Case Study: On Most General Exact Solution of Plasma Sheath Model for a Negatively Biased Probe

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Abstract

Plasma characteristics have been identified with the aid of an insulated probe analysis immersed in the plasma. An Exact solution approach coupled with initial values for the sheath thickness and wall potential can predict the overall relation between the distance from the wall and wall potential based on a positive space charge near the vicinity of the probe. The I-V plasma characteristics of the probe can identify the position of the point or potential where the total current drawn by the probe is zero, i.e. floating potential.

Keywords: Plasma sheath, Mathematical modeling, Debye sheath

I Introduction

Plasma "the fourth state of matter" is an ionized gas composed of different charges particles such as electrons and ions, and the presence of neutral atoms is important to generate sufficient ions and electrons in the discharge. A simple plasma can be identified by equal number density of ions and electrons, i.e. $N_e = N_i$, electron temperature $T_e$, ion temperature $T_i$, and Debye length $\lambda$. Plasma research was
developed from the study of low density ionized gases contained in discharged tubes. The first ionized medium was named by W. Crooks who paved the way towards understanding the discharge [1]. However, it was not until 82 years ago that the term Plasma was introduced in connection with their studies of oscillations in an ionized gas [2]. In space, Plasmas play an important role in stars in a process called fusion, where hot, dense plasmas release a large amount of energy that, partly, comes to us as visible light. Lower temperature plasmas, or cold plasmas, have many applications on earth such as pasteurization of foods, sterilization of medical products, environmental cleanup, gas discharges for lighting and lasers, isotope separation, and switching and welding technology. Plasma is mainly utilized for the fabrication of semiconductors, the plasma etching of semiconductors, surface modification, and growth of wide range of films and materials. More interesting, Plasmas for spacecraft propulsion was also explored [3]. A plasma sheath is the localized electric field that separates plasma from a material boundary. It confines the more mobile species in the plasma and accelerates the less mobile species out of the plasma and toward the walls. For the typical case where the electrons are more mobile than the positively charged ions, the electric field in the sheath points toward the boundary. Measurements of ion velocity and density in the plasma sheath were investigated using laser-induced fluorescence (LIF) [4]. The most straightforward way to measure the I-V characteristic of plasma is with a single probe, consisting of one electrode biased with voltage ramp relative to the vessel.

In the present article, most general exact solution has been obtained and analyzed with respect to the physical acceptability to predict the overall relation between distance from the wall and wall potential based on positive space charge near the vicinity of the probe.

II Plasma Sheath Model

A plasma sheath model is defined when a probe is immersed in plasma which acquires a negative potential and near the wall of the probe there is a boundary layer in which the potential increases monotonically from a negative value on the wall to a zero value to correspond the unperturbed plasma; this boundary layer is called the plasma sheath. The transition from macroscopic electrical neutrality caused by the presence of a charge induced field exists over a distance or radius called the Debye length [5]. Figure1 illustrates the plasma sheath and variation of the potential and the number densities inside the sheath.
The number densities of ions $N_i$ and electrons $N_e$ are obtained from a Maxwellian distribution as follows:

$$N_e(x) = N_0 e^{\frac{-eV(x)}{kT}}$$

$$N_i(x) = N_0 e^{\frac{-eV(x)}{kT}}$$

where $N_0$ is electron density in the bulk plasma far from sheath edge, and $e$, $V$, $K$, and $T$ are electron charge, potential, Boltzmann constant and temperature respectively.

The current densities $J_e$ and $J_i$ for the electrons and the ions caused by their random motions can be obtained from the kinetic gas theory [6]; these are expressed as:

$$J_e = \frac{I}{4} eN_e U_e$$

$$J_i = \frac{I}{4} eN_i U_i$$

where $U_i$ and $U_e$ are the average velocity of ions and electrons.

Under equilibrium conditions,

$$I_e = I_i \text{ at } x = 0$$

where $I_e$ and $I_i$ are the electron and ion currents respectively and $x$ is the distance from the wall.
If the average energies of the random ions and electrons are equal:

\[
\frac{1}{2} m_i U_i^2 = \frac{1}{2} m_e U_e^2
\]

where \( U = \left( \frac{8KT}{\pi m} \right)^{1/2} \) \( \quad (5) \)

From equations (1) – (4), it yields:

\[
e^{\left( \frac{2eV}{kT} \right)} = \left( \frac{m_i}{m_e} \right)^{1/2}
\]

This could be simplified to give the wall potential as:

\[
V(w) = -\frac{KT}{4e} \ln \left( \frac{m_i}{m_e} \right)
\]

The absolute ion velocity is expressed based on work done equivalence to kinetic energy, hence:

\[
U_i = \left( \frac{2eV}{m_i} \right)^{1/2}
\]

The total ion current crossing a surface area \( A \) around the probe is:

\[
I_i = eAN_i U_i
\]

Now, Poisson’s equation leads to,

\[
\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0} = -\frac{e}{\varepsilon_0} (N_i - N_e)
\]

This is to be solved exactly as a most general exact solution in the forthcoming section.

III Most general exact solution

The sheath thickness can be approximately determined assuming that the electron density is negligible compared with the ion density. The sheath that is formed on the anode as well as that on the cathode is a positive space charge; therefore the major population in the sheath is ions. The schematic diagram for the variation of the electric potential near the surface of a negatively biased probe have been presented and shown in the figure (2) below [7]. The same sheath mechanism but in a much thicker is formed near the cathode surface [8].
Now, for a positive space charge, \( N_e = 0 \); and substituting equation (7) into equation (8), to get:

\[
\frac{d^2 V}{dx^2} = -\frac{\rho}{\varepsilon_0} = -\frac{I}{\varepsilon_0 A} \left( \frac{m_i}{2eV} \right)^{1/2} 
\]

\[
\frac{d^2 V}{dx^2} = -\frac{I}{\varepsilon_0 A} \left( \frac{m_i}{2e} \right)^{1/2} V^{-1/2} 
\]

\[
\frac{d^2 V}{dx^2} = \gamma V^{-1/2} 
\]

where \( \gamma = -\frac{I}{\varepsilon_0 A} \left( \frac{m_i}{2e} \right)^{1/2} \)  

In order to solve equation (10), multiplying both sides of equation by \( \frac{dV}{dx} \) and then integrating with respect to \( x \), to get:

\[
\left( \frac{dV}{dx} \right)^2 = 4\gamma V^{1/2} + c_1 
\]

This leads to the most general exact solution and can be furnished as:

\[
\ln \left[ 2\gamma^{1/2} V^{1/4} + \sqrt{4\gamma V^{1/2} + c_1} \right] = x + c_2 
\]
IV Sensitivity Analysis of the result for Plasma Sheath Model

The Current density of the ions across the sheath can be obtained as follows: since the ions follow Maxwellian destitution, their probability of existence is proportional to the Boltzmann factor $e^{eV/kT}$:

$$J = eN_{is}U_{is} = eN_{io}e^{\left(\frac{eV_s}{kT_e}\right)}\left(-\frac{2eV_s}{m_i}\right)^{1/2}$$ (15)

At the plasma–sheath interface, and as illustrated in the figure, the voltage $V_s$ can be expressed as [6, 7]:

$$V_s = -\frac{kT_e}{2e}$$ (16)

Substituting equation (16) into equation (15) yields:

$$J = N_{io}e^{\left(-\frac{1}{2}\right)}\left(\frac{kT_e}{m_i}\right)^{1/2}$$ (17)

The floating potential $V_f$ can be obtained by setting the total current drawn by the probe to zero. Since most electrons are repelled, the random electron current reduced by Boltzmann factor is:

$$I_e = \frac{1}{4} e A_s N_o e^{kT} U_e$$ (18)

The total electric current is: $I_t = I_e + I_i$

$$I_t = \frac{1}{4} e A_s N_{io} e^{kT} \left(\frac{8kT}{\pi m_i}\right)^{1/2} - e A_s N_{io} e^{\frac{1}{2}} \left(\frac{kT}{m_i}\right)^{1/2} = 0$$ (19)

This can be written as:

$$e A_s N_{io} \left(\frac{kT_e}{m_i}\right)^{1/2} \left[\frac{1}{4} e^{kT} \left(\frac{8m_i}{\pi m_e}\right)^{1/2} - e \frac{I}{2}\right] = 0$$ (20)

Or

$$\left[\frac{1}{2} e^{kT} \left(\frac{2m_i}{\pi m_e}\right)^{1/2} - e \right]^\frac{1}{2} = 0$$ (21)

$$\frac{eV_f}{kT_e} = \frac{1}{2} (\ln \frac{2\pi m_e}{m_i} - 1)$$ (22)

Typically around a probe tip, the sheath is composed of positive ions to minimize the loss of electrons. The extension of the sheath into the plasma is determined by the Debye length, and is usually several Debye lengths in diameter [9]. The Debye length
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...depends on electron energy, and density of the plasma [10]. The Debye length

$$\lambda_D = \left(\frac{e_0 K T_e}{N_e e^2}\right)^{\frac{1}{2}}$$

where $e_0$ is the permittivity of free space and $N_e$ is the electron density.

From the probe data analysis, the following parameters have been obtained.

$T_e = 42000 K (3.62 \text{ eV})$

$N_e = 4.97 \times 10^{15} \text{ part/m}^3$

$\lambda_D = 0.2008 \text{ mm}$

Now, at boundary conditions $x = 0$, $V(0) = V(w) = 12 \text{ volts}$, $x = \infty$, $V(\infty) = 0$, and taking the potential $V_s = 0$ at the plasma-sheath interface according to equation (16). The electron temperature is determined from the slope of the plasma I-V characteristics. A plot of Equation (13) is shown in Figure (3) below.

Fig. (3), Distance from the wall as a function of potential for a negatively biased probe

If the potential at the sheath-plasma interface, i.e., $V_s = 1.81 \text{ V}$, then a plot of equation (13) is shown in Figure (4) below.
It should be noted that biasing a substrate or generating a plasma beam which strikes the substrate during film deposition at room temperature can improve the optical quality of the deposited films; bombarded by ions with a potential called "floating potential" [11].

V Conclusion

A sheath model has been presented which illustrates the behaviour of the relation between the potential of a negatively biased probe and the distance from the wall. An exact solution has been obtained and analyzed to explain that behaviour. This analysis can also predict the heat load that the probe acquires which could be used as an indication for the ions bombarding the substrate and the quality of deposited films; when the substrate is placed in place of the probe.

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