Plane Symmetric Cosmological Model with
Wet Dark Fluid in Bimetric Theory of Gravitation

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Abstract: The purpose of this paper is to investigate the role of wet dark fluid in plane- symmetric cosmological model within the frame work of bimetric theory of gravitation proposed by Rosen[16]. In this theory, it is observed that there is no contribution from wet dark fluid.

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1. Introduction

Einstein’s theory of general relativity is one of the most beautiful structures of theoretical physics which is also known as the most successful theory of gravitation in terms of geometry. In the last decades, the several theories of gravitation has / had been proposed as alternatives to the theory of general relativity. The most popular amongst them are scalar-tensor theories of gravitation and Rosen’s[16] bimetric theory of gravitation. In bimetric theory of gravitation , the physical situation is determined by means of two metric tensors, viz : the Riemannian metric tensor $g_{ij}$ and the background metric tensor $\gamma_{ij}$ corresponding to flat space-time. The Riemannian metric tensor $g_{ij}$ plays the same role as in Einstein general relativity and it interacts with matter whereas the background metric tensor $\gamma_{ij}$ is related to the geometry of the empty universe and it describes the inertial forces. The
interpretation of these two metric tensors in bimetric relativity theory is not unique. One can regard the $g_{ij}$ as a flat space time having no physical and geometrical significance but the physical metric tensor $g_{ij}$ is considered as a gravitational potential tensor which is determined by field equations or by interactions between matter and gravitation. In the absence of matter, one should have $g_{ij} = \gamma_{ij}$. The bimetric theory of gravitation, like the Einstein general relativity theory, satisfies the covariance and equivalence principle.

Rosen[17] proposed a new bimetric theory of gravitation on the cosmological basis in accordance with the perfect cosmological principle. In this new bimetric theory, the background metric tensor $g_{ij}$ is considered as describing a space time of constant curvature and not a flat space time of constant curvature but it should rather be chosen on the basis of cosmological consideration. Many authors Rosen[18], Israelit[6], Leibscher[10], Karade[7], Reddy and Venkateswarlu[13], Reddy and Venkateshwara Rao [14], Adhav et al.[1,2] and Katore et al.[8,9] have studied the bimetric theory of gravitation in different aspects with different space times.

Riess et al.[15],Perlmuttar et al.[12],Sahni[19] studied the nature of the dark energy component of the universe as one of the deepest mysteries of cosmology. We are motivated to use the wet dark fluid (WDF) as a model for dark energy which stems from an empirical equation of state proposed by Tait [22] and Hayward[4] to treat water and aqueous solution.

The equation of state for Wet Dark Fluid is

$$p_{WDF} = \gamma (\rho_{WDF} - \rho_*)$$

where the parameters $\gamma$ and $\rho_*$ are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$.

We have energy conservation equation as

$$\dot{\rho}_{WDF} + 3H (p_{WDF} + \rho_{WDF}) = 0.$$  

Using equation of state and $3H = \dot{v} / v$ in the above equation, we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma} \rho_* + \frac{c}{v(1+\gamma)} ,$$

where $c$ is the constant of integration and $v$ is the volume expansion. WDF has two components: one behaves as cosmological constant and other as standard fluid with equation of state $p = \gamma \rho$.

If we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$:

$$p_{WDF} + \rho_{WDF} = (1+\gamma) \rho_{WDF} - \gamma \rho_*$$

$$= (1+\gamma) \frac{c}{v(1+\gamma)} \geq 0. $$

In this paper we study the plane symmetric cosmological model with wet dark fluid in bimetric theory of gravitation. After solving the field equations one can conclude that plane-symmetric cosmological model does not exist in case of wet dark fluid in bimetric theory of gravitation.

2. Metric and Solutions of Field Equations:

The general cylindrically-symmetric metric is considered in the form given by Marder [11] as
\[ ds^2 = A^2 (dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2 , \]
where \( A, B \) and \( C \) are functions of \( x \) and \( t \).

In order to find a specific solution, we confine ourselves to the form of metric given by Singh and Singh[21] as
\[ ds^2 = A^2 (dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2 , \]  
where \( A, B \) and \( C \) are functions of time \( t \) alone.

This metric represents the anisotropic homogeneous universe (Aygun et al., [3]).

The background flat metric corresponding to equation (1) is
\[ d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 \]  
(2)

The field equations in bimetric theory of gravitation proposed by Rosen [16] are
\[ N'_{j} - \frac{1}{2}N\delta'_{j} = -8\pi kT_{j}^{i} \]  
(3)

Where \( N'_{j} = \frac{1}{2}\gamma^{ab}_{j}(g^{hi}_{j}g_{j}^{k})_{b} \)

and \( k = \left( \frac{g}{r} \right)^{\frac{1}{2}} \)
together with \( g = \det(g_{j}) \) and \( \gamma = \det(\gamma_{j}) \).

Here the vertical bar ( | ) denotes the covariant differentiation with respect to \( \gamma_{j} \)
and \( T_{j}^{i} \) is the energy momentum tensor of the matter fields.

The Rosen’s field equations in bimetric theory for the metric (2) are written in the form
\[ \left( \frac{B_{j}^{1}}{B} \right)_{4} + \left( \frac{C_{j}^{1}}{C} \right)_{4} = 16\pi k T_{1}^{i} \]  
(4)

\[ 2\left( \frac{A_{j}^{1}}{A} \right)_{4} - \left( \frac{B_{j}^{1}}{B} \right)_{4} + \left( \frac{C_{j}^{1}}{C} \right)_{4} = 16\pi k T_{2}^{2} \]  
(5)

\[ 2\left( \frac{A_{j}^{1}}{A} \right)_{4} + \left( \frac{B_{j}^{1}}{B} \right)_{4} - \left( \frac{C_{j}^{1}}{C} \right)_{4} = 16\pi k T_{3}^{3} \]  
(6)
\[
\left( \frac{B_t}{B} \right)_4 + \left( \frac{C_4}{C} \right)_4 = 16\pi k T^4_4
\]  \hspace{1cm} (7)

Here the suffix 4 denotes differentiation with respect to \( t \).

The energy momentum tensor (Singh and Chaubey [20]) for the Wet Dark Fluid source is given by

\[
T^i_j = (\rho_{\text{WDF}} + p_{\text{WDF}}) u^i u^j - p_{\text{WDF}} \delta^i_j
\]  \hspace{1cm} (8)

where \( u^i \) is the flow vector satisfying

\[
g^i_{\ j} u^i u^j = 1
\]  \hspace{1cm} (9)

In comoving system of coordinates, from equations (8) & (9), we get

\[
T^1_1 = T^2_2 = T^3_3 = -p_{\text{WDF}} \quad , \quad T^4_4 = \rho_{\text{WDF}}
\]  \hspace{1cm} (10)

Now using equation (10), equations (4) - (7) becomes

\[
\left( \frac{B_t}{B} \right)_4 + \left( \frac{C_4}{C} \right)_4 = -16\pi k p_{\text{WDF}}
\]  \hspace{1cm} (11)

\[
2 \left( \frac{A_4}{A} \right)_4 - \left( \frac{B_t}{B} \right)_4 + \left( \frac{C_4}{C} \right)_4 = -16\pi k p_{\text{WDF}}
\]  \hspace{1cm} (12)

\[
2 \left( \frac{A_4}{A} \right)_4 + \left( \frac{B_t}{B} \right)_4 - \left( \frac{C_4}{C} \right)_4 = -16\pi k p_{\text{WDF}}
\]  \hspace{1cm} (13)

\[
\left( \frac{B_t}{B} \right)_4 + \left( \frac{C_4}{C} \right)_4 = 16\pi k \rho_{\text{WDF}}
\]  \hspace{1cm} (14)

From Equation (11) and (12), we obtain

\[
\left( \frac{A_4}{A} \right)_4 = \left( \frac{B_t}{B} \right)_4
\]  \hspace{1cm} (15)

From equation (11) and (13), we obtain

\[
\left( \frac{A_4}{A} \right)_4 = \left( \frac{C_4}{C} \right)_4
\]  \hspace{1cm} (16)

From equation (15) and (16), we obtain

\[
\left( \frac{A_4}{A} \right)_4 = \left( \frac{B_t}{B} \right)_4 = \left( \frac{C_4}{C} \right)_4
\]  \hspace{1cm} (17)

Adding equations (11) to (14), we find the solution in the form

\[
p_{\text{WDF}} + \rho_{\text{WDF}} = 0
\]  \hspace{1cm} (18)

For reality conditions to hold we need \( p_{\text{WDF}} > 0 \) and \( \rho_{\text{WDF}} > 0 \).

For which the condition (18) is satisfied only when

\[
p_{\text{WDF}} = 0 \quad \text{and} \quad \rho_{\text{WDF}} = 0
\]

This means that the physical parameters pressure \( (p_{\text{WDF}}) \) and energy density \( (\rho_{\text{WDF}}) \) of the wet dark fluid are identically zero. Thus, in bimetric theory of gravitation...
Bianchi type I universe with wet dark fluid does not survive and hence only vacuum model is obtained. For vacuum case \( p_{DF} = \rho_{DF} = 0 \), the field equations (11) – (14) admit the solution of the form

\[
A = \exp(k_1 t + k_1) , \quad B = \exp(k_2 t + k_2) , \quad C = \exp(k_3 t + k_3)
\]

(19)

where \( k_1, k_2, k_3, k_4, k_5 \) and \( k_6 \) are the constants of integration.

Thus, in view of equation (19), the metric (1) takes the form

\[
ds^2 = \exp(k_1 t + k_1)(dt^2 - dx^2) - \exp(k_2 t + k_2)dy^2 - \exp(k_3 t + k_3)dz^2
\]

Which [for \( k_1 = k_2 = k_3 = \alpha \) and \( k_4 = k_5 = k_6 = \beta \)] reduces to

\[
ds^2 = e^{(\alpha + \beta)}(dt^2 - dx^2 - dy^2 - dz^2)
\]

(20)

With the proper choice of coordinate transformations and constants, the above line element (20) reduces to

\[
ds^2 = e^T(dT^2 - dX^2 - dY^2 - dZ^2)
\]

(21)

It is interesting to note that, the model is conformally flat and free from singularity. At \( T = 0 \), the model reduces to flat one.

3. Conclusion

Here, we have constructed a plane symmetric cosmological model in Rosen[16] bimetric theory of gravitation with a new equation of state for the dark energy component of the universe (known as Wet Dark Fluid). It is observed that the plane symmetric cosmological model does not exists in Bimetric theory of gravitation with wet dark fluid as a source of gravitation and hence only vacuum model is obtained.

References


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