Dynamic Response of Sandwich Panels under Impulsive Loading

H. Garehbabaee

Department of Mechanical Engineering
Shahr-e-Rey Branch, Islamic
Azad University, Tehran, Iran
garehbabaee@gmail.com

Abstract
This paper presents analytical method for determining deflection of sandwich panel with structure of fully clamped circular and rectangular plates subjected to impulsive loading. The panel response was modeled in three consecutive phases. Calculations of the cases indicate that the proposed analytical model is based on reasonable assumption. The solutions obtained are in very good agreement with different set of experimental results. This shows the merit of the model and it is hoped that the analysis will be complementary to existing simplified model in providing a quick solution on the mode of plastic deformation of sandwich panel subjected to impulsive loading.

Keywords: Sandwich Panel, Impulsive load, Deformation

1. Introduction
The dynamic response of structures subjected to blast-induced shock load is a complex phenomenon. Many researchers have investigated the nonlinear dynamic behavior of simple structures both experimentally and theoretically and a detailed review of these efforts can be found in the recent book by Jones [1]. Clamped sandwich plates are representative of the structures used in the design of
commercial and military vehicles. For example, the outermost structure on a ship comprises plates welded to an array of stiffeners. The superior performance of sandwich plates relative to monolithic solid plates is well known for applications requiring high quasi-static strength. However, the resistance of sandwich plates to dynamic loads remains to be fully investigated in order to quantify the advantages of sandwich design over monolithic design for application in shock resistant structures.

Now these structures have been increasingly used as energy absorbers in a wide range of impact/blast protective applications, such as vehicle, aircraft, ship, packaging and military industries. Unlike conventional structures which undergo only small elastic deformation, energy absorbers have to sustain intense impact loads, so that their deformation and failure may involve large geometry changes, strain-hardening effects, strain-rate effects and various interactions between different deformation modes such as bending and stretching. The elastic behavior of sandwich panels has been extensively studied and well documented in several technical books [2–5].

The present paper attempts to explain a theoretical formulation based on upper bound solution. The major objective is to predict high rate plastic deformation of circular and rectangular sandwich panels subjected to transverse impulsive loading and for clamped boundary condition. The key assumption employed in the method is that effects of circumferential and radial strains are dominate during deformation process and thickness strain is negligible which in turn simplifies the formulation and reduces the mathematical complexity of the problem.

2. Analysis

The whole deformation process can be split into three stages as shown in Figure(1):

![Figure 1: Three stage in the response of a sandwich panel subjected to the blast loads.](image)

Stage I – The blast impulse \( I \) is transmitted to the front face of sandwich structure, and the front face is assumed to have instantly obtained a velocity \( V_1 \) while the rest of the structure is stationary.

Stage II – The core is compressed while the back face is stationary.
Stage III – The back face starts to deform and the components of the plate obtain the identical velocity $v_2$, and finally the structure is brought back to rest by plastic bending and stretching.[6]

2.1. Stage I – front face deformation
The impulse delivered onto the sandwich structure (I) is assumed to have a uniform distribution over the front face. With the impulse transmission, the front face has an initial velocity as [6],

$$V_1 = \frac{I}{\pi \rho_f h_f R^2}$$  \hspace{1cm} (1)

where $R$ is radius of the circular plate, $\rho_f$ and $h_f$ are material density and thickness of front face plate, respectively.

The corresponding kinetic energy of the front face is obtained by

$$KE_1 = \frac{I^2}{2\pi \rho_f h_f R^2}$$  \hspace{1cm} (2)

2.2. Stage II – core compression
At the end of this stage, the front and back faces as well as the core structure all have an identical velocity

$$V_2 = \frac{I}{\pi R^2 (2\rho_f h_f + \rho_c h_c)}$$  \hspace{1cm} (3)

where $\rho_c$ is mass density of core material, and $h_c$ is initial thickness of core. Correspondingly, the kinetic energy of the entire structure at the end of Stage II can be written as [6]

$$KE_2 = \frac{I^2}{2\pi R^2 (2\rho_f h_f + \rho_c h_c)}$$  \hspace{1cm} (4)

Hence, the energy absorption during core compression is

$$E_p = KE_1 - KE_2$$  \hspace{1cm} (5)

The profile of the front face and front surface of core at the end of this stage is approximated by the following shape function [6-7]

$$w_{cf}(r) = \beta \left( 1 - \left( \frac{r}{R} \right)^2 \right)$$  \hspace{1cm} (6)

where $w_{cf}$ is the transverse displacement of the front plate and core and $\beta$, is the transverse displacement at the centre. $r$ and $R$ are the radial coordinate and the radius of the circular structure, respectively.

Then, the energy dissipation during core crushing can be obtained by

$$E_p = 2\pi \sigma_c^e \int_0^R \beta \left( 1 - \left( \frac{r}{R} \right)^2 \right) rdr$$  \hspace{1cm} (7)

where $\sigma_c^e$ is the transverse compressive strength of core. Its value is estimated
using the formulae given in reference [6, 4, 3]. Therefore,

$$
\beta = \frac{2E_p}{\pi \sigma; R^2}
$$

(8)

2.3. stage III – overall bending and stretching

In this phase, both front and back faces are considered that have large deflection. For simplification, the initial flat plate is considered.[7]

It is assumed that, suitable mathematical functions to describe deflection profile of front and back circular plate are following formulas;

$$
\begin{align*}
  w(r)_{\text{front}} &= (W_o + \beta) \left( 1 - \left( \frac{r}{R} \right)^2 \right) \\
  w(r)_{\text{back}} &= W_o \left( 1 - \left( \frac{r}{R} \right)^2 \right)
\end{align*}
$$

(9)

(10)

where \( w(r)_{\text{front}} \) and \( w(r)_{\text{back}} \) are transverse displacement of the front and face plate, respectively. and \((W_o + \beta)\) and \((W_o)\) are transverse displacement at the center of the front and face plate, respectively. \(r, R\) are the radial coordinate and outer radius of the plate, respectively. The values of curvatures and strains by using equation (9) and (10) can be obtained as;

$$
\begin{align*}
  \kappa_r &= -\frac{\partial^2 w}{\partial r^2} = W_o \frac{a^2}{R^2} \left[ J_0 \left( \frac{ar}{R} \right) - \frac{R}{ra} J_1 \left( \frac{ar}{R} \right) \right] \\
  \kappa_\theta &= - \frac{1}{r} \frac{\partial w}{\partial r} = W_o \frac{a}{rR} J_1 \left( \frac{ar}{R} \right)
\end{align*}
$$

(11)

(12)

and,

$$
\begin{align*}
  \varepsilon_r &= \varepsilon_m + \varepsilon_{r\theta} = \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2 + z \kappa_r \\
  \varepsilon_\theta &= \varepsilon_{r\theta} = z \kappa_\theta
\end{align*}
$$

(13)

(14)

where in above equations, \( \kappa_r \) and \( \kappa_\theta \) are the radial and circumferential curvatures of the plate, respectively. \( \varepsilon_m, \varepsilon_{r\theta} \) and \( \varepsilon_{r\theta} \) are the radial membrane strain, radial bending and circumferential bending strains, respectively, \(z\) is the transverse coordinate.[8-10]

The total plastic work done during the deformation of the sandwich structure is:

$$
W_p = W_{P(\text{front})} + W_{P(\text{back})} = \int_V (\sigma_r \varepsilon_r + \sigma_\theta \varepsilon_\theta)_{\text{front}} dV + \int_V (\sigma_r \varepsilon_r + \sigma_\theta \varepsilon_\theta)_{\text{back}} dV.
$$

(15)

where, \( W_{P(\text{front})} \) and \( W_{P(\text{back})} \) are plastic work done during the deformation of front and back faces, respectively. It is assumed that radial and circumferential strains are significant and thickness strain \( \varepsilon_z \) is negligible.
Furthermore, it is assumed that the mean dynamic flow stress can be expressed as a scalar multiple of the quasi-static yield stress.\[11\]
\[
\sigma_d = \lambda \sigma_y
\]  \hspace{1cm} (16)
where $\sigma_d$ is the mean dynamic flow stress, $\lambda$ is a constant, and $\sigma_y$ is the quasi-static yield stress. By introducing flow relation and Tresca yield criterion for rigid plastic material, the following relation can be obtained for circular plate:
\[
\sigma_r = \sigma_\theta = \sigma_d
\]  \hspace{1cm} (17)

For determining value of $\lambda$, the ratio between static and dynamic yield stresses, the well known Cowper–Symonds constitutive equation is used. Therefore, equation (16) can be rewritten in the form;
\[
\frac{\sigma_d}{\sigma_y} = 1 + \left(\frac{\dot{\varepsilon}_m}{D}\right)^{\frac{1}{q}}
\]  \hspace{1cm} (18)
where $\dot{\varepsilon}_m$ is the mean strain rate, $D$ and $q$ are material constants, typical values for mild steel are given as $D = 40.4 \text{ s}^{-1}$ and $q = 5$\[1\].

As an estimate of mean strain rate $\dot{\varepsilon}_m$, taking a moderate value for deflection of 15 mm, equation (13) implies maximum strain of approximately 0.089. Bodner and Symonds [12] and Nurick [13-14] reported that blast loaded plates reach their maximum deflection in approximately 120 $\mu$s which implies a mean strain rate in the order of $742 \text{ s}^{-1}$. Substituting the values for the mean strain rate and material constants into equation (18) yields an average value for $\lambda$ of 2.8.

Equation (15) can be converted into following equation:
\[
W_p = 2\pi \lambda \sigma_y \left[ \int_0^{h_f/2} \int_0^{h_f/2} (\varepsilon_r + \varepsilon_\theta)_{\text{front}} \, rdz \, dr \right] + \left[ \int_0^{h_f/2} \int_0^{h_f/2} (\varepsilon_r + \varepsilon_\theta)_{\text{back}} \, rdz \, dr \right]
\]  \hspace{1cm} (19)
where $h_f$ and $h_b$ are thickness of front and back plate, respectively. and $z$ is the transverse coordinate. It should be noted that the direction of principle stresses and principle strains are the same. So with substituting the relevant values into equation (19) and integrating with respect to $z$ and $r$ gives:
\[
W_p = \pi \lambda \sigma_y \left[ h_f + h_b \right] \left[ h_f^2 + h_b^2 + 2h_f \beta \right] W + h_f \beta \left[ \beta + h_f \right]
\]  \hspace{1cm} (20)
The energy dissipated through plastic work($W_p$) is equal to the initial kinetic energy of the structure($KE$).
\[
W_i = \frac{1}{2} \left( h_f^2 + h_b^2 + 2h_f \beta \right)^2 - \frac{I^2}{2\pi^2 \lambda \sigma_y R^4 (2h_f + \rho f h_f)} - \left( h_f^2 + h_b^2 + 2h_f \beta \right)
\]  \hspace{1cm} (21)
where $W_i$ is maximum deflection of the back face. Above equation may be applicable to the deformation of an impulsively loaded clamped rectangular plate with length $a$ and width $b$ ($a \times b$) by replacing $R$ in Eqs. (2, 4, 5, 8 and 21) with $R = \sqrt{ab}/\pi$ \hspace{1cm} (22)
3. Comparison with the experimental data

The model obtained in the preceding section (equation (21)) is compared with the experimental results which were reported in References [6, 15-16] for both foam core and honeycomb core panels. Figure(2) shows the comparison of the present theoretical predictions with the experimental data for the back face deflections of clamped circular plates. Figure(3) also shows the comparison between theoretically predicted back face deflections and the experimental results, for clamped rectangular plates. It is clear from Figure (2) and (3) that Equation (21) agrees well with the experimental data for the back face deflections of clamped circular and rectangular plates.

![Figure 2: Comparison between the experimental and predicted maximum deflection of the back face of circular plate for two types of core.](image-url)
4. Conclusion

The model developed in the present work accounts for the energy dissipation through plastic work. The solution is determined completely by material and geometrical parameter. Analytical predictions of present model for central deflection of fully clamped impulsively loaded circular and rectangular plates are in good agreement with numerous experimental data. This is despite of fact that calculation based on the analytical solution of present model involved only values of radial and circumferential strains. It is also shown that, to a first approximation, the theory developed for circular plates can be applied to rectangular plates.

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