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Inertial Field Energy

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Abstract

The phenomenon of Inertia may be explained if the four-dimensional scale of spacetime were to change during acceleration. Although a dynamically changing scale cannot be modeled by coordinate transformation it is compatible with General Relativity, and it is observationally indistinguishable from Special Relativity, except that it admits cosmological simultaneity. The new theory explains the origin of the inertial force as being a curved spacetime phenomenon, with the implication that accelerating matter might influence the metrical scale of space and time and consequently also by General Relativity the energy-density of spacetime. A close correspondence is found between an inertial line-element and a gravitational line-element, both expressing energy density by Poisson's equation. Under certain conditions the field energy induced by acceleration could become negative.

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1. Introduction

In two recent papers the author proposes that Inertia, like Gravitation, could be a curved spacetime phenomenon caused by accelerating motion [Masreliez, 2007a and Masreliez, 2008]. If the four spacetime metrics of the Minkowskian line element change dynamically during acceleration, it could explain Inertia without violating the two postulates upon which Einstein based his paper on Special Relativity (SR) [Einstein, 1905]:

1. All inertial frames are physically equivalent.

2. The velocity of light in these inertial frames is the same.

The dynamic scale-factor, which multiplies all four metrical components in the Minkowskian line-element and explains Inertia, is $1-(v/c)^2$, where v is the relative velocity and c the speed of light.

When Einstein derived the Lorentz transformation in this 1905 paper he implicitly made an additional assumption, which perhaps should be seen as a third postulate; he assumed that coordinate locations in inertial frames are related via the Lorentz transformation. However, if inertial frames were to differ by metrical scale-factors that depend on their relative velocities they would not be related via the Lorentz transformation, but via Voigt's transformation [Voigt, 1887] that was derived prior to the Lorentz transformation. The resulting dynamic scale theory would be observationally indistinguishable from SR, but would allow simultaneity regardless of motion. And, it would allow us to treat Inertia as a gravitation-type phenomenon [Masreliez, 2008].
The present paper shows that there is a close correspondence between an inertial line-element and a gravitational line-element, both expressing energy density by Poisson's equation.

2. Inertia and energy

Energy is intimately related to motion and to Inertia; kinetic energy would not exist without Inertia, since a force would not be required to accelerate or decelerate an object. The recent discovery that motion might change the metrics of spacetime, including the temporal metric, opens up the possibility that the energy density of spacetime might be influenced by accelerating motion.

The proposition that accelerating motion might cause spacetime curvature is consistent with the general understanding that Inertia is a phenomenon similar to Gravitation. Accordingly accelerating particles might cause local changes of their spacetime metric, and we will assume that an accelerating stream of particles correspond to an *inertial field* similar to a gravitational field describing how the induced spacetime curvature changes with spatial location. By this approach the spacetime curvature becomes a function solely of a velocity that depends on position. This point of view is allowed, if we assume that the inertial properties of spacetime do not change with time, since in this case the temporal dependence is implicit when the velocity profile as a function of location is known [Masreliez, 2007a].

According to this view, a flow of accelerating particles creates an inertial field with scalar field potential $v^2/2$, which according to GR might generate spacetime energy. This possibility is investigated in this paper.

3. Deriving the inertial energy density

Consider the inertial line element found in [Masreliez, 2007a]:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = (1-v^2)\left(dx_0{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2\right) \\ v &= v(x,y,z) = v(x_1,x_2,x_3) \end{aligned} \tag{3.1}$$

In this expression the speed of light is $c=1$ and $x_0=t,\ x_1=x,\ x_2=y,\ x_3=z$. Also, repeated indices indicate summation according to Einstein's convention unless otherwise noted.
With this line element all trajectories are geodesics of general relativity, which would explain Inertia.
Einstein's GR equations are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{3.2}$$

$R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ the energy-momentum tensor and G the gravitational constant. The components of the Ricci tensor are given by:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu} \tag{3.3}$$

The Christoffel symbols are:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}\right) \tag{3.4}$$

The Ricci scalar is formed by contracting the Ricci tensor:

$$R = g^{\mu\nu}R_{\mu\nu} \tag{3.5}$$

In GR the energy density is not defined in general, since it depends on the chosen reference frame. I will assume that the energy is measured in relation to an inertial reference frame, which could be arbitrary. I will also assume that the total (net) Inertial Energy Density (IED) relative to this reference frame is given by:

$$IED = T_0^0 - T_1^1 - T_2^2 - T_3^3 \tag{3.6}$$

This definition is used by Richard Tolman [Tolman, 1930].
Making use of the relations (3.2) and (3.5) we have:

$$IED = \frac{1}{4\pi G}R_0^0 \tag{3.7}$$

The expression for the Ricci tensor components (3.3) contains derivatives of the metrics with respect to the four spacetime coordinates. All derivatives with respect to time (x_0) disappear because the velocity is modeled as a function solely of the three spatial coordinates.

Let us evaluate the contribution to R_{00} from differentiation of one of the spatial coordinates denoted by the index i. In the expressions below the index i is kept fixed. Thus, here repeated indices do *not* imply summation.

$$\Gamma^i_{00,i} = \frac{\partial}{\partial r}\left[\frac{1}{2}\left(g^{ii}\right)\left(-g_{00,i}\right)\right] = \frac{1}{2}\frac{\partial}{\partial r}\left[\left(-\frac{1}{(1-v^2)}\right)\left(2vv_i\right)\right] = \frac{-vv_{ii}-v_i^2}{(1-v^2)} - \frac{2\left(vv_i\right)^2}{(1-v^2)^2} \tag{3.8}$$

$$\Gamma^0_{00} = 0 \tag{3.9}$$

$$\Gamma^i_{00} = \frac{1}{2}\left(g^{ii}\right)\left(-g_{00.i}\right) = -\frac{vv_i}{(1-v^2)} \tag{3.10}$$

$$\Gamma^0_{i0} = \Gamma^1_{i1} = \Gamma^2_{i2} = \Gamma^3_{i3} = \frac{1}{2}\left(g^{00}\right)\left(g_{00,i}\right) = -\frac{vv_i}{(1-v^2)} \tag{3.11}$$

Using (3.3) this gives a contribution to R_{00} from each of the three spatial components i=1, 2, and 3:

$$\begin{aligned} R_{00}(i) &= \left(\Gamma^i_{00,i} + \Gamma^i_{00}\left(\Gamma^0_{i0} + \Gamma^1_{i1} + \Gamma^2_{i2} + \Gamma^3_{i3}\right) - \Gamma^i_{00}\Gamma^0_{i0} - \Gamma^0_{0i}\Gamma^i_{00}\right) = \\ &= \left(\Gamma^i_{00,i} + 2\Gamma^i_{00}\Gamma^0_{i0}\right) = \\ &= \left(\frac{-vv_{ii}-v_i^2}{(1-v^2)} - \frac{2\left(vv_i\right)^2}{(1-v^2)^2} + \frac{2\left(vv_i\right)^2}{(1-v^2)^2}\right) = \\ &= \left(\frac{-vv_{ii}-v_i^2}{(1-v^2)}\right) = -\frac{1}{(1-v^2)}\frac{\partial^2(v^2/2)}{dx_i^2} \end{aligned} \tag{3.12}$$

Adding all three contributions the inertial energy density becomes after raising one index and using (3.7):

$$\begin{aligned} IED &= -\frac{1}{4\pi G(1-v^2)^2}\left[\frac{\partial^2(v^2/2)}{dx_1^2} + \frac{\partial^2(v^2/2)}{dx_2^2} + \frac{\partial^2(v^2/2)}{dx_3^2}\right] = \\ &= -\frac{1}{4\pi G(1-v^2)^2}\Delta\left(\frac{v^2}{2}\right) \end{aligned} \tag{3.13}$$

Where Δ is the Laplace operator.

The *gravitational* field potential satisfies a similar relation, the Poisson gravitational equation:

$$\rho_{matter} = \frac{1}{4\pi G}\Delta(\Phi) \tag{3.14}$$

Φ is the gravitational potential, which outside a spherically symmetric mass M is MG/r.

4. The Poisson equation for gravitational matter energy density

We can derive a geometric expression for *gravitational* matter energy density based on Tolman's energy density relation (3.6) using the same approach as in section 3.

Consider the line-element:

$$ds^2 = e^{N(r)} \cdot dt^2 - e^{-N(r)} \cdot dr^2 - r^2 \cdot \left(d\theta^2 + \sin^2(\theta)\, d\varphi^2\right) \qquad (4.1)$$

This line-element has been investigated extensively with various metrics *exp(N)*, the most well known being Schwarzschild's exterior solution with *exp(N)=1/(1-2MG/r).* It is also used in the Einstein-Born-Infeld solution for an electromagnetic spherically symmetric spacetime field [Born-Infield, 1934], and in Hoffman's monopole solution [Hoffman, 1935]. The same form of the line-element was used by Reissner (1916) and Nordstrom (1918) for an electrically charged mass accumulation. More recent examples are Bekenstein's (1975) line-element and the 'geon' of Demianski (1986), which also under certain conditions reduces to the line-element (4.1). Therefore, this 'gravitational line-element' may be used to model gravitational energy density.

The components of Einstein's tensor for this line element may be found in many books on GR. Using these results Einstein's tensor components for the line element (4.1) become, letting the lower index *r* denote differentiation with respect to *r*:

Temporal T_0^0 component:

$$R_0^0 - \frac{1}{2}R = -e^{-L}\left[\frac{N_r}{r} + \frac{\left(1-e^{-N}\right)}{r^2}\right] = 8\pi T_0^0 \qquad (4.2)$$

Radial T_1^1 component:

$$R_1^1 - \frac{1}{2}R = 8\pi T_1^1 = 8\pi T_0^0 \qquad (4.3)$$

Angular T_2^2 and T_3^3 components:

$$R_\alpha^\alpha - \frac{1}{2}R = -e^{N}\cdot\left[\frac{N_{rr}}{2} + \frac{N_r}{r} + \frac{N_r^2}{2}\right] = 8\pi T_\alpha^\alpha \qquad (4.4)$$

Forming the Tolman energy density expression (3.6) we get:

$$e^N\left[N_{rr} + \frac{2N_r}{r} + N_r^2\right] = 8\pi\left(T_0^0 - T_1^1 - T_2^2 - T_3^3\right) \qquad (4.5)$$

Making the substitution:

$$e^{N(r)} = 1 - \varphi(r) \qquad (4.6)$$

We find that $\varphi(r)$ satisfies:

$$\begin{aligned} &-\left(\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr}\right) = 8\pi\left(T_0^0 - T_1^1 - T_2^2 - T_3^3\right) \\ &-\frac{1}{4\pi}\Delta\left(\frac{\varphi}{2}\right) = T_0^0 - T_1^1 - T_2^2 - T_3^3 \end{aligned} \qquad (4.7)$$

With $\varphi(r)=2GM/r$ *this is Poisson's equation but with opposite sign.*

The sign convention in the field equations of the present paper agrees with Tolman [Tolman, 1930], with the one used by Miser, Thorne and Wheeler [Miser, Thorne and Wheeler, 1973] and by Peeble [Peeble, 1993]. It yields a Poisson-type equation with *negative* matter-energy density. A possible explanation to this sign reversal may be found in the Appendix.

5. Inertial matter energy density

Assuming that Inertia and Gravitation are closely related, we might speculate that inertial energy density corresponding to matter energy density also might be generated by the inertial field. This new kind of energy density denoted $\rho_{inertial}$ would be the negative of the inertial spacetime energy density IED defined by (3.7).
From (3.13) we would then have:

$$\rho_{inertial} = \frac{1}{4\pi G(1-v^2)^2}\Delta\left(\frac{v^2}{2}\right) \tag{5.1}$$

Reintroducing c this relation becomes:

$$\rho_{inertial} = \frac{1}{4\pi G\left[1-(v/c)^2\right]^2}\Delta\left(\frac{v^2}{2}\right) \tag{5.2}$$

And the corresponding Inertial Matter Energy Density (IMED) is:

$$IMED = \frac{c^2}{4\pi G\left[1-(v/c)^2\right]^2}\Delta\left(\frac{v^2}{2}\right) \tag{5.3}$$

6. Inertial matter energy

The 'Inertial Matter Energy (IME)' within a spatial volume may be derived by integrating (5.3):

$$IME = \iiint_V \frac{c^2}{4\pi G\left[1-(v/c)^2\right]^2}\Delta\left(\frac{v^2}{2}\right)\sqrt{-g}\cdot dV = \frac{c^2}{4\pi G}\iiint_V \Delta\left(\frac{v^2}{2}\right)dV \tag{6.1}$$

Therefore the *effective* inertial mass density is:

$$\rho_{inertial} = \frac{1}{4\pi G}\Delta\left(\frac{v^2}{2}\right) = \frac{1}{4\pi G}div(a) \tag{6.2}$$

The acceleration vector $a(x_1, x_2, x_3)$ satisfies:

$$a = grad\left(\frac{v^2}{2}\right) \tag{6.3}$$

This implies:

$$IME = \frac{c^2}{4\pi G}\iiint_V div(a)\,dV \tag{6.4}$$

Furthermore, using the Divergence Theorem:

$$IME = \frac{c^2}{4\pi G}\iint_S grad\left(\frac{v^2}{2}\right)\overline{dS} = \frac{c^2}{4\pi G}\iint_S \overline{a}\cdot\overline{dS} \tag{6.5}$$

According to relation (6.1) the inertial spacetime energy could be huge; in fact it appears to be unrealistically large. In deriving it we assumed that the inertial spacetime curvature applies to *all* accelerating volume elements. However, Inertia is a property of *particles*, and it is likely that the volume affected by the inertial scale factor is related to the volumes of particles rather than to the total spatial volume of the flow. This could very significantly reduce the energy that might be generated. However, since the constant c^2/G is very large, even a small effective volume might correspond to a large amount of energy.

7. Negative field energy

The expression for (6.4) for inertial field energy suggests that it might be possible to generate both positive *and negative* spacetime 'matter energy' from accelerating matter. If the accelerating flow is such that the divergence for the acceleration is negative, the induced field energy could also become negative. Although this is something new and totally unexpected, it should theoretically be possible if acceleration curves spacetime by the inertial scale factor. However, if the acceleration is proportional to $-1/r^2$ as in a gravitational field around a central mass accumulation or in an electrostatic field of a central charge, the inertial field energy disappears, since the Laplace operator acting on $1/r$ disappears.

But, negative field energy might be induced with other inertial field potentials. Consider for example a static spherically symmetric inertial field potential $v^2/2$ proportional to r^a where a is a constant. The Laplace operator in spherical coordinates is:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} \tag{7.1}$$

We find from (6.2) that the inertial mass energy density has the same sign as $a(1+a)$; it is negative in the range $-1<a<0$ and is minimum for $a=-1/2$. However, with cylindrical coordinates the same inertial field potential r^a always yields positive inertial energy, since the sign is the same as for a^2.

Furthermore, it appears that negative energy also might be induced by charged particles accelerated by time-varying voltages and by certain kinds of rotating motion.

8. Concluding comments

At first it might seem untenable, or perhaps even objectionable, that negative spacetime energy may be artificially generated. Nevertheless, if Inertia and Gravitation have the same origin, it is not unreasonable that inertial energy corresponding to gravitational energy exists.

Two previous papers suggest that the phenomenon of Inertia is caused by changing spacetime metrics during acceleration [Masreliez, 2007a and Masreliez, 2008]. The present paper follows up on this proposition by deriving the energy-momentum tensor that would be induced by acceleration if the proposed theory of inertia were valid. A close correspondence is found between an inertial line-element and a gravitational line-element, both expressing energy density by Poisson's equation. This suggests that it might be possible to generate inertial spacetime energy from accelerating motion.

Summarizing:

If the spacetime metrics during acceleration change in a relative sense it would according to the proposed theory imply that an accelerating particle experiences spacetime curvature, which would explain Inertia. As a consequence of this curvature a corresponding energy-momentum tensor could be induced, with field energy density that could be positive or negative. If the motion is such that the mathematical divergence of the acceleration is negative, the induced inertial field energy density could become negative.

The possibility of artificially generating negative spacetime energy would be of great theoretical as well as practical interest.

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Appendix: A comment on matter-energy and the energy-momentum tensor

In his book 'Out of my later years' (1919) Einstein makes the following statement regarding his field equation:

But, it is similar to a building, one wing of which is made from fine marble (left part of the equation), but the other wing of which is made from low grade wood (right side of the equation).

The energy-momentum tensor, which usually is found on the right hand side of the field equation, has to be 'put in by hand'; it must be postulated, giving the impression that it contains matter or radiation energy, possibly of some other kind than what is given by the spacetime geometry on the left hand side. General relativity does not tell us how this matter energy is generated and sustained, which might be why Einstein calls it 'low grade wood'. It would be much more satisfying if the right hand also were made from the fine marble of spacetime geometry.

This idea is not new. Tullio Levi-Civita [Levi-Civita, 1917] and Hendrik Lorentz [Lorentz, 1916] independently proposed that Einstein's tensor implicitly defines gravitational

field energy density. With this interpretation Einstein's equations is an identity that simply says that the gravitational field energy density is such that it always matches the source energy field, but with opposite sign. Quoting Levi-Civita [Loinger, 2002]:
'The nature of ds^2 *is always such as to balance all mechanical actions; in fact the sum of the energy tensor and the inertial (spacetime) one identically vanishes.'*

We might therefore speculate that since the inertial spacetime field energy derived above appears to be of the same kind as the gravitational matter energy, it seems possible that the tensor $T_{\mu\nu}$ on the right hand side of Einstein's equations (3.2) actually might be generated by hitherto unknown processes that involve *metrics as expressed by the geometry on the left hand side*. Processes that modulate these metrics in time and/or space could according to GR generate energy, which might be the essence of matter. And, if the geometry on the left hand side models this matter its energy density *should* appear with negative sign. In this case the energy-momentum tensor on the right hand side of Einstein's equations could correspond to a spacetime tensor on the left hand side *with opposite sign*. The geometry on the left hand side would then consist of two parts; a first part corresponding to the energy-momentum tensor with negative sign, $-8\pi GT_{\mu\nu}$, and a second part matching the traditional gravitational spacetime curvature tensor (Einstein's tensor $G_{\mu\nu}$). In this case both sides of Einstein's equations would be spacetime geometry. In Einstein's words, both sides of his equations would be made 'from fine marble' and the essence matter would be nothing but spacetime geometry.

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