Two Qubit von Neumann Entropy in Ising Chain
with Added Dzyaloshinskii – Moriya
and Blume – Capel Interaction

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Abstract

In this paper, the Dzyaloshinskii – Moriya(DM) and Blume – Capel(BC) interaction effect in Von Neumann entropy of spin- 1 Ising model has been considered. The spin -1 Ising model has the lack of Von Neumann entropy in the absence of interaction(DM) and (BC) interaction. In the presence of DM and BC interaction, the Von Neumann entropy started to increase by increasing the power of DM interaction and it shows that the pure states change into mixed states.

Keywords: Ising model, Dzyaloshinskii – Moriya interaction, Blume – Capel interaction

Introduction

The study of Von Neumann entropy has been much considered in recent decade, and this one of the most important and wonderful concepts in quantum mechanics.[1,2] The quantum entanglement is very important in solid state physics and spin systems. [3,4] also, the entanglement has been seen naturally in quantum many body systems and quantum phenomenon like super conductivity[5], Hall quantum effect[6] and quantum transition [7]. It is obvious that one qubit system doesn’t have the possibility of the entanglement progress in a N qubit system since it arrives from a separated state to another separated state. So, we
can use interaction between two qubit for accessing to an entangled state. The two qubit entanglement of spin $\frac{1}{2}$ XX,XYZ model has been examined.[8,9,] In one hand , the two qubit entanglement of XYZ spin $\frac{1}{2}$ has completely been studied in the different states, in the presence of longitudinal magnetic field. Also, the thermal entanglement in different states of XYZ model has been exactly studied in the presence of DM.[10,11]. The DM interaction [15,16] is used for studying the weak ferromagnetic and anti ferromagnetic crystals like $Yb_4As_3$, $K V_2O_3$, $LaMnO_2$, $Fe\alpha$.[12,13,14]. The DM interaction is actually that orbit-spin interaction and by using the phenomenological approach, DM interaction was shown as: $\vec{D} \times (\vec{S}_i \times \vec{S}_j)$ which $\vec{D}$ is the DM vector. In the interaction discussion, we can examine Von Neumann entropy.[17] Von Neumann entropy is the modified of Shannon entropy (classical information theory).[18] Von Neumann entropy is zero for pure states and for mixed states is non-zero. If we write the density matrix in the terms of ground state as: $\rho = |\psi_0\rangle \langle \psi_0|$ which in that $|\psi_0\rangle$ is the ground state ket, in this case , the Von Neumann entropy will be as $E = -\sum_i \rho_{ii} \log \rho_{ii}$, that $\rho_{ii}$ will be the elements on the diagonal of the density matrix and the logarithm base according to the quantum number $m_z$. For two qubit system of spin-1, Von Neumann entropy is as: $E = -\sum_i \rho_{ii} \log \rho_{ii}$. For studying of two qubit interaction , we can use spin-1-particle Ising model. The Ising model , lonely, hasn’t any entanglement since the states of that are pure but if we add DM interaction of to the Ising model, it is expected that the mixed state created and the entanglement is observed. For this purpose, we study the entanglement of Ising model in the presence of DM interaction.

2- spin-1 Ising model in the presence of spin - orbit interaction

The Hamiltonian of Ising model for spin-1 is as below :

$$H_0 = J \sum_j S_j^z S_{j+1}^z$$

(1)

$J$ is the coupling constant ( antiferromagnetic $J\langle 0$ , ferromagnetic $J\rangle 0$ ), the Hamiltonian of DM interaction is as below:

$$H_{DM} = \vec{D} \sum_j (\vec{S}_j \times \vec{S}_{j+1})$$

(2)

In which $\vec{D}$ is the DM vector and we choose as $\vec{D} = D\vec{e}$ and in this case, the Hamiltonian of Ising model will be as below figure in the presence of DM interaction:

$$H = J \sum_j S_j^z S_{j+1}^z + D \sum_j (S_j^z S_{j+1}^z - S_j^y S_{j+1}^y)$$

(3)
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And for two particles system, we can write the Hamiltonian as below figure:

\[ H = J S_i^z S_j^z + D(S_i^x S_j^y - S_i^y S_j^x) \]  

(4)

And we choose the base kets of the Hilbert space as eigenstates of the \( S_i^z \otimes S_j^z \) operator. In this case, we used the Hamiltonian matrix representation (4) we examine the Von Neumann entropy for anti ferromagnetic state (ferromagnetic). The Von Neumann entropy in the anti ferromagnetic state will be as below:

\[ E(\rho) = -(\frac{4D^2}{8D^2 + (J-q)^2})\log_2(\frac{(J-q)^2}{8D^2 + (J-q)^2}) - (\frac{4D^2}{8D^2 + (J-q)^2})\log_2(\frac{(J+q)^2}{8D^2 + (J+q)^2}) \]

(6)

Which \( q = \sqrt{J^2 + 8D^2} \). We must attend that in the anti ferromagnetic, per all D values, the system has got the entanglement.

In ferromagnetic state, if \( |J| \langle D \), we have got entanglement and the Von Neumann entropy will be:

\[ E(\rho) = -(\frac{4D^2}{8D^2 + (J+q)^2})\log_2(\frac{(J+q)^2}{8D^2 + (J+q)^2}) - (\frac{4D^2}{8D^2 + (J+q)^2})\log_2(\frac{(J-q)^2}{8D^2 + (J-q)^2}) \]

(7)

And in \( |J| \rangle D \) area, we don’t have any entanglement.

3- spin-1 Ising model in the presence of BC and DM interaction

The Hamiltonian of Ising model in the presence of DM and BC interaction will be as below:

\[ H = J \sum_j S_i^z S_{j+1}^z + D \sum_j (S_i^x S_{j+1}^y - S_i^y S_{j+1}^x) + b \sum_j (S_i^z)^2 \]  

(8)

Which for two particles state will be as below:

\[ H = J S_i^z S_j^z + D(S_i^x S_j^y - S_i^y S_j^x) + b(S_i^z)^2 \]

(9)

And we have chosen the \( S_i^z \otimes S_j^z \) bases as the Hilbert space base kets. For the Von Neumann entropy anti ferromagnetic state, it will be:

\[ E(\rho) = -(\frac{2}{2+a^2})\log_2(\frac{1}{2+a^2}) - (\frac{a^2}{2+a^2})\log_2(\frac{a^2}{2+a^2}) \]

(10)

Which in that \( a = \frac{1}{2D}(-J + 2b + \sqrt{8D^2 + (J-2b)^2}) \) and for all the D,b values, we have got entanglement. The Von Neumann entropy changes diagram is given in figure 1.

Also, for ferromagnetic state, per value of \( 0 \leq b \leq \frac{1}{2} \), \( |J| \rangle \sqrt{1-2b} \) we have got entanglement and the Von Neumann entropy will be:
\[ E(\rho) = -\left(\frac{2}{2+a'^2}\right) \log_2\left(\frac{1}{2+a'^2}\right) - \left(\frac{a'^2}{2+a'^2}\right) \log_2\left(\frac{a'^2}{2+a'^2}\right) \] (11)

which \( a' = \frac{1}{2D} (J + 2b + \sqrt{8D^2 + (J + 2b)^2}) \). The Von Neumann entropy changes diagram is given in figure 2 according to D, b changes. According to figure 1and 2, the system is lack of Von Neumann entropy in the absence of DM and BC interaction means that the states are completely pure. If we also enter the BC interaction in the absence of DM, the states will also stay pure. By affecting DM interaction, the states will change into mixed states from pure ones that this makes the Von Neumann entropy starts to increase from zero. So that by increasing the power of DM interaction, the Von Neumann entropy also starts to increase. In the anti ferromagnetic, per \( b \leq 0.7 \), the Von Neumann entropy will maximize and then decreases.

**Conclusion**

We have considered the Von Neumann entropy of spin-1 Ising model in the presence of DM and b-k interaction. The Von Neumann entropy is zero in the absence of DM interaction that indicates the existence of pure states in the system. The Von Neumann entropy gets out of zero by DM interaction effect and it started to increase by increasing the power of DM interaction, that indicates the existence of mixed states.

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**References**


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Figure 1: The Von Neumann entropy diagram for Ising model in the presence of DM and b-k interaction in the anti ferromagnetic state of spin1 Ising model according to Dm interaction power (D) and b-k interaction power (b).
Figure 2: The Von Neumann entropy diagram for Ising model in the presence of DM and b-k interaction in the ferromagnetic state of spin 1 Ising model according to Dm interaction power (D) and b-k interaction power (b).

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