Sequence of Fuzzy Topographic Topological Mapping and k -Fibonacci Sequence

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Abstract

Fuzzy Topological Topographic Mapping (*FTTM*) is a model for solving neuromagnetic inverse problem. *FTTM* consists of four components and connected by three algorithms. *FTTM* version 1 and *FTTM* version 2 were designed to present 3D view of an unbounded single current and bounded multicurrent sources, respectively. In 2008, Suhana proved the conjecture posed by Liau in 2005 such that if there exist n number of *FTTM*, then $n^4 - n$ new elements of *FTTM* will be generated from it. In this paper, a new proof of sequence of *FTTM* is presented using k -Fibonacci sequence.

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1 Introduction

Generally, $FTTM$ [1–5]. is a 4-tuple of topological spaces which are homeomorphic to each other and can be represented as

$$FTTM = \{(M, B, F, T) : M \cong B \cong F \cong T\}$$

The exact arrangement for the sequential elements of $FTTM$ [6] is presented in Figure 1.

Figure 1: Sequence of $FTTM_n$

Yun [4] noticed that if there are two elements of $FTTM$ that are homeomorphic to each other component-wise, it would generate more homeomorphisms. The number of generating new elements of $FTTM$ is

$$\left[\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \right] - 2 = 14 \text{ elements} \quad (1)$$

Consequently, Yun proposed a conjecture such that, if there exist n elements of $FTTM$, then the number of new elements are $n^4 - n$ [4]. Jamaian et. al proved the conjecture in [6] and introduced several new concepts in order to achieve it. These concepts are presented as follows.

2 Some Definitions on Sequence of FTTM

The following definitions are utilized from [7].

Definition 2.1 *Sequence of FTTM [7]*

$FTTM_1, FTTM_2, FTTM_3$ and $FTTM_4$ are presented respectively in Figure 2. $FTTM_1$ can be viewed as a square whereby MC , BM , FM and TM are its vertices and the homeomorphisms are its edges. $FTTM_1$ has 4 vertices and 4 edges. $FTTM_2$ contains 8 vertices, 12 edges, 6 faces and 1 cube. Generally a cube is a combination of two sets of $FTTM$. $FTTM_3$ consists of 12 vertices,

24 edges, 15 faces and 3 cubes. $FTTM_4$ has 16 vertices, 28 edges, 16 faces and 6 cubes. Consequently, some patterns for the number of vertices, edges, faces and cubes are observed from sequences of $FTTM$ as listed in Table 1.

Figure 2: (a): $FTTM_1$, (b): $FTTM_2$, (c): $FTTM_3$, (d): $FTTM_4$

Definition 2.2 Sequence of Vertices of $FTTM_n$ [7]

The sequence for the number of vertices in $FTTM_n$ which is $vFTTM_n$ is defined recursively by the equation

$$vFTTM_n = vFTTM_{n-1} + 4 \text{ for } n \geq 1 \text{ and } vFTTM_0 = 0. \quad (2)$$

Definition 2.3 Sequence of Edges of $FTTM_n$ [7]

The sequence of edges for the number of edges $FTTM_n$ which is $eFTTM_n$ is defined recursively by equation

$$eFTTM_n = eFTTM_{n-1} + 8 \text{ for } n > 1 \text{ and } eFTTM_1 = 4. \quad (3)$$

Definition 2.4 Sequence of Faces of $FTTM_n$ [7]

The sequence for the number of faces for $FTTM_n$ which is $fFTTM_n$ is defined recursively by the equation

$$fFTTM_n = fFTTM_{n-1} + 5 \text{ for } n > 1 \text{ and } fFTTM_1 = 1. \quad (4)$$

A cube is a combination of two or more $FTTM$ in $FTTM_n$ (Figure 3) [7]. A cube of $FTTM$ is denoted by $FTTM_{k/n}$ where k is a combination of k number of terms in $FTTM_n$. The sequence of cubes i.e, $FTTM_{2/n}$ begins with the integers 0, 1, 3, 6, 10, 15, 21,...and so forth. Cubes of $FTTM$; $FTTM_{2/n}$ is defined as follows.

Definition 2.5 [7] Sequence of cubes for $FTTM_{2/n}$ is given recursively by equation

$$FTTM_{2/n} = \frac{n(n-1)}{2!}. \quad (5)$$

Figure 3: k-FTTM

Cubes of $FTTM$ also exist from the combination of three $FTTM$ in $FTTM_n$ [7]. The sequence of cubes produced from the combination of three $FTTM$ i.e, $FTTM_{3/n}$ begins with integers 0, 0, 1, 4, 10, 20, 35, 56,...and so forth. $FTTM_{3/n}$ is defined as follows.

Definition 2.6 [7] The sequence of cubes for $FTTM_{3/n}$ is given by equation

$$FTTM_{3/n} = \frac{n(n-1)(n-2)}{3!}. \quad (6)$$

Cubes of $FTTM$ are generated from the combination of four $FTTM$ in $FTTM_n$ [6]. The sequence of cubes produced from the combination of four $FTTM$ i.e, $FTTM_{4/n}$ begin with integers 0, 0, 0, 1, 5, 15, 35, 70,...and so forth. $FTTM_{4/n}$ is defined as follows.

Definition 2.7 [7] The sequence of cubes for $FTTM_{4/n}$ is given by equation

$$FTTM_{4/n} = \frac{n(n-1)(n-2)(n-3)}{4!} \quad (7)$$

Definition 2.8 Cubes of $FTTM_{k/n}$

In general, a cube of $FTTM_{k/n}$ defines a combination of k number of $FTTM$ in $FTTM_n$, namely (see Figure 3)

$$FTTM_{k/n} = \begin{cases} \frac{n(n-1)(n-2)(n-3) \cdots (n-k+1)}{k!} & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases} \quad (8)$$

Table 1 lists the number of vertices, edges, faces and cubes for some sequences of FTTM [6]

Table 1: Vertices, Edges, Faces and Cubes for Sequences of $FTTM$ for $n = 1, 2, 3, \dots, 10$

$FTTM_n$	Vertices	Edges	Faces	$FTTM_{2/n}$	$FTTM_{3/n}$	$FTTM_{4/n}$
$FTTM_1$	4	4	1	0	0	0
$FTTM_2$	8	12	6	1	0	0
$FTTM_3$	12	20	11	3	1	0
$FTTM_4$	16	28	16	6	4	1
$FTTM_5$	20	36	21	10	10	5
$FTTM_6$	24	44	26	15	20	15
$FTTM_7$	28	52	31	21	35	35
$FTTM_8$	32	60	36	28	56	70
$FTTM_9$	36	68	41	36	84	126
$FTTM_{10}$	40	76	46	45	120	210

3 Alternative Proof on Sequence of FTTM

Proofs for sequences of FTTM can become tedious as it involves large sequence of numbers. The most efficient way to prove sequences of FTTM is by using constructive method [8]. This method required one to develop geometrical features for all sequences of FTTM as presented in Section 1. A mathematical model [9] for n dimension of sequences of FTTM can be obtained using difference equations [10].

Theorem 3.1 *The sequence of number of edges in $FTTM_n$; $eFTTM_n$ can be represented as*

$$eFTTM_n = 8n - 4$$

Proof of Theorem 3.1

Recall Definition 3, where the sequence for edges is presented as

$$eFTTM_n = eFTTM_{n-1} + 8$$

Equivalently,

$$eFTTM_{n+1} - eFTTM_n = 8 \quad (9)$$

Equation (9) can be viewed as a non-homogenous ordinary difference equation in the form of

$$eFTTM_n = S_n + T_n \quad (10)$$

with S_n is the general solution, as such

$$S_{n+1} - S_n = 0 \quad (11)$$

and T_n is particular solution such that

$$T_{n+1} - T_n = 8 \quad (12)$$

Equation (11) can be solved by finding the related polynomial for equation (9). In this case, the related polynomial for (9) is

$$\begin{aligned} P(r) &= r - 1 \\ &= 0 \end{aligned} \quad (13)$$

which implies $r = 1$. The solution for (11) is given by $S_n = Ar$

$$\therefore S_n = A \quad (14)$$

since $r = 1$. To get solution for (12), first let

$$T_n = Bn$$

Substituting T_n into (12) will gives

$$\begin{aligned} T_{n+1} - T_n &= 8, \\ B(n+1) - (Bn) &= 8, \\ B &= 8 \\ \therefore T_n &= 8n \end{aligned} \quad (15)$$

From (13) and (15), the solution for (10) can be written as

$$eFTTM_n = A + 8n$$

From Table 1, initial values for $eFTTM_1 = 4$ which implies $A = -4$.

$$\therefore eFTTM_n = -4 + 8n$$

■

Theorem 3.2 *The sequence for the number of faces of $FTTM_n$; $fFTTM_n$ can be represented as*

$$fFTTM_n = 5n - 4$$

Proof of Theorem 3.2

Recall Definition 4, where the sequence of faces can be defined as

$$fFTTM_n = fFTTM_{n-1} + 5$$

Equivalently,

$$fFTTM_{n+1} - fFTTM_n = 5 \quad (16)$$

Similarly, Equation (16) can be viewed as a non-homogenous ordinary difference equation as follows

$$fFTTM_n = U_n + V_n \quad (17)$$

with U_n is the general solution

$$U_{n+1} - U_n = 0 \quad (18)$$

and V_n is the particular solution such that

$$V_{n+1} - V_n = 5 \quad (19)$$

Equation (18) can be solved by finding the related polynomial for equation (16). In this case, the related polynomial for (16) is

$$\begin{aligned} P(r) &= r - 1 \\ &= 0 \end{aligned} \quad (20)$$

which implies $r = 1$. Solution for (18) is given by $U_n = Ar$.

$$\therefore U_n = A \quad (21)$$

To get solution for (19), first let

$$V_n = Bn$$

Substituting V_n into (19) will gives

$$\begin{aligned} V_{n+1} - V_n &= 5 \\ (Bn + 1) - (Bn) &= 5 \\ B &= 5 \\ \therefore V_n &= 5n \end{aligned} \quad (22)$$

from (20) and (22), the solution for (17) can be written as

$$fFTTM_n = A + 5n$$

From the Table 1, initial values for $FTTM_1 = 1$ which implies $A = -4$.

$$\therefore fFTTM_n = -4 + 5n$$

■

4 FTTM as a Pascal Triangle

Jamaian [7] found that there exist a relation on sequence of $FTTM$ and Pascal's Triangle. She showed that the sequence of $FTTM_{2/n}$, $FTTM_{3/n}$, and $FTTM_{4/n}$ are presented in the third, the fourth, and the fifth main diagonals of Pascal's Triangle, respectively as shown in Figure 4

Figure 4: *FTTM* in Pascal's Triangle

5 New approach using *k-Fibonacci* sequence

The number of cubes produces by the combination of any three terms of *FTTM* in $FTTM_n$ is presented in Table 2.

Table 2: Sequence of $FTTM_{3/n}$

$cFTTM_n$	$FTTM_{3/n}$
n=1	0
n=2	0
n=3	1
n=4	4
n=5	10
n=6	20
n=7	35
n=8	56

A generalization for the sequence based on Table 2 is not obvious since n tends to be very large. One way to overcome this problem is by using extended Fibonacci Cubes [11] which has resemblance to sequence of cubes of *FTTM*. A *k-Fibonacci* sequence [12–14] can also be used to obtain cubes of *FTTM*.

Fibonacci numbers are the terms of the sequence $\{0, 1, 1, 2, 3, \dots\}$ wherein each term is the sum of the two previous term [15, 16]. Generally, Fibonacci

numbers [17] $F_0, F_1, F_2, F_3, \dots$, are designed recursively by the equations ,

$$F_n = \begin{cases} 0 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ F_{n-1} + F_{n-2} & \text{if } n > 1. \end{cases}$$

In general, a k -Fibonacci sequence [18] is defined recursively by the equation

$$F_{k,n} = \begin{cases} 0 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ kF_{k,n} + F_{n-1} & \text{if } n > 1. \end{cases}$$

Falcon et. al [16] introduced a formula for the general term of the k -Fibonacci sequence as follows

$$F_{k,n} = \sum_{i=0}^{\frac{n-1}{2}} \left[\binom{n-1-i}{i} k^{n-1-2i} \right] \text{ for } n \geq 2. \quad (23)$$

From equation (23), Definition 2.5, Definition 2.6, and Definition 2.7, some new theorems on cubes of $FTTM$ are deduced.

Theorem 5.1 *The number of cubes produced by the combination of any two terms in $FTTM_n$; $FTTM_{2/n}$ can be represented as*

$$FTTM_{2/n} = \sum_{i=2}^n \left[\binom{n+2-i}{i} - \binom{n+1-i}{i+1} \right] = \frac{n(n-1)}{2} \text{ for } n \geq 2 \quad (24)$$

Proof of Theorem 5.1

Recall equation (23) and let $k=1$,

$$\begin{aligned}
 F_{1,5} &= \binom{4}{0} + \binom{3}{1} + \binom{2}{2} \\
 F_{1,6} &= \binom{5}{0} + \binom{4}{1} + \binom{3}{2} \\
 F_{1,7} &= \binom{6}{0} + \binom{5}{1} + \binom{4}{2} + \binom{3}{3} \\
 F_{1,8} &= \binom{7}{0} + \binom{6}{1} + \binom{5}{2} + \binom{4}{3} \\
 F_{1,9} &= \binom{8}{0} + \binom{7}{1} + \binom{6}{2} + \binom{5}{3} + \binom{4}{4} \\
 &\vdots \\
 F_{1,n} &= \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \binom{n-4}{3} + \binom{n-5}{4} + \cdots
 \end{aligned} \tag{25}$$

Notice that the number of cubes produced by the combination of any two terms in $FTTM_n$; $FTTM_{2/n}$ match the third term of $F_{1,n}$ such that $F_{1,n}^{(i)}$ where the i th coefficient in the expression $F_{1,n}$ is a polynomial on k , with

$$F_{1,n}^{(i)} = \binom{n-1}{i-1} \tag{26}$$

We can prove Theorem 1 by mathematical induction. Let

$$P(n) = FTTM_{2/n} = \sum_{i=2}^n \left[\binom{n+2-i}{i} - \binom{n+1-i}{i+1} \right] = \frac{n(n-1)}{2}; n \geq 2. \tag{27}$$

For $n = 2$,

$$\begin{aligned}
 FTTM_{2/2} &= \frac{(2)(2-1)}{2} \\
 &= 1
 \end{aligned} \tag{28}$$

However

$$\begin{aligned}
 FTTM_{2/2} &= \binom{2+2-2}{2} - \binom{2+1-2}{2+1} \\
 &= \binom{2}{2} - \binom{1}{3} \\
 &= 1 - 0 \\
 &= 1
 \end{aligned} \tag{29}$$

It is shown that $P(n)$ is true when $n = 2$. Next, assume $P(n)$ is true. Then it is required to show $P(n+1)$ is true given by

$$\begin{aligned}
P(n+1) = FTTM_{2/n+1} &= \sum_{i=2}^{n+1} \left[\binom{(n+1)+2-i}{i} - \binom{(n+1)+1-i}{i+1} \right] \\
&= \frac{n(n+1)}{2}
\end{aligned} \tag{30}$$

Notice,

$$\begin{aligned}
P(n+1) &= \sum_{i=2}^{n+1} \left[\binom{n+3-i}{i} - \binom{n+2-i}{i+1} \right] \\
&= \left[\binom{n+1}{2} + \binom{n}{3} + \binom{n-1}{4} + \cdots + \binom{3}{n} + \binom{2}{n+1} \right] \\
&\quad - \left[\binom{n}{3} + \binom{n-1}{4} + \cdots + \binom{2}{n+1} + \binom{1}{n+2} \right] \\
&= \binom{n+1}{2} - \binom{1}{n+2} \\
&= \binom{n+1}{2}
\end{aligned} \tag{31}$$

as $\binom{1}{n+2} = 0$. Thus, by a property of binomial coefficient [19],

$$\begin{aligned}
P(n+1) &= \binom{n+1}{2} \\
&= \frac{(n+1)!}{2!((n+1)-2)!} \\
&= \frac{(n+1)(n)(n-1)!}{2(n-1)!} \\
&= \frac{n(n+1)}{2}
\end{aligned} \tag{32}$$

■

Similarly, we can prove for $FTTM_{3/n}$ given by

Theorem 5.2 *The number of cubes produced by the combination of any three terms in $FTTM_n$; $FTTM_{3/n}$ can be represented as*

$$FTTM_{3/n} = \sum_{i=3}^n \left[\binom{n+3-i}{i} - \binom{n+2-i}{i+1} \right] = \frac{n(n-1)(n-2)}{3!} \tag{33}$$

for $n \geq 3$

Proof of Theorem 5.2

Recall Equation (23), by using $k = 1$,

$$\begin{aligned}
 F_{1,7} &= \binom{6}{0} + \binom{5}{1} + \binom{4}{2} + \binom{3}{3} \\
 F_{1,8} &= \binom{7}{0} + \binom{6}{1} + \binom{5}{2} + \binom{4}{3} \\
 F_{1,9} &= \binom{8}{0} + \binom{7}{1} + \binom{6}{2} + \binom{5}{3} + \binom{4}{4} \\
 F_{1,10} &= \binom{9}{0} + \binom{8}{1} + \binom{7}{2} + \binom{6}{3} + \binom{5}{4} \\
 F_{1,11} &= \binom{10}{0} + \binom{9}{1} + \binom{8}{2} + \binom{7}{3} + \binom{6}{4} + \binom{5}{5} \\
 &\vdots \\
 F_{1,n} &= \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \binom{n-4}{3} + \cdots \\
 &\quad + \binom{n-1-i}{i}
 \end{aligned} \tag{34}$$

The number of cubes produced by the combination of any three terms in $FTTM_n$; $FTTM_{3/n}$ exist at the fourth term in $F_{1,n}$ such that $i=4$ for equation (26). Theorem 2 can be proven by mathematical induction. Let

$$\begin{aligned}
 P(n) = FTTM_{3/n} &= \sum_{i=3}^n \left[\binom{n+3-i}{i} - \binom{n+2-i}{i+1} \right] \\
 &= \frac{n(n-1)(n-2)}{3!}
 \end{aligned} \tag{35}$$

For $n = 3$, $P(3) =$

$$FTTM_{3/3} = \frac{(3)(3-1)(3-2)}{3!} = 1 \tag{36}$$

However

$$\begin{aligned}
 FTTM_{3/3} &= \binom{3+3-3}{3} - \binom{3+2-3}{3+1} \\
 &= \binom{3}{3} - \binom{2}{4} \\
 &= 1
 \end{aligned} \tag{37}$$

Since $P(3)$ is true, next we assume $P(n)$ is true. It is required to show $P(n+1)$ is true given by

$$\begin{aligned}
P(n+1) = FTTM_{3/n+1} &= \sum_{i=3}^{n+1} \left[\binom{(n+1)+3-i}{i} - \binom{(n+1)+2-i}{i+1} \right] \\
&= \frac{(n+1)(n)(n-1)}{3!}
\end{aligned} \tag{38}$$

Therefore,

$$\begin{aligned}
P(n+1) &= \sum_{i=3}^{n+1} \left[\binom{n+4-i}{i} - \binom{n+3-i}{i+1} \right] \\
&= \left[\binom{n+1}{3} + \binom{n}{4} + \cdots + \binom{3}{n+1} \right] \\
&\quad - \left[\binom{n}{4} + \binom{n-1}{5} + \cdots + \binom{3}{n+1} + \binom{2}{n+2} \right] \\
&= \binom{n+1}{3} - \binom{2}{n+2} \\
&= \binom{n+1}{3}
\end{aligned} \tag{39}$$

as $\binom{2}{n+2} = 0$. Thus, by a property of binomial coefficient [19],

$$\begin{aligned}
P(n+1) &= \binom{n+1}{3} \\
&= \frac{(n+1)!}{3!((n+1)-3)!} \\
&= \frac{(n+1)(n)(n-1)(n-2)!}{3!(n-2)!} \\
&= \frac{(n+1)(n)(n-1)}{3!}
\end{aligned} \tag{40}$$

■

Theorem 5.3 *The number of cubes produced by the combination of any four terms in $FTTM_n$; $FTTM_{4/n}$ can be represented as*

$$FTTM_{4/n} = \sum_{i=4}^n \left[\binom{n+4-i}{i} - \binom{n+3-i}{i+1} \right] = \frac{n(n-1)(n-2)(n-3)}{4!} \tag{41}$$

for $n \geq 4$.

Proof of Theorem 5.3

Recall equation (23), and using $k = 1$,

$$\begin{aligned}
 F_{1,9} &= \binom{8}{0} + \binom{7}{1} + \binom{6}{2} + \binom{5}{3} + \binom{4}{4} \\
 F_{1,10} &= \binom{9}{0} + \binom{8}{1} + \binom{7}{2} + \binom{6}{3} + \binom{5}{4} \\
 F_{1,11} &= \binom{10}{0} + \binom{9}{1} + \binom{8}{2} + \binom{7}{3} + \binom{6}{4} + \binom{5}{5} \\
 F_{1,12} &= \binom{11}{0} + \binom{10}{1} + \binom{9}{2} + \binom{8}{3} + \binom{7}{4} + \binom{6}{5} \\
 &\vdots \\
 F_{1,n} &= \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \binom{n-4}{3} \\
 &\quad + \binom{n-5}{4} + \binom{n-6}{5} \cdots + \binom{n-1-i}{i}
 \end{aligned} \tag{42}$$

The number of cubes produced by the combination of any four terms in $FTTM_n$; $FTTM_{4/n}$ exists at the fifth term in $F_{1,n}$ such that $i = 5$ for Equation (26). Theorem 3 can be proven by mathematical induction. Let

$$\begin{aligned}
 P(n) = FTTM_{4/n} &= \sum_{i=4}^n \left[\binom{n+4-i}{i} - \binom{n+3-i}{i+1} \right] \\
 &= \frac{n(n-1)(n-2)(n-3)}{4!}
 \end{aligned} \tag{43}$$

For $n = 4$, $P(4) =$

$$FTTM_{4/4} = \frac{(4)(4-1)(4-2)(4-3)}{4!} = 1 \tag{44}$$

On the other hand,

$$\begin{aligned}
 FTTM_{4/4} &= \binom{4+4-4}{4} - \binom{4+3-4}{4+1} \\
 &= \binom{4}{4} - \binom{3}{5} \\
 &= 1
 \end{aligned} \tag{45}$$

From equation(45), the condition satisfied when $n = 4$. Assume $P(n)$ is true. Then it is required to show $P(n+1)$ is true given by

$$\begin{aligned}
 P(n+1) = FTTM_{4/n+1} &= \sum_{i=4}^{n+1} \left[\binom{n+5-i}{i} - \binom{n+4-i}{i+1} \right] \\
 &= \frac{(n+1)(n)(n-1)(n-2)}{4!}
 \end{aligned} \tag{46}$$

Furthermore,

$$\begin{aligned}
 P(n+1) &= \sum_{i=4}^{n+1} \left[\binom{n+5-i}{i} - \binom{n+4-i}{i+1} \right] \\
 &= \left[\binom{n+1}{4} + \binom{n}{5} + \cdots + \binom{4}{n+1} \right] \\
 &\quad - \left[\binom{n}{5} + \binom{n-1}{6} + \cdots + \binom{4}{n+1} + \binom{3}{n+2} \right] \quad (47) \\
 &= \binom{n+1}{4} - \binom{3}{n+2} \\
 &= \binom{n+1}{4}
 \end{aligned}$$

as $\binom{3}{n+2} = 0$. Thus, by a property of binomial coefficient [19],

$$\begin{aligned}
 P(n+1) &= \binom{n+1}{4} \\
 &= \frac{(n+1)!}{4!((n+1)-4)!} \\
 &= \frac{(n+1)(n)(n-1)(n-2)(n-3)!}{4!(n-3)!} \quad (48) \\
 &= \frac{(n+1)(n)(n-1)(n-2)}{4!}
 \end{aligned}$$

■

6 Conclusion

In this paper, alternative proof for sequence of cubes of FTTM are introduced by using k -Fibonacci sequence. Table 3 summarized the number of cubes produced for $FTTM_n$ for some n .

Table 3: Cubes for Sequences of FTTM for $n = 1, 2, 3, \dots, 10$

n	$FTTM_{2/n}$	$FTTM_{3/n}$	$FTTM_{4/n}$
$n=1$	0	0	0
$n=2$	1	0	0
$n=3$	3	1	0
$n=4$	6	4	1
$n=5$	10	10	5
$n=6$	15	20	15
$n=7$	21	35	35
$n=8$	28	56	70
$n=9$	36	84	126
$n=10$	45	120	210

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