The Ordered Weighted Geometric Averaging Algorithm to Multiple Attribute Decision Making within Triangular Fuzzy Numbers Based on the Mean Area Measurement Method

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Abstract

In this paper, the triangular fuzzy number is expressed by its Mean Area. So the Ordered Weighted Geometric Averaging Algorithm is used to solve multiple attribute decision making with attribute values within triangular fuzzy numbers based on the Mean Area Measurement Method.

Keywords: Mean area measurement method, Ordered Weighted Geometric Averaging Algorithm, Multiple attribute, Triangular fuzzy number

1 Introduction

In general, multiple attribute decision making is characterized by a decision maker, who is called to rank all the alternatives as well as select the best. In many
situations, attribute values are given in the form of triangular fuzzy number. So far a lot of research has been done to it, see [1-5]. In this paper, firstly Mean Area Measurement is adopted to express the corresponding triangular fuzzy number. Secondly the Ordered Weighted Geometric Averaging Algorithm (OWGA) is used to solve multiple attribute decision making with attribute values within triangular fuzzy numbers based on the Mean Area Measurement Method.

2 Decision making Method

Definition 1 \( \tilde{M} = [l, m, u] \) is called a triangular fuzzy number, and its membership function is the following:

\[
\mu_{\tilde{M}}(x) = \begin{cases} 
\frac{x-l}{m-l}, & x \in [l, m]; \\
\frac{u-x}{u-m}, & x \in [m, u]; \\
0, & \text{others.}
\end{cases}
\]

Here, \( 0 < l < m < u \) are real numbers, as in [1, 3, 4]. Triangular fuzzy numbers can express uncertain information very well.

For some \( \alpha \in [0,1] \), we denote \( \alpha \)–cut set of \( \tilde{M} \) as

\[
\tilde{M}_{\alpha} = [m'(\alpha), m''(\alpha)],
\]

So

\[
m(\tilde{M}_{\alpha}) = \frac{m'(\alpha) + m''(\alpha)}{2}
\]

is the mean of \( \alpha \)–cut set of \( \tilde{M} \)

Definition 2 \( s(\tilde{M}) = \int_{0}^{h_\alpha} m(\tilde{M}_{\alpha})d\alpha \) is called the Mean Area of \( \tilde{M} \).

Here, \( h_\alpha \) is the height of \( \tilde{M} \).

For any triangular fuzzy number \( \tilde{M} = [l, m, u] \), according to the definition of the Mean Area of \( \tilde{M} \), we can get

\[
s(\tilde{M}) = \frac{(l+2m+u)}{4}
\]

Definition 3 let \( \text{OWGA} : R^n \rightarrow R^* \), if

\[
\text{OWGA}_\omega(\alpha_1, \alpha_2, \cdots, \alpha_n) = \prod_{j=1}^{n} b_j^{\alpha_j},
\]

where \( \omega=(\omega_1, \omega_2, \cdots, \omega_n)^T \) is the exponential weighted vector according to
**Ordered weighted geometric averaging algorithm**

**OWGA**, \( \omega_j \in [0,1], j \in N \), and \( \sum_{j=1}^{n} \omega_j = 1 \). and \( b_j \) is the \( j \)th number of \((\alpha_1, \alpha_2, \ldots, \alpha_n)\), \( R^+ \) is the set of all positive real numbers, then **OWGA** is called as the Ordered Weighted Geometric Averaging Algorithm operator.

So OWGA can be adopted to solve multiple attribute decision making with attribute values within triangular fuzzy numbers based on the Mean Area Measurement Method.

### 3 Decision Making Steps

**Step 1** Replace each triangular fuzzy number \( \tilde{M} = [l, m, u] \) of the original decision matrix with its Mean Area \( s(\tilde{M}) \), so the original matrix is transformed into the matrix composed by their Mean Area values \( A = (a_{ij}) \), here \( a_{ij} = s(\tilde{M}) \).

**Step 2** Let \( I_1, I_2 \) represent the subscript sets of the benefit type and the cost benefit attributes. Standard \( A = (a_{ij}) \) into standardization matrix \( R = (r_{ij}) \) as:

\[
r_{ij} = \frac{a_{ij}}{\max(a_{ij})}, \quad j = 1, 2, \ldots, n; \quad j \in I_1
\]

\[
r_{ij} = \frac{\min(a_{ij})}{a_{ij}}, \quad i = 1, 2, \ldots, n; \quad j \in I_2
\]

**Step 3** Obtain \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) according to the importance of each attribute, and **OWGA** can be adopted to get overall value for each alternative

\[
z_i(\omega) = \text{OWGA}_\omega (r_{i1}, r_{i2}, \ldots, r_{in}) = \prod_{j=1}^{n} b_j^{\omega_j}
\]

**Step 4** Rank the alternatives and select the best by \( z_i(\omega) \).

### 4 Illustrative Example

Consider the following example. We will evaluate four textbooks \( x_i (i = 1, 2, 3, 4) \) against four attributes:

- \( u_1 \): Difficult level,
- \( u_2 \): Novelty level,
- \( u_3 \): Price,
- \( u_4 \): Printing quality

And the evaluating results are shown in Table 1 with attribution values in the form of triangular fuzzy numbers, where \( u_1, u_2 \) and \( u_4 \) are benefit attributes, and \( u_3 \) is cost attribute. Which textbook is the best?
TABLE 1. Decision Making Matrix Table

<table>
<thead>
<tr>
<th></th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>[7,8,10]</td>
<td>[5,6,8]</td>
<td>[7,8,9]</td>
<td>[5,7,8]</td>
</tr>
<tr>
<td>$x_2$</td>
<td>[5,8,10]</td>
<td>[4,5,7]</td>
<td>[4,6,7]</td>
<td>[6,8,10]</td>
</tr>
<tr>
<td>$x_3$</td>
<td>[6,7,9]</td>
<td>[6,7,8]</td>
<td>[7,8,9]</td>
<td>[6,8,9]</td>
</tr>
<tr>
<td>$x_4$</td>
<td>[6,8,9]</td>
<td>[6,7,8]</td>
<td>[6,8,10]</td>
<td>[5,7,9]</td>
</tr>
</tbody>
</table>

**Step 1** Replace each attribute value of triangular fuzzy number of $\tilde{M}=[a_l, a_m, a_u]$ in the original decision matrix in Table 1 with its Mean Area and get the matrix composed by the Mean Area,

$$A=(a_j) = \begin{pmatrix} 8.25 & 6.25 & 7.75 & 6.75 \\ 7.75 & 5.25 & 5.75 & 8 \\ 7.25 & 7 & 8 & 7.75 \\ 7.75 & 7 & 8 & 7 \end{pmatrix}$$

**Step 2** Calculate the standardization matrix by (1) and (2)

$$R = (r_j) = \begin{pmatrix} 1 & 0.8929 & 0.7419 & 0.8438 \\ 0.9394 & 0.75 & 1 & 1 \\ 0.8788 & 1 & 0.7188 & 0.9688 \\ 0.9394 & 1 & 0.7188 & 0.8750 \end{pmatrix}$$

**Step 3** Here let $\omega=(0.3,0.4,0.2,0.1)$ based on the importance of the four attributes and calculate $z_i(\omega)$ by (3)

$$z_1(\omega)=0.8851, \ z_2(\omega)=0.9596, \ z_3(\omega)=0.9310, \ z_4(\omega)=0.9188,$$

**Step 4** Utilize $z_i(\omega)$ to rank the alternatives: $x_2 \succ x_3 \succ x_4 \succ x_1$, and thus the best alternative is $x_2$, so the 2th textbook is the best.

**References**


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