A Note about the Physics within
One Electromagnetic Wave Length

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Abstract

This brief note represents our further pursuit of some of the implications of our previously constructed "combined spacetime 4-manifold M3." Here, we derive from Maxwell Equations and Einstein Relativity, with the quantum probability waves reinterpreted as waves of real energies, eight simple propositions that together give a renewed description of the dynamics of our composite energy entity: (photon in the particle universe M1, electromagnetic wave in the wave universe M2), within one wave length.

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1 Introduction

This note presents an analysis of the physics within one wave length of an electromagnetic radiation. We will derive eight propositions from Maxwell Equations coupled with Einstein Relativity, with the underlying spacetime geometry to be our previously constructed combined spacetime 4-manifold $\mathcal{M}^3$ (cf. [8]). To avoid any unnecessary repetition from our previous work, we will now immediately extract those elements of our $\mathcal{M}^3$ that will be pertinent to this paper, referring any details to our above cited paper.

To begin with, $\mathcal{M}^3$ is a product spacetime of a particle universe $\mathcal{M}^1$ and a wave universe $\mathcal{M}^2$ (which predated $\mathcal{M}^1$ and is devoid of any matter), with $\mathcal{M}^1$ and $\mathcal{M}^2$ related in the fashion of a diagonal map. As a direct consequence, we related the quantum probability waves to the energy waves from
Maxwell Equations. $\mathcal{M}^{[1]}$ and $\mathcal{M}^{[2]}$ are separately based on two independent sets of Einstein Field Equations, each with its own gravitational constant, $G^{[i]}$, $i = 1, 2$. We unified the two sets of gravitational motions into one, resulting in the recognized universal gravitational constant

$$G^{[3]} = \frac{G^{[1]} G^{[2]}}{G^{[1]} + G^{[2]}}. \tag{1}$$

We also derived the value of $G^{[2]}$:

$$G^{[2]} = \frac{c^5 \cdot \text{second}^2}{1.6\hbar} \quad (\hbar \equiv \text{the Planck constant}) \tag{2}$$

$$\approx 2.3 \times 10^{75} \times \frac{\text{meter}^3}{\text{kilogram} \times \text{second}^2} \approx 10^{85} G^{[3]} \tag{3}.$$

As a consequence, $G^{[3]} \approx G^{[1]}$. Since $G^{[2]}$ is so large, the pre-existing pure electromagnetic waves contained in $\mathcal{M}^{[2]}$ easily formed an astronomically large Black Hole B. The center of B by its infinite gravity could transform an electromagnetic wave in $\mathcal{M}^{[2]}$ into a (photon, wave), existing in a composite universe $\mathcal{M}^{[3]} = \mathcal{M}^{[1]} \times B \subset \mathcal{M}^{[1]} \times \mathcal{M}^{[2]}$, which was the Big Bang of $\mathcal{M}^{[1]}$. By Feynman’s analysis, we distributed the energy $E$ of any (particle, wave) to be $(\frac{3}{4} E, \frac{1}{4} E)$. Electromagnetic waves in the Black Hole B exist in a quotient-spacetime structure due to the complex structure of $\{(it, ix, iy, iz)\}$ inside the Schwarzschild radius, which accounts for the quantum mechanics therein.

Without the existence of our $\mathcal{M}^{[2]}$ that contains (real) energy, quantum mechanics is based on probabilities, and our topic here has long been well addressed (cf. e.g., [11], pp. 812-822). Consequently few new research studies exist in this regard. Within the quantum framework, Maxwell Equations in free space refer to a vacuum, where the creation and annihilation operators ensure the probabilistic existence of a photon of energy $E = h\nu$ in this ground state (cf. e.g., [9]). Solving the Schrödinger wave equation for the photon’s position states, one would derive a wavefunction that gives positive probabilities throughout the entire space (cf. e.g., [3,5,6]), where the commutator of position and momentum operators obeys the Heisenberg relation. The photon’s spins are then the eigenvalues of the spin operator (cf. e.g., [1]). Our work here will provide new perspectives of these quantum mechanical descriptions.

2 Analysis

**Theorem 1** (Proof omitted) Maxwell Equations admit the following electromagnetic wave in a spacetime free from matter:
(1) an electric field $E = (0, A \cos (kx - \omega t), 0),\n(2) a magnetic field $B = (0, 0, \frac{A}{c} \cos (kx - \omega t))$, and
(3) the Poynting vector $S := \varepsilon_0 c^2 E \times B = (\varepsilon_0 c A^2 \cos^2 (kx - \omega t), 0, 0)$, as referenced by an arbitrary inertial frame $S$.

Notation 1 Henceforth we will denote the above (1), (2), and (3) by $\Phi$, and all the following propositions will be based on $\Phi$.

Proposition 1 The energy $E$ contained in $\Phi \ \forall t \in \mathbb{R}$ has its projection on the $x$–axis : $[x_1 (t), x_2 (t)]$, with $x_2 (t) - x_1 (t) < \infty$.

Proof. Without loss of generality, we set $t = 0$. Since $E$ has density $\frac{\|E\|}{c} = \varepsilon_0 c A^2 \cos^2 (kx)$, $x$ necessarily varies within $[x_1 (0), x_2 (0)]$ with $x_2 (0) - x_1 (0) \in (0, \infty)$, for otherwise $E = \infty$.

Proposition 2 Continuing Proposition 1, one has $\forall t \in \mathbb{R}$

$$x_2 (t) - x_1 (t) \equiv b (t) = \lambda = \frac{2\pi}{k}$$

$$\equiv \frac{c}{\nu} \equiv \frac{2\pi c}{\omega}.$$

Proof. By Planck’s well-known energy formula, $h \omega = \hbar \nu = E$; thus, $\Phi$ defines exactly one photon with wave length $\lambda \equiv \frac{c}{\nu}$. For any given length $l_o$ there exists an inertial frame $S_o$ of velocity $v = (-v_o, 0, 0)$ such that relative to $S_o$, by Lorentz transformation, one has $\lambda > l_o$. Since $\lambda$ contains exactly one photon of $E = \frac{h \omega}{\lambda}$ and $\lambda$ can be arbitrarily large, one has $b (t) \neq m \lambda \ \forall m \geq 2$: Consider $b (t) = 2\lambda$: then two photons exists in $b (t)$. However, there exists a frame $S_\alpha$ such that $b (t) = 1 \cdot \lambda$ so that $S_\alpha$ observes only one photon, a contradiction. By the quantum wavefunction, any fraction of $\lambda$ may reveal 0 or 1 photon. Thus, for $\Phi$ to reveal one photon with probability 1 we have $\forall t \in \mathbb{R}$, $x_2 (t) - x_1 (t) \equiv b (t) = \lambda$.

Proposition 3 Continuing Proposition 2, one has $\forall t \in \mathbb{R}$

$$[x_1 (t), x_2 (t)] \equiv [0, \lambda] \equiv [0, 2\pi].$$

(Cf. e.g., [2].)

Proof. With loss of generality, we set, by Proposition 2, $[x_1 (0), x_2 (0)] = [0, \lambda]$. Then $\forall t \in \mathbb{R}$,

$$[x_1 (t), x_2 (t)] = [0 + ct, \lambda + ct] \equiv [0, \lambda] + ct.$$

(Cf. e.g., [2].)
so that a frame at rest with \([0, \lambda]\) renders \(t \equiv 0\) and thus \([x_1(t), x_2(t)] \equiv [0, \lambda]\).

Since \(\lambda k = 2\pi\), one has \(\forall \xi \in [0, 1]\)

\[
(L\lambda) \cdot k = \xi \cdot 2\pi \equiv \phi, \text{ i.e.,}\]

(8)

\[
\xi \lambda \equiv \phi(\xi) \in [\phi_\beta, \phi_\beta + 2\pi]
\]

(9)

for some \(\phi_\beta \in \mathbb{R}\).

(10)

We show \(\phi_\beta = 0\): Since 0 and \(\lambda\) are of the identical phase in \([0, 2\pi]\), \(\lambda^2\) is necessarily the returning point of a simple harmonic motion from 0 to \(\frac{\lambda}{2}\) and \(\frac{\lambda}{2}\) to 0; as a result, 0 and \(\frac{\lambda}{2}\) possess the maximum potential energy, proportional to

\[
1 = \cos^2(0 \equiv 2\pi) \equiv \cos^2(\phi(\xi = 0, 1))
\]

(11)

\[
= \cos^2\left(\phi\left(\xi = \frac{1}{2}\right)\right) = \cos^2\phi.
\]

(12)

Thus, \(\phi_\beta = 0\) and \([0, \lambda] \equiv [0, 2\pi]\). ■

**Proposition 4** Continuing Proposition 3, one has \(\forall t \in \mathbb{R}\)

\([x_1(t), x_2(t)] \equiv S^1\), with radius \(r = \frac{\lambda}{2\pi}\).

(13)

**Proof.** This is a corollary to Proposition 3. ■

**Proposition 5** Continuing Proposition 4, one has \(\forall t \in \mathbb{R}, [x_1(t), x_2(t)] \equiv S^1\) spin in the motion of

\[
\begin{pmatrix}
  x(t) \\
y(t) \\
z(t)
\end{pmatrix}
= \begin{pmatrix}
  ct + \xi \lambda \\
r \rho r \cos\left(\frac{ct + \xi \lambda}{r}\right) \\
r \rho r \sin\left(\frac{ct + \xi \lambda}{r}\right)
\end{pmatrix},
\]

(14)

with \(0 \leq \xi, \rho \leq 1\). (Cf. e.g., [7])

**Proof.** Consider the frame in the proof of Proposition 3 that is at rest with the traveling wave \([0, \lambda] + ct \equiv [0, \lambda]\). By Special Relativity, the photon in \(\lambda\) (nevertheless) travels at speed \(c\) from \(x = 0\) to \(x = \lambda \equiv 0\). Yet from frame \(S\) in the initial setup \(\Phi\), the photon travels at \(x(t) = ct + \xi_0 \lambda\) for some \(\xi_0 \in [0, 1]\), or at \(\phi(t) = \phi_0 = \frac{\xi \lambda}{r} \equiv \xi_0 \cdot 2\pi\). As such, for frame \(S\) to account for the photon moving from \(\phi = 0\) to \(\phi = 2\pi\), it is necessary that the photon rotates on the \((y, z) - \text{plane}\) in the motion of

\[
\begin{pmatrix}
y(t) \\
z(t)
\end{pmatrix}
= \begin{pmatrix}
r \cos\left(\frac{ct + \xi \lambda}{r}\right) \\
r \sin\left(\frac{ct + \xi \lambda}{r}\right)
\end{pmatrix}.
\]

(15)
However, since the photon is represented by a point \( x = \xi_0 \lambda \in [0, \lambda] \) on the \( x \)-axis (with \( y = z = 0 \)), the above radius \( r \) of rotation necessarily refers to the wave energy contained in \( \Phi \) that is distributed from

\[
\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},
\]

(16)

to

\[
\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} r \cos \left( \frac{ct + \xi_0 \lambda}{r} \right) \\ r \sin \left( \frac{ct + \xi_0 \lambda}{r} \right) \end{pmatrix},
\]

(17)

or

\[
\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} \rho r \cos \left( \frac{ct + \xi_0 \lambda}{r} \right) \\ \rho r \sin \left( \frac{ct + \xi_0 \lambda}{r} \right) \end{pmatrix}, \quad \forall \rho \in [0, 1].
\]

(18)

Lastly, since \( \xi_0 \in [0, 1] \) is arbitrary, the conclusion follows. □

**Remark 1** In the above, if one changes \( ct + \xi_\lambda \) to \( (-c)t + \xi (-\lambda) \), then the spin changes its orientation. As such, a photon has two orientations of spins.

**Proposition 6** The photon results in a (mini) black hole in \( M^{[1]} \), which is a white hole in \( M^{[2]} \), with a Schwarzschild radius

\[
r_\gamma = \frac{2G^{[3]} \hat{E}}{c^4},
\]

(19)

where \( G^{[3]} \equiv \) the recognized gravitational constant, and \( \hat{E} \equiv \) the laboratory-measured energy of the photon. (Cf. e.g.,[4].)

**Proof.** The photon being a point particle in \( M^{[1]} \) has infinite energy density, which punctures \( M^{[1]} \) with a black hole of radius \( r_\gamma \). Since the black hole leads to \( M^{[2]} \), it is by definition a white hole in \( M^{[2]} \). □

**Proposition 7** The probability density \( f(t, x) \) for the photon to emerge at \( (t, x) \) is

\[
f(t, x) = a \cos^2 (kx - \omega t), \quad \text{with}
\]

(20)

\[
\int_{t_o}^{t_o + \frac{\lambda}{c}} \int_{ct_o + \lambda}^{ct_o + \lambda} f(t, x) \, dx \, dt = 1, \forall t_o \in \mathbb{R}, \quad \text{with}
\]

(21)

\[
x \in [0, \lambda] \text{ set at } t = 0.
\]

(22)
Proof. The wavefunction (see [8]) is such that
\[ \Psi(t, x, y, z) = z_o \cdot \|E(t, x, y, z)\|_{C^3}, \]
so that the probability density of the photon is
\[ |\Psi(t, x, y, z)|^2 = a \cos^2(kx - \omega t). \]
By Proposition 2, we have the conclusion.

Proposition 8 Continuing Proposition 7, one has the photon appearing in \( \mathcal{M}[1] \) for a time duration
\[ \Delta t \in \left[ \frac{5}{16} \frac{\lambda}{c}, \frac{\lambda}{c} \right]. \]

Proof. In [8], we showed that the distribution of energy \( E \) over the photon in \( \mathcal{M}[1] \) and its electromagnetic wave in \( \mathcal{M}[2] \) is \( \frac{\lambda}{4} E \) and \( \frac{\lambda}{4} E \) respectively, and that accordingly the laboratory-measured energy of the photon is \( \hat{E} = \frac{10}{16} E. \) As such, the laboratory-measured Poynting vector
\[ \hat{S} = \frac{10}{16} S = \left( \frac{10}{16} \varepsilon_0 c A^2 \cos^2(kx - \omega t), 0, 0 \right) \]
and its average over \([0, \lambda]\) is thus
\[ \hat{S}_{avg} = \left( \frac{5}{16} \varepsilon_0 c A^2, 0, 0 \right). \]
This implies that if the photon carries an average energy density of \( \left( \frac{5}{16} \varepsilon_0 A^2 \right) \) over \([0, \lambda]\), then it must travel \( 3 \times 10^8 \text{meters} \) in one second; however, if the photon carries an average energy density of \( (\varepsilon_0 A^2) \), then it only travels \( \frac{5}{16} \times 3 \times 10^8 \text{meters} \), or \( \frac{5}{16} \) second. Thus, \( \Delta t \in \left[ \frac{5}{16} \frac{\lambda}{c}, \frac{\lambda}{c} \right]. \)

Remark 2 When a photon is observed, by the above analysis it must be located in its last fractional wave length \( \lambda. \) This implies the quantum uncertainties in its energy, phase, and position (cf. e.g., [10]).
3 Summary

In this paper, we have shown that Maxwell Equations and Einstein Relativity plus $\Psi(t, x, y, z) = z_o \cdot \|\mathbf{E}(t, x, y, z)\|_{C^2}$ imply, to any inertial frame $S$, a traveling cylinder of spinning electromagnetic waves of (real) energy (in joule) localized in $[0, \lambda] \times S^1 + ct$, which remains invisible in the wave universe $\mathcal{M}^{[2]}$ until a point $x_o \in [0, \lambda]$ randomly appears as a photon $\gamma$ in $\mathcal{M}^{[1]}$. Then (photon $\gamma$, electromagnetic wave $\tilde{\gamma}$) exists in $\mathcal{M}^{[3]}$ for a duration $\Delta t$, with $\gamma$ presenting itself as a spinning (black, white) hole $\Omega$, puncturing $[0, \lambda] \times S^1 + c ([t, t + \Delta t])$ with $\Omega$ that is centered at $x_o + c ([t, t + \Delta t])$ with a small Schwarzschild radius $r_{\gamma}$; $\gamma$ carries $\frac{3}{4}$ of the energy $E$ of $[0, \lambda] \times S^1 + c ([t, t + \Delta t])$ in $\Omega$, and $\tilde{\gamma}$ carries the remaining $\frac{1}{4}E$ in $[[0, \lambda] \times S^1 - \Omega] + c ([t, t + \Delta t])$ in $\mathcal{M}^{[2]}$. After $t + \Delta t$, $\gamma$ may disappear from $\mathcal{M}^{[1]}$, leaving the entire $E$ in $\mathcal{M}^{[2]}$, but in any case within a distance of $d = n \cdot \lambda$ the photon $\gamma$ must appear $n$ times, so that $\nu \lambda = c$ and the photon $\gamma$ has energy $\hat{E} = h \nu$.

References


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