Epochs of Discontinuity for the Standard Model of Cosmology with Supernovae Observational Data

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Abstract

We recently predicted the standard model for cosmology, based upon the Friedmann expansion of spacetime using the Robertson-Walker metric (FRW), shall exhibit discontinuities as the Universe passes through one or more epochs of low matter density. We now present evidence for this when tested with the most recently available, combined, self-consistent data from astronomical observations. The standard model predicts large "jumps" for the values of normalized dark energy $\Omega_\Lambda$ and spacetime curvature $\Omega_k$ parameters at about $\Omega_m$ of 0.30, 0.22 and 0.18. The discontinuity is not found when the FRW model is used without a term for $\Omega_\Lambda$ or when a term allowing for the observed bluing of ancient emissions is added in linear combination to the standard model. Consideration of this failure when checked against supernovae data, along with the discords with common physics by many orders of magnitude, suggests models based upon large values for $\Omega_\Lambda$ should be examined quite carefully.

Keywords: Cosmic Constant, Dark Energy, FRW, Supernova, Discontinuity

1 Introduction

Use of very large ground based telescopes and the Hubble Space Telescope (HST) have allowed close examination of the light decay curves from ancient...
supernovae, type Ia (SNe Ia) and the redshifts of associated galaxies to nearly 11 Gyears [1, 2]. The data from several groups composing dozens of investigators have been recently combined into a list of 192 SNe Ia redshifts, moduli, moduli errors and are available on-line [3]. These groups have proposed the data are best explained as describing a flat Universe composed of about 25% matter (common matter and exotic dark matter) with the remaining 75% being dark energy [4, 5]. This ”dark energy” is a term first coined by Perlmutter, M. Turner, White [6] after the usefulness of the cosmic constant had been well-expounded by Carroll, Press and E. Turner in a review [7]. Invention of the cosmic constant is credited to Einstein, however, to explain the static Universe prior to 1920, which was thought to be only the Milky Way [8]. He purposed this constant described an energy of an unknown and undiscovered type to operate in spacetime. A little later, Friedmann informed the world that two convenient solutions to the Einstein equation were contracting and expanding Universes [9]. After Hubble published his correlation for an expanding Universe of galaxies, Einstein seems to have abandoned the idea of a cosmic constant.

We have recently examined the mathematics [10] of the FRW Universe based upon the popular ideas of the combined normalization of the matter ($\Omega_m$), spacetime ($\Omega_k$) and dark energy ($\Omega_\Lambda$) as unitless parameters [7]. This analysis uncovered discontinuous solutions at low matter densities and large dark energies which are likely scenarios in the future of such a Universe. We did not test our analysis with the data then available; we thought the data slightly too noisy. Here we report our results, using the 192 SNe Ia available on-line [3] as the best, self-consistent data set yet, and we also use a Hubble constant of 62.3 km s$^{-1}$ Mpc$^{-1}$ as the latest and best estimate for this parameter [11]. We have found discontinuities of spacetime and dark energy at $\Omega_m$ of 0.30, 0.22 and 0.18, in Universes with positive curvature. These facts, along with the physics for dark energy requiring special pleading and disagreeing with common physics by many orders of magnitude [12], leads us to purpose dark energy should be carefully considered before being allowed a portion of our present situation.

2 Theory

The relationship normally used to describe our Universe when using SNe Ia data, including redshifts, is

$$D_L = \frac{c(1+z)}{H_0 \sqrt{\Omega_k}} \sin n\left\{ \sqrt{\Omega_k} \int_0^z \frac{1}{\sqrt{(1+z)^2(1 + \Omega_{m}z) - z(2 + z)\Omega_\Lambda}} dz \right\}$$

(1)

When $\Omega_k$ is positive $\sin n$ becomes sinh and when negative, sin is used in Eq. (1); the integral cannot be integrated analytically so numerical methods must
be used with real data. This useful equation is based upon the realization that
the parameters for matter, spacetime curvature and the cosmic constant can
be normalized, as an aid for evaluation
\[ \Omega_m + \Omega_k + \Omega_\Lambda = 1. \] (2)

Equation (2) is simplified when dark energy is negligible becoming simply
\( \Omega_m + \Omega_k = 1 \) and because a flat Universe is only realized when \( \Omega_k = 0 \),
requiring the very special case of \( \Omega_m = 1 \) which is definitely not favored
by astronomers, this case describes the infinitely and gently curved spacetime
favored by Einstein [13]. We realized the complicated term in the denominator
of the integral could lead to some mathematical problems, some of which we
have presented [10]. It is common for investigators to take the log of one or
both sides of Eq. (1) and use log(data) to cast this in a linear manner. We
will avoid this because such a convenience sometimes discards results which
may be interesting.

Equation (1) can then be simplified for analysis when the term including the
cosmic constant is allowed to become 0 as presented below
\[ D_L = \frac{c(1 + z)}{H_0\sqrt{\Omega_k}} \sinh\{\sqrt{\Omega_k}[2(\text{arctanh}\sqrt{\Omega_k} - \text{arctanh}\frac{\sqrt{\Omega_k}}{\sqrt{1 - \Omega_m z}})]\} \] (3)

with similar conditions for \( \sinh \) and \( \Omega_k \) as before and presuming a non-zero
\( \Omega_k \).

We now recast the above two relationships (1) and (3) into forms demanding
the frequency ratio, \( \xi \), of retreating galaxies, rather than redshift with \( \xi = \frac{1}{(1+z)} \), which are mathematically simpler to use for modeling data. Equation
(1) can be recast as
\[ D_L = \frac{c}{\xi H_0\sqrt{\Omega_k}} \sinh\left\{\sqrt{\Omega_k}\int_{\xi_1}^1 \frac{1}{\xi \sqrt{\Omega_m} + \frac{\Omega_\Lambda}{\Omega_m} + \frac{\Omega_k}{\Omega_m} d\xi}\right\} \] (4)

and here we presume a closed Universe with \( \Omega_k > 0 \). We will use this form for
modeling a Universe which includes dark energy but not presuming a flat Uni-
verse, though we are aware that most empirical data from the past 200 hundred
years (since Gauss) indicate a flat Universe. Einstein thought otherwise [13],
while others currently lean towards an infinite, though curved Universe [14].

To double-check for the presence of discontinuities we also fit the data to the
next equation which is the derivation, using \( \xi \) in place of \( \frac{1}{(1+z)} \), most related
to Eq. (16) from Öztas and Smith
\[ D_L = \frac{c}{\xi H_0\sqrt{\Omega_k}} \sinh\left\{\sqrt{\frac{\Omega_k}{\Omega_m}}\int_{\xi_1}^1 \frac{1}{\sqrt{\xi + \frac{\Omega_\Lambda \xi^4 + \Omega_k \xi^2}{\Omega_m}}} d\xi\right\} \] (5)
which differs from Eq. (4) by bringing the constant $1/\sqrt{\Omega_m}$ out from the integration.

For a Universe with negligible dark energy we will rewrite Eq. (3) in terms of the frequency ratio decline as the light source recedes, again presuming curved spacetime

$$D_L = \frac{c}{\xi H_0 \sqrt{\Omega_k} \sinh \left\{ 2 \left( \tanh \sqrt{\Omega_k} - \tanh \left( \frac{\sqrt{\Omega_k}}{\sqrt{\Omega_m + \Omega_k}} \right) \right) \right\}}.$$  \hfill (6)

The usefulness of Eq. (6) becomes apparent as we consider reports that emissions from supernovae and associated galaxies seem bluer than they should [15, 16]. From these observations we have suggested a term be added to Eq. (6) to account for an intrinsic bluing of emissions as these retreat in spacetime [17]

$$D_{L(\nu)} = \frac{c}{H_0} \left( \xi^\frac{1}{2} - 1 \right)$$ \hfill (7)

where $D_{L(\nu)}$ is the portion of the apparent distance due to the difference in emission frequencies between ancient and recent SNe. The observed luminar distance, $D_{L(tot)}$, is the sum of the geometric distance, $D_L$, in equations 1 through 6, and the apparent distance due to change in emission frequency as a function of lookback time, $D_{L(\nu)}$

$$D_{L(tot)} = D_L + D_{L(\nu)}.$$ \hfill (8)

The new unitless constant $U$ describes the frequency decline of emissions with increased passing of local time, but is a timeless constant; the rational for Eqs. (7,8) has been presented previously [17].

3 Model fitting

We first calculated the frequency ratios and associated distances of SNe Ia with geometric errors, in Mpc, from the on-line redshifts and associated distance moduli data [3]. We used a commercial program (TableCurve 2D, SYSTAT corporation) to fit all models to all 192 SNe Ia using a robust minimization routine which discriminates against outliers and with data weighed, normalized with respect to errors. We presumed a Hubble constant of 62.3 km s$^{-1}$ Mpc$^{-1}$ as the most recent and best value for this local constant [11], which remained fixed throughout all data minimization. Since the most recent SNe Ia data pair in this set was emitted over 230 million years ago, galaxy peculiar velocities should be considered as negligible additions to luminar distance errors. Two and three dimensional plots were made using Mathematica (Wolfram Research) and TableCurve 3D (SYSTAT corporation).
Values of the normalized matter density, $\Omega_m$, were entered step-wise and only one degree of parameter freedom was allowed for all fits. This still means that both parameters, $\Omega_k$ and $\Omega_\Lambda$ were floated simultaneously for Eqs. (4) and (5) but only $U$ was allowed to float when the linear combination of Eqs. (6, 7) were fit, since values for $\Omega_k$ are automatically determined by the selection of $\Omega_m$. Likewise, when we used the data to constrain the classic FRW model; we only allowed $\Omega_k$ to float.

Each goodness of fit was sorted using the resulting fit standard error and the best fits presented as the lowest $\chi^2$ values calculated using the common relationship

$$\chi^2 = \sum_i \frac{(s_{D_L,i})^2}{\sigma_{D_L,i}^2}$$

where $\sigma_{D_L,i}$ is the uncertainty in the individual distance observations and $s_{D_L,i}$ is the calculated individual errors from the fit. Note the use of this transformation, $z = \frac{1}{1+\xi}$, also allows one to insert an addition data pair, at $[0,0]$ with no error if we wished, while realizing these models must become asymptotic towards the ordinate axis. If we also wish we can select an age for the Universe, say 14.5 Gyears and allow $\xi$ to become 0 at this time. This latter presumption is good only when all data are normalized with respect to the age of the Universe as predicted by geologists [18], which has not been done.

4 Results

We used Eqs. (4) and (5) to model the data for inclusion of the cosmic constant, beginning with $\Omega_m$ of 0.35 and moving to 0.10 in steps of 0.01, and results were identical using both variations. The steps were shortened to 0.003 from $\Omega_m$ of about 0.30, 0.22 and 0.18 in attempted refinements of our results as presented in Fig. 1. We observed serious and reproducible discontinuities at $\Omega_m$ of 0.30, 0.22 and 0.18 with $\Omega_\Lambda$ declining stepwise and $\Omega_k$ jumping in the opposite direction. While we stepped $\Omega_m$ down towards 0.10 we observed $\Omega_\Lambda$ to decline to nearly zero below $\Omega_m$ of 0.18 with $\Omega_k$ becoming the remainder of the normalization; 0.88 at $\Omega_m$ of 0.12. A $\Omega_m$ of either 0.22 or 0.18 are both within the important range as currently thought by many, for while many astronomers believe $\Omega_m$ is presently about 0.25, some people think values much lower, current $\Omega_m$ down to about 0.05 are more reasonable [19, 20].

When we evaluated the FRW relationship without the $\Omega_\Lambda$ term, allowing only $\Omega_k$ to float and varying $\Omega_m$ from 0.01 to 0.30, Eq. (6), we observed only smoothly varying solutions with $\Omega_k$ easily settling on the predicted, if trivial, values for normalization. We found the minimum for $\chi^2$ to settle around an $\Omega_m$ from 0.03 to 0.07 and we cannot narrow the range for the best fit of the simple FRW model any further.
We finally evaluated the FRW relationship, without the cosmic constant but with the allowance for emission bluing with lookback time, Eqs. (7) and (8), varying $\Omega_m$ from 0.03 to 0.30. The results were similar to the classic FRW model with the best fits for $\Omega_m$ from 0.05 to 0.08 and we did not observe any indication of discontinuity between analyses. We present results for these fits in Fig. 2, note the "error" bars are not those for the various values for $U$, but are the relative magnitudes for $\chi^2$ of the fits. It is obvious that the goodness of fits decrease nearly monotonically as the value for $\Omega_m$ increases from 0.08 to 0.30. On the other hand, the values for $U$ are quite scattered, with a simple average of $618 \pm 75$ and the straight line is shown as the best fit but with a correlation coefficient of $< 0.01$. We believe the standard deviation of 75 is deceivingly small, because the error for $U$ from individual fits is usually about 5X the value of $U$.

5 Discussion

We have found using the frequency ratio, $\xi$, rather than the redshift, $z$, and fitting model functions using SNe Ia data directly, rather than the log trans-
formations to be instructive. We have uncovered serious discontinuities using observational data and the standard model (FRW) for cosmology when a term including the cosmic constant $\Omega_\Lambda$ is evaluated - even when using a second form of this relationship. Two other models, the classic FRW model and that model but including an extra term for emission bluing with increasing lookback time [17], do not yield discontinuous solutions with this data, when tested at matter densities currently thought important. We have previously warned of this catastrophic solution for dark energy models but had thought this would occur at solutions of very low matter densities [10].

This leads us to suggest that while some form of energy, available in massive quantities, was perhaps responsible for our inflationary Universe immediately after singularity, this energy is no longer available in large amounts. We think it an error to assign the very small perturbations in the cosmic microwave background as evidence for some form of dark energy presently responsible for 75% of all ”stuff” in the Universe. The use of the cosmic constant to describe the possible effects of vacuum energy within the FRW metric, as commonly done, has other problems, too. Higher matter densities than present, for instance those densities which possibly occurred soon after early galaxy formation, from about $z$ of 5 to 8, demand the cosmic constant become negative, attractive in nature, for real solutions. Other large problems arise when the strength of the
vacuum energy (anti-gravity) are compared with more accepted values for the expectation energy of a vacuum and Newton’s G; the discrepancies are many orders of magnitude [12]. Here we find the cosmic constant declines to zero at lower matter densities using the SNe Ia data, thought to be the eventual fate of our Universe which differs from the expectations of some astronomers with a future Universe made primarily of vacuum energy.

Since we cannot rely on the current FRW model which includes the cosmic constant we are left with FRW models relying upon significant amounts $\Omega_k$ in a Universe past the critical density. Discrimination between several models which describe the Universe as curved, that is with a non-negligible $\Omega_k$, need to wait for much more and better SNe Ia data. Collection of this data is heavily dependent on more acquisition time using the HST. We note that most data beyond $z = 0.2$ are quite noisy even from the HST, when compared with data from more recent explosions. At these ancient lookback times it seems difficult to sometimes properly designate electronic transitions for determination of the redshift. (Published S/N ratios would not impress many laboratory spectroscopists, but we are reminded that astronomy is an observational science without the common possibilities of reproducible experiments.) But all this means that those astronomers dedicated to collecting SNe Ia data need significantly more HST time to better determine SNe Ia light decay curves and redshifts, especially past $z$ of 0.2.

Since both the classic FRW and the bluing models gave reasonable solutions without discontinuities while having very large contributions from the spacetime curvature parameter, we should suggest a modern meaning for this. Our interpretation of $\Omega_k$ is that it represents both curvature and the rough, relative magnitude of spacetime. While solutions of the FRW model with $\Omega_m = 1.0$ and $\Omega_k = 0.0$ are theoretically possible and interesting, this state is impossible with real data from ancient sources. The Universe is primarily spacetime with very slight curvature and our qualitative perception is tainted from our initial frame located within a solar system dense with matter.

References


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