

Fsusy and Field Theoretical Construction

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Abstract

Following our previous work on fractional spin symmetries (FSS) [6, 7], we consider here the construction of field theoretical models that are invariant under the $D = 2(1/3, 1/3)$ supersymmetric algebra.

1 Superspace setup

Fractional supersymmetry [1, 2, 3, 4, 5, 6, 7] is once again considered in this work. The $D = 2(1/3, 1/3)$ superalgebra discussed in [2, 3, 6, 7] is generated by the left and right conserved charges $Q_{1/3}^-$, $Q_{1/3}^+$, P and $\overline{Q}_{-1/3}^-$, $\overline{Q}_{-1/3}^+$, \overline{P} respectively together with four topological charges $\Delta^{(-,-)}$, $\Delta^{(-,+)}$, $\Delta^{(+,-)}$ and $\Delta^{(+,+)}$ relating the two sectors. The \pm and 0 charges carried by these objects are those of the $Z_3 \times \overline{Z}_3$ automorphism symmetry acting as:

$$\begin{aligned}\Gamma Q^+ &= qQ^+, \Gamma Q^- = \overline{q}Q^-, \Gamma P = P, \\ \overline{\Gamma} Q^+ &= q\overline{Q}^+, \overline{\Gamma} Q^- = \overline{q}\overline{Q}^-, \overline{\Gamma} \overline{P} = \overline{P}, \\ \overline{\Gamma} Q^\pm &= \overline{Q}^\pm, \overline{\Gamma} \overline{Q}^\pm = Q^\pm, \overline{\Gamma} \overline{P} = \overline{P},\end{aligned}\tag{1}$$

where Γ and $\overline{\Gamma}$ are the generators of the Z_3 and \overline{Z}_3 group and where we have used the convention notations $Q_{-1/3}^\pm = \overline{Q}^\pm$ and $P_{-1} = \overline{P}$ in addition to $Q_{1/3}^\pm = Q^\pm$ and $P_1 = P$ used in [7]. The $D = 2(1/3, 1/3)$ supersymmetric algebra admits moreover an extra $Z_2 \times \overline{Z}_2$ symmetry, generated by $C \otimes \overline{C}$, acting as follows:

$$\begin{aligned}CQ^- &= Q^+C, \\ \overline{C}\overline{Q}^- &= \overline{Q}^+\overline{C}.\end{aligned}\tag{2}$$

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Note that under the complex conjugation (*) of complex variables $z^* = \bar{z}$, we have the obvious relations:

$$\begin{aligned} \bar{Q}^+ &= (Q^-), \\ \bar{P} &= P^*, \end{aligned} \tag{3}$$

showing that the left and right sectors are related by the complex conjugation of the two dimensional world sheet parametrized by z and \bar{z} . For later use, we quote hereafter the different automorphisms of the $D = 2(1/3, 1/3)$ superalgebra

	Q^+	Q^-	\bar{Q}^+	\bar{Q}^-	P	\bar{P}	$\Delta^{(-,-)}$
C	Q^-	Q^+	\bar{Q}^+	\bar{Q}^-	P	\bar{P}	$\Delta^{(+,-)}$
\bar{C}	Q^+	Q^-	\bar{Q}^-	\bar{Q}^+	P	\bar{P}	$\Delta^{(-,+)}$
$*$	\bar{Q}^-	\bar{Q}^+	Q^-	Q^+	\bar{P}	P	$\Delta^{(+,+)}$

(4)

A differential representation of the $D = 2(1/3, 1/3)$ superalgebra respecting (4) may be obtained by introducing a large superspace $(z, \theta^\pm, \bar{z}, \bar{\theta}^\pm, x^{++}, x^{--}, x^{+-}, x^{-+})$ with $(\theta^\pm)^3 = 0$ and $(\bar{\theta}^\pm)^3 = 0$. We find

$$\begin{aligned} D^- &= D^- + \alpha\bar{\theta}^- \partial^{(-,+)} + \alpha\bar{\theta}^+ \partial^{(-,-)}, \\ D^+ &= D^+ + \alpha\bar{\theta}^+ \partial^{(+,-)} + \alpha\bar{\theta}^- \partial^{(+,+)}, \\ \bar{D}^- &= \bar{D}^- + \bar{\alpha}\theta^- \partial^{(+,-)} + \bar{\alpha}\theta^+ \partial^{(-,-)}, \\ \bar{D}^+ &= \bar{D}^+ + \bar{\alpha}\theta^+ \partial^{(-,+)} + \bar{\alpha}\theta^- \partial^{(+,+)}, \\ P &= -\bar{q} \frac{\partial}{\partial z}, \bar{P} = -q \frac{\partial}{\partial \bar{z}}, \\ \Delta^{(+,+)} &= (\bar{\alpha} - \alpha q) \partial^{(+,+)}, \Delta^{(-,-)} = (\bar{\alpha} - \alpha \bar{q}) \partial^{(-,-)}, \\ \Delta^{(+,-)} &= (\bar{\alpha} - \alpha \bar{q}) \partial^{(+,-)}, \Delta^{(-,+)} = (\bar{\alpha} - \alpha q) \partial^{(-,+)}, \end{aligned} \tag{5}$$

where the derivatives along the extra directions, realizing the topological charges as translation generators, are defined as: $\partial^{(+,+)} = \frac{\partial}{\partial x^{--}}$ and so on. \bar{D}^- and \bar{D}^+ are the spin $\frac{1}{3}$ charge operators realizing the right sector of the $D = 2(1/3, 1/3)$ superalgebra without topological charges. As shown on the table (4), \bar{D}^- and \bar{D}^+ read as :

$$\begin{aligned} \bar{D}^+ &= \partial/\partial\bar{\theta}^- + \bar{\theta}^{-2} \partial/\partial\bar{z} \\ \bar{D}^- &= \partial/\partial\bar{\theta}^+ + \bar{\theta}^{+2} \partial/\partial\bar{z} \end{aligned} \tag{6}$$

Note that \bar{D}^- and \bar{D}^+ are related to each other the \bar{Z}_2 automorphism group acting on the superspace variables θ^\pm and z as

$$\begin{aligned} \bar{C}\theta^- &= \bar{\theta}^+ \bar{C} \\ \bar{C}\bar{z} &= \bar{z} \bar{C} \end{aligned} \tag{7}$$

Superfields describing off shell representations of the $D = 2(1/3, 1/3)$ superalgebra are superfunctions defined on the generalized superspace $(z, \theta, \bar{z}, \bar{\theta}, x)$. They consist of $3^4 = 81$ component fields depending on the bosonic variable z, \bar{z} and x . This is a big number of degrees of freedom that renders very difficult the elaboration of invariant field theoretical models under the $D = 2(1/3, 1/3)$ symmetry. However, forgetting about some automorphism symmetries, one may construct models which are invariant under subalgebras of $D = 2(1/3, 1/3)$. To do that, various possibilities are in order. The simple way is to ignore all the automorphisms given by (4). This is the case of the subalgebra:

$$\begin{aligned} Q^{-3} &= P, \\ \bar{Q}^{-3} &= \bar{P}, \\ Q^{-}\bar{Q}^{-} - \bar{q}\bar{Q}^{-}Q^{-} &= \Delta^{(-,-)} \end{aligned} \tag{8}$$

generated by non hermitian charge operators. Non unitary invariant models under this symmetry will be considered here. The same thing may be said about the subalgebra generated by $(Q^{+}, \bar{Q}^{+}, P, \bar{P})$ as it is related to (8) by the $Z_2 \times \bar{Z}_2$ symmetry generated by $C \otimes \bar{C}$. The second kind of models, which will be studied later, are based on the subalgebra:

$$\begin{aligned} \bar{Q}^{-} &= P, \\ \bar{Q}^{+} &= \bar{P}, \\ Q^{-}\bar{Q}^{+} - q\bar{Q}^{+}Q^{-} &= \Delta. \end{aligned} \tag{9}$$

These equations are stable under the complex (*) as shown by (4). The field theory invariant under (9) is real and may describe unitary $D = 2(1/3, 1/3)$ supersymmetric models. In what follows, we first study those theories that are invariant under (8). Introducing the superspace $(z, \theta^{+}, \bar{z}, \bar{\theta}^{+}, x^{++})$ with $\theta^{+3} = 0$ and $\bar{\theta}^{+3} = 0$ a representation of this algebra reads as:

$$\begin{aligned} D^{-} &= D^{-} + \alpha\bar{\theta}^{+}\partial^{(-,-)}, \\ \bar{D}^{-} &= \bar{D}^{-} + \beta\theta^{+}\partial^{(-,-)}, \\ P &= -\bar{q}\frac{\partial}{\partial z}, \bar{P} = -q\frac{\partial}{\partial \bar{z}}, \\ \Delta^{(-,-)} &= (\beta - \alpha\bar{q})\partial^{(-,-)}, \end{aligned} \tag{10}$$

where D^{-} and \bar{D}^{-} are given by (6). To check that the above relations form indeed a representation of (8), we follow the same strategy as before. First we calculate the the square of $D^{-}(\bar{D}^{-})$. We find:

$$D^{-2} = D^{-2} + \bar{\theta}^{+2}\partial^{(-,-)2} + (1 + \bar{q})\bar{\theta}^{+}\partial^{(-,-)}D^{-}, \tag{11}$$

where D^{-2} is given by [7]

$$D^{-2} = \partial^2/\partial\theta^{+2} + (1+q)\theta^+\partial/\partial\theta^+\partial/\partial z + (1+q^2)\theta^{+2}\partial^2/\partial\theta^{+2}\partial/\partial z \quad (12)$$

Repeating the some procedure, we get:

$$D^{-3} = (1+q)\frac{\partial}{\partial z} + \alpha(1+\bar{q}+\bar{q}^2)[\bar{\theta}^+D^- + \alpha\bar{\theta}^{+2}\partial^{(-,-)}]D^-\partial^{(-,-)}, \quad (13)$$

which reduces to P because of the identity $1+\bar{q}+\bar{q}^2=0$. A similar proof is valid for \bar{D}^- . Moreover using the commutation rules:

$$\begin{aligned} D^-\bar{D}^- &= q\bar{D}^-D^- \\ D^-\bar{\theta}^+ &= \bar{q}\bar{\theta}^+D^-, \\ \bar{D}^-\theta^+ &= \bar{q}\bar{\theta}^+\bar{D}^- \\ P\bar{D}^- &= \bar{D}^-P \\ \Delta^{(-,-)}\bar{D}^- &= \bar{D}^-\Delta^{(-,-)}, \end{aligned} \quad (14)$$

It is not difficult to see that the second equality of (8) is also satisfied.

2 Field theoretical construction

Superfields defined on the superspace $(z, \theta^+, \bar{z}, \bar{\theta}^+, x^{++})$ are usually complex. These off shell representations, consisting of 3^2 complex degrees of freedom, may carry both a spin $s = h - \bar{h}$ and $Z_3 \times \bar{Z}_3$ charge (m, n) with $m, n = 0, \pm 1 \pmod{3}$. The $\theta^+_{-1/3}$ and $\bar{\theta}^+_{1/3}$ expansion of a generic superfield $\phi^{(m,n)}_{h,\bar{h}}$ reads as:

$$\begin{aligned} \phi^{(m,n)}_{h,\bar{h}} &= \varphi^{(m,n)}_{h,\bar{h}} + \theta^+_{-1/3}\psi^{(m-1,n)}_{(h+\frac{1}{3},\bar{h})} + \bar{\theta}^+_{1/3}\eta^{(m,n-1)}_{(h,\bar{h}+\frac{1}{3})} \\ &+ \theta^+_{-1/3}\bar{\theta}^+_{1/3}F^{(m-1,n-1)}_{(h+\frac{1}{3},\bar{h}+\frac{1}{3})} + \theta^{+2}_{-1/3}\chi^{(m-2,n)}_{(h+\frac{2}{3},\bar{h})} \\ &+ \bar{\theta}^{+2}_{1/3}\lambda^{(m,n-2)}_{(h,\bar{h}+\frac{2}{3})} + \theta^{+2}_{-1/3}\bar{\theta}^+_{1/3}\xi^{(m-2,n-1)}_{(h+\frac{2}{3},\bar{h}+\frac{1}{3})} \\ &+ \bar{\theta}^{+2}_{1/3}\theta^+_{-1/3}V^{(m-1,n-2)}_{(h+\frac{1}{3},\bar{h}+\frac{2}{3})} + \theta^{+2}_{-1/3}\bar{\theta}^{+2}_{1/3}D^{(m-2,n-2)}_{(h+\frac{2}{3},\bar{h}+\frac{2}{3})} \end{aligned} \quad (15)$$

Taking $h = \bar{h} = -1$ and $n = m = -1$, we that the fields $\varphi^{(-,-)}$, $\chi^{(0,-)}_{\frac{2}{3}}$, $\lambda^{(-,0)}_{-\frac{2}{3}}$ and $D^{(-,-)}$ are exactly those appearing in the realization of thee critical spin $\frac{1}{3}$ supersymmetry of the TPM namely

$$\begin{aligned} \varphi^{(-,-)} &= \phi^{(-,-)}_{\frac{1}{21},\frac{1}{21}}, \quad D^{(0,0)} = \phi^{(0,0)}_{\frac{1}{21},\frac{5}{7}} \\ \chi^{(0,-)}_{\frac{2}{3}} &= \phi^{(0,-)}_{\frac{5}{7},\frac{1}{21}}, \quad \lambda^{(0,-)}_{-\frac{2}{3}} = \phi^{(-,0)}_{\frac{1}{21},\frac{5}{7}} \end{aligned} \quad (16)$$

The remaining fields $\psi_{\frac{1}{3}}^{(-,-)}$, $\eta_{-\frac{1}{3}}^{(-,+)}$, $F^{(+,+)}$, $\xi_{\frac{1}{3}}^{(0,+)}$ and $V_{-\frac{1}{3}}^{(+,0)}$ which are identified with:

$$\begin{aligned} \psi_{\frac{1}{3}}^{(-,-)} &= \phi_{\frac{8}{21}, \frac{1}{21}}^{(+,-)}, \quad \eta_{-\frac{1}{3}}^{(-,+)} = \phi_{\frac{1}{21}, \frac{8}{21}}^{(-,+)} \\ \xi_{\frac{1}{3}}^{(0,+)} &= \phi_{\frac{5}{7}, \frac{8}{21}}^{(0,+)}, \quad F^{(+,+)} = \phi_{\frac{8}{21}, \frac{8}{21}}^{(+,+)} \\ V_{-\frac{1}{3}}^{(+,0)} &= \phi_{\frac{8}{21}, \frac{5}{7}}^{(+,0)} \end{aligned} \tag{17}$$

Are extra conformal fields since they are not predicted by the $C = \frac{6}{7}$ conformal theory. They are however indispensable in the building of a manifestly $D = 2(1/3, 1/3)$ supersymmetric theory eventually invariant under the $Z_3 \times \bar{Z}_3$ discrete symmetry of (8). As for the left sector considered previously, here also the highest θ -component terms of superfields (14) transform as a total space time derivative under the change $\delta\theta^+ = \varepsilon^+$ and $\delta\bar{\theta}^+ = \bar{\varepsilon}^+$. Invariant actions S are then constructed as in $D = 2(1/2, 1/2)$ supersymmetric theories. We have:

$$S = \int d^2z d^4\theta L^{(-,-)}, \tag{18}$$

where $\int d^4\theta \sim \bar{D}^{-2} D^{-2}$ and where the super-Lagrangian $L^{(-,-)}$ carries a $(-, -)Z_3 \times \bar{Z}_3$ charge and scales as $1/3 + 1/3$ dimensional quantity since the integral measure scales as $(\text{length})^{1/3+1/3}$. Note that we have ignored the x -dependence realizing the topological charge $\Delta^{(-,-)}$. Other details will given when examining hermitian models. Using dimensional arguments, it is not difficult to see that $L^{(-,-)}$ is of the form:

$$L^{(-,-)} \sim D^- \phi_1^{(m,n)} \bar{D}^- \phi_2^{(-m,-n)} + W^{(-,-)}(\phi_1, \phi_2), \tag{19}$$

where the integers m, n may take the values $0, \pm 1$. As pointed out from the beginning of this work, the action S and the lagrangian (18-19) are not hermitian. Setting $n = m = -1$ as suggested by the thermal deformation of the TPM [6] and using the θ -expansion of the complex superfields $\phi_1^{(-,-)}$ and $\phi_2^{(+,+)}$ as well as the expression of the derivatives D^- and \bar{D}^- and (6), one may calculate the component fields contribution to the action S of the first term of (18). Straightforward algebra leads to:

$$\begin{aligned} D^- \phi_1^{(-,-)} &= \psi_{\frac{1}{3}}^{(+,-)} + \bar{\theta}^+ F^{(+,+)} + q^2 \bar{\theta}^{+2} V_{-\frac{1}{3}}^{(+,0)} \\ &\quad - q \bar{\theta}^+ \left(\chi_{\frac{2}{3}}^{(0,+)} + \bar{\theta}^+ \xi_{\frac{1}{3}}^{(0,+)} + \bar{\theta}^{+2} D^{(0,0)} \right) \\ &\quad + \theta^{+2} \left(\partial\varphi + \bar{\theta}^+ \partial\eta_{-\frac{1}{3}}^{(-,+)} + \bar{\theta}^{+2} \partial\lambda_{-\frac{2}{3}}^{(-,0)} \right), \end{aligned} \tag{20}$$

$$\begin{aligned}
\overline{D}^- \phi_1^{(+,+)} &= \overline{\eta}_{-\frac{1}{3}}^{(+,0)} + \overline{q}\theta^+ \overline{F}^{(0,0)} + q\theta^{+2} \overline{\xi}_{\frac{1}{3}}^{(-,0)} \\
&\quad - q\overline{\theta}^+ \left(\overline{\lambda}_{-\frac{2}{3}}^{(+,-)} + \theta^+ \overline{V}_{-\frac{1}{3}}^{(0,-)} + q\theta^{+2} \overline{D}^{(-,-)} \right) \\
&\quad + \overline{\theta}^{+2} \left(\overline{\partial}\overline{\varphi}^{(+,+)} + \theta^+ \overline{\partial}\overline{\psi}_{\frac{1}{3}}^{(0,-)} + \theta^{+2} \overline{\partial}\overline{\chi}_{\frac{2}{3}}^{(-,+)} \right), \quad (21)
\end{aligned}$$

where we have put a bar on the component fields of the superfield $\phi_2^{(+,+)}$ in order to avoid the confusion with the $\phi_1^{(-,-)}$ component fields. Evidently, ∂ and $\overline{\partial}$ mean $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial \bar{z}}$ respectively. Integrating with respect to $d^4\theta$ the superfield kinetic term taking into account the commutation rule $\overline{\theta}^+ \theta^+ = q\theta^+ \overline{\theta}^+$, we get:

$$\begin{aligned}
L_0 &\sim \partial\varphi^{(-,-)} \overline{\partial}\overline{\varphi}^{(+,+)} + (\partial\lambda_{-\frac{2}{3}}^{(-,0)} \overline{\eta}_{-\frac{1}{3}}^{(+,0)} - \overline{q}\chi_{\frac{2}{3}}^{(0,-)} \overline{\partial}\overline{\psi}_{\frac{1}{3}}^{(0,-)}) \\
&\quad - \left(\partial\eta_{-\frac{1}{3}}^{(-,+)} \overline{\lambda}_{-\frac{2}{3}}^{(+,-)} - \overline{q}\psi_{\frac{1}{3}}^{(+,-)} \overline{\partial}\overline{\chi}_{\frac{2}{3}}^{(-,+)} \right) \\
&\quad - \overline{q}\psi - qD^{(0,0)} \overline{F}^{(0,0)} - \overline{q}F^{(+,+)} \overline{D}^{(-,-)} + \overline{q}V_{-\frac{1}{3}}^{(+,0)} \overline{\xi}_{\frac{1}{3}}^{(-,0)} + \overline{q}\xi_{\frac{1}{3}}^{(0,+)} \overline{V}_{-\frac{1}{3}}^{(0,-)} \quad (22)
\end{aligned}$$

Note that this relation contains two kinds of fields. Dynamical fields namely $\varphi^{(-,-)}$, $\psi^{(+,-)}$, $\overline{\eta}^{(-,+)}$, $\lambda^{(-,0)}$ and $\overline{\varphi}^{(+,+)}$, $\overline{\psi}^{(0,-)}$, $\overline{\eta}^{(+,0)}$, $\overline{\lambda}^{(+,0)}$. They obey free field equations of motion whose solutions factorise into analytic and antianalytic parts. Auxiliary fields $F^{(+,+)}$, $V^{(+,0)}$, $\xi^{(0,+)}$, $D^{(0,0)}$ and $\overline{F}^{(0,0)}$, $\overline{\xi}^{(-,0)}$, $\overline{V}^{(0,-)}$, $\overline{D}^{(-,-)}$. They appear linearly in L_0 and lead then to constraint equations. Details on the role of these fields will be given later. Note moreover that (22) is invariant under the following transformations:

$$\begin{aligned}
\delta\varphi^{(m,n)} &= \varepsilon_{-\frac{1}{3}}^+ \psi_{\frac{1}{3}}^{(m-1,n)} + \overline{\varepsilon}_{\frac{1}{3}}^+ \eta_{-\frac{1}{3}}^{(m,n-1)} \\
\delta\psi_{\frac{1}{3}}^{(m-1,n)} &= -q\varepsilon_{-\frac{1}{3}}^+ \chi_{\frac{2}{3}}^{(m-2,n)} + \overline{\varepsilon}_{\frac{1}{3}}^+ F^{(m-1,n-1)} \\
\delta\eta_{-\frac{1}{3}}^{(m,n-1)} &= -\overline{\varepsilon}_{\frac{1}{3}}^+ \lambda_{-\frac{2}{3}}^{(m,n-2)} + \overline{q}\varepsilon_{-\frac{1}{3}}^+ F^{(m-1,n-1)} \\
\delta F^{(m-2,n-1)} &= (1+q)\varepsilon_{-\frac{1}{3}}^+ \xi_{\frac{1}{3}}^{(m-2,n-1)} - \overline{q}\overline{\varepsilon}_{\frac{1}{3}}^+ V_{-\frac{1}{3}}^{(m-1,n-2)} \\
\delta\chi_{\frac{2}{3}}^{(m-2,n)} &= q\varepsilon_{-\frac{1}{3}}^+ \partial\varphi^{(m,n)} + \overline{\varepsilon}_{\frac{1}{3}}^+ \xi_{\frac{1}{3}}^{(m-2,n-1)} \quad (23) \\
\delta\lambda_{-\frac{2}{3}}^{(m,n-2)} &= \overline{q}\overline{\varepsilon}_{\frac{1}{3}}^+ \overline{\partial}\varphi^{(m,n)} + \varepsilon_{-\frac{1}{3}}^+ V_{-\frac{1}{3}}^{(m-1,n-2)} \\
\delta\xi_{\frac{1}{3}}^{(m-2,n-1)} &= \overline{q}\varepsilon_{-\frac{1}{3}}^+ \partial\eta_{-\frac{1}{3}}^{(m,n-1)} - q\overline{\varepsilon}_{\frac{1}{3}}^+ D^{(m-2,n-2)} \\
\delta V_{-\frac{1}{3}}^{(m-1,n-2)} &= \overline{q}\overline{\varepsilon}_{\frac{1}{3}}^+ \overline{\partial}\psi_{\frac{1}{3}}^{(m-1,n)} - q\varepsilon_{-\frac{1}{3}}^+ D^{(m-2,n-2)} \\
\delta D^{(m-2,n-2)} &= \varepsilon_{-\frac{1}{3}}^+ \partial\lambda_{-\frac{2}{3}}^{(m,n-2)} - \overline{q}\overline{\varepsilon}_{\frac{1}{3}}^+ \overline{\partial}\chi_{\frac{2}{3}}^{(m-2,n)}
\end{aligned}$$

The spin $\pm 4/3$ supersymmetric conserved current G^- and \overline{G}^- generating these transformations are obtained by using the Noether method. They read as:

$$G^- = \partial\varphi^{(-,-)} \overline{\psi}^{(0,+)} - q\psi^{(+,0)} \overline{\partial}\overline{\varphi}^{(+,+)} + \overline{q}\chi^{(0,-)} \overline{\lambda}^{(-,+)}, \quad (24)$$

$$\overline{G}^- = \overline{q}\overline{\partial}\varphi^{(-,-)} \overline{\eta}^{(+,0)} + \overline{q}\overline{\partial}\overline{\varphi}^{(+,+)} \eta^{(-,+)} + \lambda^{(-,0)} \overline{\lambda}^{(+,-)} \quad (25)$$

Finally, observe that starting from (23, 24) and using the Z_2 -symmetries generated by C and \overline{C} , we can build the field realisations of the dual current G^+ and \overline{G}^+ as follows:

$$G^+ = CG^-C^{-1}, \overline{G}^+ = \overline{C}\overline{G}^-\overline{C}^{-1}. \tag{26}$$

We have for G^+ for instance:

$$G^+ = \partial\varphi^{(+,-)}\psi^{(0,+)} + \psi_{1/3}^{(-,-)}\partial\overline{\varphi}^{(-,+)} + \overline{q}\chi_{2/3}^{(0,-)}\overline{\chi}_{2/3}^{(+,+)} + \dots, \tag{27}$$

where $C\varphi^{(+,-)} = \varphi^{(-,-)}C$ and so on. The superpotient term $W^{(-,-)}$ is a priori an arbitrary function of the superfields ϕ_1 and ϕ_2 which may be restricted by requiring convariance under the $Z_3 \times \overline{Z}_3$ transformations. The most general form of $W^{(-,-)}$ respecting the $Z_3 \times \overline{Z}_3$ symmetry reads then as

$$\begin{aligned} W^{(-,-)} = & \sum_m \left(g_m \phi_1^{(-,-)^{3m+1}} + g'_m \phi_2^{(+, +)^{3m+2}} \right) \\ & + \sum_m g''_m \phi_1^{(-,-)^{m+2}} \phi_2^{(+, +)^{m+1}} \end{aligned} \tag{28}$$

where g_m, g'_m and g''_m are coupling constants. Note that leading linear term in the above equation $g_0\phi_1^{(-,-)}$, integrated with respect to $d^4\theta$, give $g_0D^{(0,0)}$. It describes exactly the $\phi_{1,3}$ thermal perturbation of the $c = 6/7$ critical theory as shown by (16) and (14). We expect that this term is the mediator of the spontaneous breaking of the $D = 2(1/3.1/3)$ supersymmetry of the TPM. Recall that in the case of the TIM, the $\phi_{1,3}$ field scaling as $3/5 + 3/5$ conformal object breaks spontaneously the $(1/2, 1/2)$ supersymmetry of the $c = 7/10$ model. Unfortunately, this feature cannot be checked directly on the scalar potential $V(\varphi, \varphi^*)$ since the theory we are considering in this section is non unitary .we shall not pursue this direction. $D = 2(1/3.1/3)$ supersymmetry breaking will be discussed on the following unitary model.

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