) is a locally asymptotically stable equilibrium point.*

It is observed that the refuge function proposed by Almanza-Vásquez presents the best alternative to conserve the species, since it relates the size of the refuge, the carrying capacity and the land fraction that can take off and maintain the original stability conditions without the intervention.

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