Longitudinal and Transverse Elastic Waves in
One-Dimensional Phononic Crystals

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Abstract

We have fabricated one-dimensional phononic crystals making a manual arrangement of alternate layers of aluminum and epoxy. The experimental realization is designed to present longitudinal and transverse phononic band gaps in the range of megahertz frequencies. Detailed studies of mechanical waves transmission have been experimentally performed using the Fourier transform of short duration pulses. The band structure of the phononic crystal is calculated using an analytical formula. There is a good agreement between the experimental results and the theoretical dispersion relation.

Keywords: Phononic crystal, elastic wave, longitudinal, transverse.

1 Introduction

Phononic crystals are periodic structures made of two materials with different elastic properties [1]. Phononic crystals with band gaps for mechanical waves are counterparts -by analogy- to photonic crystals with forbidden gaps for photon propagation [2]. Phononic crystals are one-, two- or three-dimensional arrangements with periodicity on the same order of magnitude with the gap wavelength. In these systems, the main consequence of the periodicity is the creation of phononic band gaps that are frequency intervals over which the propagation of mechanical waves is forbidden. Phononic crystals can be used as isolators, perfect acoustic mirrors, noise suppression or by the introduction

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of defects, they can be used as resonators, cavities or guidance of elastic waves. Phononic crystals in the megahertz (MHz) range are suitable for applications in biomedical ultrasound or acoustic microscopy [3].

Recently special attention has been paid to the study of one-dimensional phononic crystals [4-12]. One of the authors has reported the experimental verification of an omni-directional mirror in one-dimensional phononic crystals in the range of kHz [8]. More recently have been reported the fabrication of one-dimensional phononic crystals with periods in the range of the micrometer where is possible to attain gaps in the gigahertz range [12]. Independently of the frequency range of the band gaps, to study the mechanical propagation in these periodic structures is of great interest since new effects related to the spatial periodicity have been recently reported, such as the modeling of heterostructures with giant photonic band gaps [13,14], the existence of surface Tamm states [15,16] or the observation of phononic Bloch oscillations [17,18].

In this work, we present the experimental fabrication and characterization of a one-dimensional phononic crystal. The transmission spectra shows evidence of the first twelve longitudinal bands, which are in the low megahertz range. We have also measured some transverse bands. We have performed theoretical calculations of the band structure using an analytical formula. We have found an excellent agreement between the experimental results and the theoretical dispersion relation.
2 Theory

The band structure is calculated for an infinite one-dimensional phononic crystal illustrated in Fig. 1(a). This structure is composed by the repetition of a unit cell of period \( d \) composed by two thin films of aluminum (dark gray) and epoxy (light gray) of width \( d_{al} \) and \( d_{ep} \), respectively. The phononic dispersion relation can be obtained from the analytical formula [12]

\[
\cos(kd) = \cos \left( \frac{2\gamma d_{al}}{c_{al}} \right) \cos \left( \frac{2\gamma d_{ep}}{c_{ep}} \right) - \frac{1}{2} \left[ \frac{\rho_{al} c_{al}^{L}}{\rho_{ep} c_{ep}} + \frac{\rho_{ep} c_{ep}^{L}}{\rho_{al} c_{al}} \right] \sin \left( \frac{2\gamma d_{al}}{c_{al}} \right) \sin \left( \frac{2\gamma d_{ep}}{c_{ep}} \right)
\]

where \( k \) is the phononic wave vector, \( \gamma \) is the phonon frequency, \( c_{al}^{L} \) and \( c_{ep}^{L} \) are the constituent layer longitudinal or transverse phonon velocities, and finally \( \rho_{al} \) and \( \rho_{ep} \) are the densities for the aluminum and epoxy, respectively.

3 Longitudinal waves

Fig. 1(b) shows a schematic draw of a finite multilayer of aluminum (5052-H32) and epoxy (Epotek 302). The multilayer is composed of five layers of aluminum and four layers of epoxy. The aluminum layer has a width of \( d_{al} = 3 \) mm and the epoxy layer has a width of \( d_{ep} = 4.5 \) mm. The multilayer period is \( d = d_{al} + d_{ep} = 7.5 \) mm. The width of the multilayer is \( D = 4d + d_{al} \). The incidence and transmission media are illustrated in black at both ends of the multilayer and are made of high temperature bakelite delay line. The sample was fabricated by a careful manual arrangement of the subsequent layers of aluminum and epoxy. We have take care to avoid to produce air bubbles in the epoxy during the fabrication process. However, is inevitable to produce a certain amount of air bubbles. Anecdotally, we have found that all our attempts to remove the air bubbles have produced more air bubbles.

The constituent layer densities for the aluminum and epoxy are \( \rho_{al} = 2680 \text{ kg/m}^3 \) and \( \rho_{ep} = 1100 \text{ kg/m}^3 \), respectively. The longitudinal velocities are \( c_{al}^{L} = 6234 \text{ m/s} \) and \( c_{ep}^{L} = 2500 \text{ m/s} \). The comparison for the theoretical dispersion relation (left side) and the experimental transmission (right side) are presented in Fig. 2. In the right side of panels (a), (b), (c) and (d) we present the results obtained for the transmission (in arbitrary units, au) for the transducers of 250 kHz, 500 kHz, 1 MHz and 1.5 MHz. We observe that in all the cases, it exist a good agreement between the high transmission ranges with the allowed bands. Conversely, exist a clear relation between the low transmission zones and the band gaps. It is important to note that in panel (d) we verify the existence of the 11th and 12th bands which is a prove of the good crystalline quality of the sample.
Figure 2: Comparison of the band structure (left side) with the experimental transmission (right side). In panels (a), (b), (c) and (d) we present the results for transducers of 250 kHz, 500 kHz, 1 MHz and 1.5 MHz, respectively.

4 Transversal waves

Now we consider the transmission of transverse waves. The transverse velocities are \( c_{al} = 3100 \text{ m/s} \) and \( c_{tp} = 1160 \text{ m/s} \). In Fig. 3 we present in the left side the band structure for longitudinal and transverse waves with dotted and dashed lines, respectively. In the right side, we present the experimental transmission obtained for a 500 kHz normal incidence shear wave transducer. We observe that for the case of transverse waves, it exist a certain degree of acoustical contamination from the longitudinal bands over the transverse bands which can be observed in the transmission. It has been reported [19] that for an homogeneous medium the contamination between the longitudinal amplitude \( A_l \) over the transverse amplitude \( A_t \) is on the order of \(-30dB\).

The equivalence in the lineal scale can be calculated in the form

\[
-30dB = 20 \log \left( \frac{A_l}{A_t} \right) \tag{2}
\]

\[
A_l \approx 0.03A_t \tag{3}
\]

We have found that the contamination in the transmission can be much more important for the multilayer structure than for an homogeneous medium due to the existence of multiple reflections for the spurious longitudinal modes which can cause constructive interference. On the other hand, it has been
Figure 3: Comparison of the band structure and the transverse wave transmission. In the left, we present the band structure for longitudinal and transverse bands with dotted and dashed line respectively. In the right side we present the transverse transmission, where the existence of the transverse bands is verified, but also can be observed the contamination of longitudinal waves.

reported [20] that the absorption in the polymer is twice greater for transverse waves than for longitudinal waves. These two effects contributed to observe a transverse transmission that is the result of the combination of both bands, transverse and longitudinal.

In the right side of Fig. 3 we present with thick arrows the maximums related with transverse waves. Conversely, we present with tiny arrows the maximum related with longitudinal waves. We observe in the range from 250-350 kHz a clear sign of two transverse bands, but also we observe the sign of a longitudinal band in the middle of these transverse bands. In the range of 450-550 kHz, the transmission verify the existence of two transverse bands. However, the existence of the band gap between is not well defined. This probably is due that the transductor is centered in the 500 kHz and the maximum power is at this frequency. Nevertheless, it exist a small attenuation in the band gap just below the 500 kHz, which can be considered as the sign of the band gap. Finally, in the range of 600 kHZ we observe a tiny manifestation of a transverse band.
5 Conclusions

We have presented experimental results of the transmission for longitudinal and transverse waves in a finite one dimensional phononic crystal. We have compared these results with the theoretical band structure. We have found that the existence of bands and band gaps is easily demonstrated for longitudinal waves and it has been possible to verify the existence up to the 12th. band. In contrast, we have found that the verification of transverse bands is difficult due to the presence of contamination of longitudinal waves in the transverse transducer. We have found that this contamination can be much more important in multilayers than in homogeneous media due to the existence of two different phenomena. The first is related to the multiple reflections of the spurious longitudinal modes that increases the longitudinal coherent transmission. The second is related to the bigger absorption that suffers the transverse waves with respect to the longitudinal waves. We considered that these two phenomena can be a limiting factor in study of phenomena related with the propagation of transverse waves in phononic crystals.

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References


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