Temperature Distribution Profiles during Vertical Continuous Casting Process

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Abstract

A vertical continuous casting model is considered to analysis the temperature distribution profiles during the casting process. Heat flux is applied at the interface between the liquid layer and the solid layer to overcome the deficient of insufficiency boundary conditions. By combining the general solution of time dependent heat transfer partial differential equation and heat flux at the interface generate linear systems, and linear systems determine the temperature distribution profiles at each layer. The width of liquid layer and enough time interval are important factors in controlling the casting process and yield profound influence on the mechanisms.

Keywords: Casting, Heat flux, Linear system, Temperature distribution profiles

1 Introduction

The determination of the temperature distribution profiles in the industrial continuous casting of metals and metal alloy has been an important stage to investigate intensively failure mechanisms occurring during the casting process. A schematic of vertical continuous casting is given in Fig. 1. Mirshahi [1] investigated
the tensile properties on high temperature for in situ composites. The variation of temperature influences on the tensile properties, and the increase of temperature decreases greatly the tensile strength. The effects of temperature on fracture behavior were studied by Li et al. [2]. Ultimate tensile strengths of the composites first increase then decrease with the increase of temperature, and the interface debonding undergoes the dominant fracture mechanism with increasing temperature.

The casting temperature is a crucial factor to determine the microstructure and mechanical properties of an in-situ bulk metallic glass matrix composite [3,4]. The behavior of bulk metallic glasses (BMGs) is investigated by Zhu et al. [3]. They used different casting temperature and announced that the microstructures of BMGs are closely affected by the casting temperatures. Sha et al. [4] studied the effects of casting temperature on the microstructure and mechanical properties of an in-situ bulk metallic glass matrix composite using X-ray diffraction. The partial crystallization of the amorphous matrix and the variation of the morphology of the crystalline dendrites were influenced by the cooling rate and oxygen content.

Moreover, casting and mould temperature provided significant influence on the base material [5,6]. A reduction of the mould temperature leads to a decrease in both the average dendrite arm space and the average grain size. Fischer et al. [7] studied the influence of the casting and mould temperatures on the microstructure and compression behaviour of investment-cast open-pore aluminium foams. They found that a decreased mould temperature and an increased casting temperature lead to increased mechanical properties during the static compression testing of open-pore 10 and 15 pores per inch A356 foams. However, it is difficult to, in most of previous works, find technical methods to determine detailed temperature distribution profiles during casting process for composite materials. In present study, heat flux at the interface is combined to heat equation and a linear system is obtained to clarify temperature distribution profiles for each layer. Temperature distribution profiles are displayed for various radius, longitudinal, and time variables. The results performed here contribute further understanding to the failure mechanisms during casting process.

### 2 Mathematical Modelling

The partial differential equation for temperature distribution profile is

\[
\frac{\partial T}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2},
\]  

(1)

where the \( \tau \) is time variable (see Fig. 1).
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For the steady state, Eq (1) is reduced to
\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = 0, \tag{2} \]
and the general solution is
\[ T(r, z) = C_1 J_0 (\sqrt{\lambda} r) e^{-\sqrt{\lambda} z} + C_2 Y_0 (\sqrt{\lambda} r) e^{-\sqrt{\lambda} z}, \tag{3} \]
where \( J \) and \( Y \) are Bessel functions of order *, \( \lambda \) is an unknown scalar, and \( C_1 \) and \( C_2 \) are integral constants.

(i) Along the radial direction \( r \)

For the fixed \( z = z_0 \), the temperature distribution profile based on the boundary conditions below
\[ T(r_c, z_0) = T_c(z_0), \quad T(r_b, z_0) = T_b(z_0) \]
is
\[ T(r, z_0) = \frac{e^{\pi i [J_0 (\sqrt{\lambda} r) - Y_0 (\sqrt{\lambda} r)]}}{J_0 (\sqrt{\lambda} r) Y_0 (\sqrt{\lambda} r) - J_0 (\sqrt{\lambda} r) Y_0 (\sqrt{\lambda} r)} J_0 (\sqrt{\lambda} r) e^{-\sqrt{\lambda} z_0} + \frac{e^{\pi i [J_0 (\sqrt{\lambda} r) - Y_0 (\sqrt{\lambda} r)]}}{J_0 (\sqrt{\lambda} r) Y_0 (\sqrt{\lambda} r) - J_0 (\sqrt{\lambda} r) Y_0 (\sqrt{\lambda} r)} Y_0 (\sqrt{\lambda} r) e^{-\sqrt{\lambda} z_0} \tag{4} \]

(ii) Along the longitudinal direction \( z \)

At the fixed point \( r = r_0 \), the boundary conditions to determine integral constants for each layer are given below;
\[ T(r_0, z_1) = T_l(r_0), \quad T(r_0, z_2) = T_3(r_0), \quad T(r_0, z_3) = T_3(r_0). \]
However, since only two boundary values \( T_l(r_0) \) and \( T_3(r_0) \) are known, additional necessary information are required to obtain a unique temperature distribution profile for each layer. Taking into consideration heat flux at the interface layer point, the equations for the layer is expressed as
where \( q_{\text{Liq}} \) and \( q_{\text{Sol}} \) are the heat flux of the liquid and the solid layer, respectively, \( k_{\text{Liq}} \) is the conductivity and \( L_{\text{Liq}} \) is the length of the liquid layer. Then, the integral constants for cylinder temperature distribution profile at the liquid layer can be determined by solving the following linear system:

\[
\begin{align*}
C_{\text{Liq}} J_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_1} + C_{\text{Liq}} Y_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_1} &= T_1(r_0) \\
C_{\text{Liq}} J_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_2} + C_{\text{Liq}} Y_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_2} - T_2(r_0) &= 0
\end{align*}
\]

\[
q_{\text{Liq}} + \frac{k_{\text{Liq}}}{L_{\text{Liq}}} T_2(r_0) = \frac{k_{\text{Liq}}}{L_{\text{Liq}}} T_1(r_0)
\]

\[
q_{\text{Liq}} - q_{\text{Sol}} = 0
\]

\[
q_{\text{Sol}} - \frac{k_{\text{Sol}}}{L_{\text{Sol}}} T_2(r_0) = - \frac{k_{\text{Sol}}}{L_{\text{Sol}}} T_3(r_0)
\]

under the constraints

\[
\begin{align*}
C_{\text{Liq}} J_0(\sqrt{\lambda} r_0) + C_{\text{Liq}} Y_0(\sqrt{\lambda} r_0) &= T_1(r_0) e^{\sqrt{\lambda} z_1} \\
T_2(r_0) e^{\sqrt{\lambda} z_2} &= T_1(r_0) e^{\sqrt{\lambda} z_1} \quad (7)
\end{align*}
\]

The integral constants for the solid layer can be obtained by solving the linear system below:

\[
\begin{align*}
C_{\text{Sol}} J_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_1} + C_{\text{Sol}} Y_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_1} &= T_2(r_0) \\
C_{\text{Sol}} J_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_2} + C_{\text{Sol}} Y_0(\sqrt{\lambda} r_0) e^{-\sqrt{\lambda} z_2} &= T_3(r_0)
\end{align*}
\]

\[
C_{\text{Sol}} J_0(\sqrt{\lambda} r_0) + C_{\text{Sol}} Y_0(\sqrt{\lambda} r_0) = T_2(r_0) e^{\sqrt{\lambda} z_2}
\]

\[
T_3(r_0) e^{\sqrt{\lambda} z_3} = T_3(r_0) e^{\sqrt{\lambda} z_3} \quad (9)
\]

2.2. Temperature distribution for the time dependent state

Due to the separation variables as

\[ T = H(r) R(r) Z(z), \]

the general solution for the time dependent PDE(1) is

\[ T = K_1 H_0 J_0(\sqrt{\mu + \lambda} r) e^{-\mu r - \sqrt{\lambda} z} + K_2 H_0 Y_0(\sqrt{\mu + \lambda} r) e^{-\mu r - \sqrt{\lambda} z}. \quad (10) \]

The \( H_0 \) represents the initial temperature at the point \( (\tau = 0, \ z = 0, \ r = r_0) \), and \( K_1 \) and \( K_2 \) are integral constants (see Appendix). At the fixed variables \( \tau = \tau_0 \) and \( z = z_0 \) the temperature distribution profile along the radial direction \( r \) is
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\[ T(\tau_0, r, z_0) = \frac{e^{\mu c - z_0^2} \left[ J_0(\sqrt{\mu + \lambda r_0}) - T_1(\sqrt{\mu + \lambda r_0}) \right] + e^{\mu c + z_0^2} \left[ J_0(\sqrt{\mu + \lambda r_0}) - T_1(\sqrt{\mu + \lambda r_0}) \right]}{J_0(\sqrt{\mu + \lambda r_0})Y_0(\sqrt{\mu + \lambda r_0}) - J_0(\sqrt{\mu + \lambda r_0})Y_0(\sqrt{\mu + \lambda r_0})} J_0(\sqrt{\mu + \lambda r_0})e^{-\mu c - z_0^2}, \]  

which is obtained according to the boundary conditions

\[ T(\tau_0, r, z_0) = T_1(\tau_0, z_0), \quad T(\tau_0, r_0, z_0) = T_0(\tau_0, z_0). \]

The temperature distribution profiles along the longitudinal direction \( z \), at the fixed point \( \tau = \tau_0 \) and \( r = r_0 \), are determined through the process above for the given boundary conditions;

\[ T(\tau_0, r_0, z_1) = T_1(\tau_0, r_0), \quad T(\tau_0, r_0, z_2) = T_2(\tau_0, r_0), \quad T(\tau_0, r_0, z_3) = T_3(\tau_0, r_0) \]

and the constraints;

\[ C_{Liq}J_0(\sqrt{\mu + \lambda r_0}) + C_{Liq}Y_0(\sqrt{\mu + \lambda r_0}) = T_1(\tau_0, r_0)e^{\mu r_0 + z_1^2}; \]

\[ T_2(\tau_0, r_0)e^{z_2^2} = T_1(\tau_0, r_0)e^{z_1^2} \]

\[ C_{Sol}J_0(\sqrt{\mu + \lambda r_0}) + C_{Sol}Y_0(\sqrt{\mu + \lambda r_0}) = T_3(\tau_0, r_0)e^{\mu r_0 + z_2^2}; \]

\[ T_2(\tau_0, r_0)e^{z_2^2} = T_3(\tau_0, r_0)e^{z_3^2}. \]

The conductivities 39.2 w/mK and 33.8 w/mK, respectively, for liquid and solid layers are applied to display temperature distribution profiles.

3. Results and Discussion

Temperature distribution profiles during the casting process are described based on the method developed in section 2. The analysis of the temperature distribution for the vertical continuous casting fills with the boundary conditions

\[ T_1(\tau_0, z_0) = R_1 = 1650 \quad \tau_0 \quad T_0(\tau_0, z_0) = R_0 = 1151. \]

The constant \( \mu \) is determined by considering enough cooling time \( \tau = \tau^* \), that is,

\[ H(\tau^*) = H_0e^{-\mu \tau^*} = RT \Rightarrow \mu = \frac{1}{\tau^*} \ln\left(\frac{H_0}{RT}\right). \]

The \( RT \) represents the room temperature, and \( H_0 = 1650^\circ C, \quad RT = 20^\circ C \) and \( \tau^* = 36000 \) are chosen.

The temperature distribution profiles along the radius at \( \tau = 0 \) are shown in Fig. 2. Temperature decreases exponentially as the radius increases at \( z = 0 \) (see Fig. 2(a)), and as shown in Fig. 2(b) the decreasing rate of the temperature is getting smaller according to the increase of the value \( z \). The phenomenon is due to the temperature difference label between the center and boundary of the vertical continuous casting system. Fig. 3 illustrates the temperature distribution profiles along the \( z \)-axis for various \( r \) at \( \tau = 0 \). Likewise Fig. 2, exponential decrease
occurs in temperature distribution profiles as the value \( z \) increases. Smaller rate of decreasing variation is appeared, owing to the initial temperature, as the radius increase.

\[ \tau = 0 \quad z = 0 \quad Rc = 1650 \, ^{\circ}C \quad Rb = 1151 \, ^{\circ}C \]

**Figure 2.** Temperature distribution profile: (a) along the radius at \( z = 0 \) and \( \tau = 0 \), (b) along the radius for various \( z \) at \( \tau = 0 \).

**Figure 3.** Temperature distribution profiles along the \( z \)-axis for various \( r \) at \( \tau = 0 \).

Fig. 4 represents the isothermal temperature distribution profiles at \( \tau = 0 \) for the representative temperature \( T = 1100 \), \( T = 600 \), and \( T = 300 \). The temperature decreases exponentially near the center, and the rate of change is getting slower along the radius. The graph is symmetric to the center and the distribution range is wider for higher temperature degree. At \( z = 0 \) and \( \tau = 0 \) the temperature distribution profiles is shown in Fig. 5 for various liquid layers \( z_2 = 70 \), \( z_2 = 50 \), \( z_2 = 30 \), and \( z_2 = 20 \). For the widest liquid layer \( z_2 = 70 \) the rate of temperature change is smallest, while the largest decrease is appeared the liquid layer \( z_2 = 20 \). The phenomenon is due to the narrow liquid layer for the same boundary condition, implying that enough liquid layer is necessary to reduce the failure mechanisms for the output.
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Fig. 4. Isothermal temperature distribution profiles at $\tau = 0$.

Fig. 5. Temperature distribution profiles along the radius for various $z_2$ at $\tau = 0$.

Fig. 6 presents the temperature distribution profiles for representative time values $\tau = 0$, $\tau = 600$, $\tau = 1200$, and $\tau = 3600$.

Fig. 6(a) is depicted along the radius for the chosen $z = 20$. The temperature decreases as time value increases. The patterns of temperature decrease are similar for each chosen time. Temperature distribution profiles along the $z$-axis at $r = 0.5$ is displayed in Fig. 6(b). The temperature decrease exponentially along the $z$-axis and the rate of temperature decrease is getting shaper for the lower time value.

4. Conclusion

Temperature distribution profiles during the casting process are analyzed considering time variable. Exponential decrease developed along the radius and value $z$ in the temperature distribution profiles. Isothermal line of the temperature is
symmetric to the radius and the rate of change is getting slower along the radius. Similar phenomenon occurred in time value variable. For the lower time value the rate of temperature decrease is getting shaper with exponential decay. Thoughgout the above resultst the width of liquid layer and enough time interval influence sensitively to the mechanisms, and are crucial factors to be considered in controlling the casting process. The results and the analysis performed here contribute further understanding to the behaviors of casting.

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References


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Appendix
The time dependent partial differential equation for temperature distribution profile is

$$\frac{\partial T}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{\partial z^2}. \quad (A1)$$

Due to the separation variables as for Eq(A1)

$$T = H(\tau)R(r)Z(z)$$

the partial differential equation becomes

$$\frac{H'(\tau)}{H(\tau)} = \frac{1}{rR(r)} \frac{d}{dr} \left( r \frac{dR(r)}{dr} \right) + \frac{1}{Z(z)} \frac{d^2Z(z)}{dz^2}. \quad (A2)$$

The l.h.s. depends on $\tau$ and the r.h.s. on $(r, z)$ in Eq(A2), so that both sides equal a constant $-\mu$ (the separation constant)

$$\frac{H'(\tau)}{H(\tau)} = \frac{1}{rR(r)} \frac{d}{dr} \left( r \frac{dR(r)}{dr} \right) + \frac{1}{Z(z)} \frac{d^2Z(z)}{dz^2} = -\mu. \quad (A3)$$

Thus, for $\mu > 0$,

$$H(\tau) = H_0 e^{-\mu \tau}$$

For some constant $\lambda$, the setting

$$\frac{1}{Z(z)} \frac{d^2Z(z)}{dz^2} = \lambda, \quad (A4)$$

yields the solution

$$Z(z) = C_1 e^{z \sqrt{\lambda}} + C_2 e^{-z \sqrt{\lambda}}, \quad (A5)$$

where the $C_1$ and $C_2$ represent the integral constants. By the decreasing of the temperature as $z$ increases the solution (A5) is reduced to

$$Z(z) = C_1 e^{-z \sqrt{\lambda}}$$

The equation for $R$ is then, on rearranging,

$$r^2 \frac{d^2R(r)}{dr^2} + r \frac{dR(r)}{dr} + r^2 (\mu + \lambda)R(r) = 0, \quad (A6)$$

and the general solution is

$$R(r) = C_3 J_0(\sqrt{\mu + \lambda} r) + C_4 Y_0(\sqrt{\mu + \lambda} r), \quad (A7)$$

where $J_*$ and $Y_*$ are Bessel functions of order $\ast$, and $C_3$ and $C_4$ are integral constants. Therefore, the general solution for the time dependent temperature distribution profiles is

$$T = K_1 H_0 J_0(\sqrt{\mu + \lambda} r)e^{-\mu \tau - z \sqrt{\lambda}} + K_2 H_0 Y_0(\sqrt{\mu + \lambda} r)e^{-\mu \tau - z \sqrt{\lambda}}, \quad (A8)$$

where $K_1$ and $K_2$ are integral constants.

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