The Study of Inelastic Scattering of Alpha Particles on Nuclei ^{20}Ne , ^{24}Mg by Methods Strongly-Connected Channels

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Abstract

The processes of interaction of alpha particles with collective states of nuclei ^{22}Ne , ^{24}Mg by strongly-connected channels of nuclear reactions studied. As the collective Hamiltonian able to take the expression patterns of the interacting bosons model (IBM).

Keywords: Inelastic scattering, alpha particles on nuclei ^{20}Ne , ^{24}Mg

I. Introduction

In nuclear reactions are more realistic approaches, such as the strongly-connected method of channels (SCMC). As the Hamiltonian collective states of the target nuclei, in this method, was used the Bohr-Mottelson Hamiltonian [1,2].

In this paper, the content of this theory SCMC we modify somewhat by adopting to describe the collective states of the target nuclei of interacting bosons model (IBM), because in this paper we study the scattering of nuclear particles to spherical nuclei, in the Hamiltonian IBM retained terms that describe the vibration

states with SU(5) symmetry. As calculated by the program as ECIS 88 [3] used, slightly modified with the replacement of the potential binding reaction channels on the operator of particle interaction with boson collective excitations of the target nuclei.

In the first part is describing of the general theory of particle scattering on collective states of nuclei SCMC by using the collective excitations as a state of interacting bosons.

The theory is applied to the study of the scattering of the ^4He on the ground states of light nuclei ^{22}Ne , when the incident energy $E = 48.8$ MeV and ^{24}Mg when energy particles $E = 50.5$ MeV. In this case, just consider the effect of the first excited states of nuclei on the main level of the scattering processes. Optical potential parameter is taken from systematic C.M Perey [4] and the Institute of Nuclear Physics [5]. Weak variations of the parameters do not exceed the squared errors of the specified systematic.

II. The equations connected channels of nuclear reactions

Suppose that there are excited states N_s of the target nucleus, which is strongly linked to its basic state by coupling potentials between channels V_{int} reactions. Denote spin, parity, energy n- state I_n, π_n, ω_n , respectively. If the energy of the incident particle (in the center of mass) is equal to E_1 , then the energy of the particles leaving the target of n-state is equal to $E_n = E_1 - \omega_n$.

The Hamiltonian of the whole system:

$$H = T + H_i + V(r, \theta, \varphi) = T + H_i + V_{\text{diag}} + V_{\text{int}}, \quad (1)$$

where T is the kinetic energy of the incident particle and H_i - Hamiltonian of the internal movements of the nucleons of the target nucleus.

The potential interaction of the incident particle with the target divided by the diagonal part and channel bonding scattering particles.

$$V(r, \theta, \varphi) = V_{\text{diag}} + V_{\text{int}} \quad (2)$$

Wave function can be written by the equality:

$$\psi = r^{-1} \sum_{J_n \ln j_n} R_{J_n \ln j_n}(r) \sum_{m_j M_n} (j_n I_n m_j M_n | JM) Y_{\ln j_n} \Phi_{I_n M_n}, \quad (3)$$

where $\Phi_{I_n M_n}$ is the wave function of the target nucleus in the n-state, which is determined by solving the equation:

$$H_i \Phi_{I_n M_n} = \omega_n \Phi_{I_n M_n} \quad (4)$$

Now, substituting (2) and (3) in the Shrodinger equation and multiplying (4) by $(Y_{\ln j_n} \otimes \Phi_{I_n})_{JM}^*$ from the left side and integrating over all axes, except the radial variable r , and dividing Equation E_n , we obtain the equation:

$$\left(\frac{d^2}{d\rho_n^2} - \frac{\ln(\ln+1)}{\rho_n^2} - \frac{1}{E_n} + 1 \right) R_{J_n \ln j_n} = E_n^{-1} \sum_{n \ln j_n} \left\langle (Y_{\ln j_n} \otimes \Phi_{I_n})_{JM} \left| V_{ce} \right| (Y_{\ln j_n'} \otimes \Phi_{I_n'})_{JM} \right\rangle R_{J_n \ln j_n'}(r) \quad (5)$$

where $\rho_n = k_n r$; k_n - The wave number.

This equation represents a set of n connected equations. The system (5) is known equations coupling method of channels, and the value on the right side is called the matrix elements of communication channels. If the system (5) are counted V_{12} and V_{21} , the off-diagonal matrix elements, then we obtain a system of two equations:

$$\left[\frac{d^2}{d\rho_1^2} - \frac{l_1(l_1+1)}{\rho_1^2} + \frac{1}{E_1} V_{11} - 1 \right] R_1(\vec{r}) = -\frac{1}{E_1} V_{12} R_2(\vec{r}), \quad \left[\frac{d^2}{d\rho_2^2} - \frac{l_2(l_2+1)}{\rho_2^2} + \frac{1}{E_2} V_{22} - 1 \right] R_2(\vec{r}) = -\frac{1}{E_2} V_{21} R_1(\vec{r}). \quad (6)$$

This system corresponds to the problem of describing the elastic and inelastic scattering involving other than the ground state of another excited state of the nucleus 2^+ .

In this paper, as the Hamiltonian of the collective excitations is using the model Hamiltonian of interacting bosons (IBM) $SU(3)$ symmetry and apply the theory to the scattering of particles on the vibration state of spherical nuclei.

$$H_i = H_0 + \varepsilon(d^+ d) + \sum_{L=0,2,\eta} \frac{1}{2} (2L+1) c_L [d^+ \cdot d^+]^{(L)} [d \cdot d]^{(L)(0)}. \quad (7)$$

The phenomenon of the interaction of the incident particle with the nucleus will be considered by a phenomenological optical potential, the radial dependence which assumes the form Wood-Saxons functions

$$V_{diag} = (-V + iW)(1+e)^{-1} - 4iW_D e'(1+e')^{-2} - V_{so} \left(\frac{\lambda^2}{ar} \right) (\vec{\sigma} \cdot \vec{l}) e(1+e)^{-2} + \\ + \frac{ZZ'e^2}{2R_e} \left(3 - \frac{r^2}{R_e^2} \right) \theta(R_e - r) - \frac{ZZ'e^2}{r} \theta(r - R_e) \quad (8)$$

In this equation we have introduced the notation: $e = \exp\left(\frac{r-R_0}{a}\right)$; $e' = \exp\left(\frac{r-R'_0}{a'}\right)$

where R_0, R'_0 - average radii of the optical potential $R_0 = r_0 A^{1/3}$, $R'_0 = r'_0 A^{1/3}$; V, W, W_D, V_{so} - optical parameters and the spin-orbit interaction potentials.

Potential communication channels can in general terms be written as:

$$V_{\text{int}} = \sum_{i,\lambda} v_{\lambda}^{(i)}(r) (Q_{\lambda}^{(i)} \cdot Y_x) \quad (9)$$

Q_{λ} - operator, acting only on the coordinates of the target nucleus. The calculation of the matrix elements of this operator gives:

$$\langle (Y_{lj} \otimes \Phi_I) JM | V_{\text{int}} | (Y_{l'j'} \otimes \Phi_{I'}) JM \rangle = \sum_{i,\lambda} v_{\lambda}^{(i)}(r) \langle I || Q_{\lambda}^{(i)} || I' \rangle A(l_j I, l' j' I', \lambda J), \quad (10)$$

$$A(l_j I, l' j' I', \lambda J) = \frac{1}{\sqrt{4\pi}} (-)^{I'-I+l+l'} \sqrt{(2I+1)(2I'+1)(2j+1)(2j'+1)} (ll'00)(\lambda 0) W(j l j' I'; J \lambda).$$

We give an explicit expression of the operators: $Q_{\lambda\mu}^{(1)} = d^+ d$,

$$Q_{\lambda\mu}^{(2)} = \frac{\sqrt{4\pi}}{(2\lambda_1+1)(2\lambda_2+1)} (\lambda_1 \lambda_2 00 | \lambda 0) C_L(2L+1) (d^+ d^+)^L (dd)^L \quad (11)$$

The wave functions of different vibration states with spin and IM with one and two bosons:

$$|1; IM \rangle = d_{IM}^+ |0 \rangle, \quad \left| 2; IM \right\rangle = \frac{1}{\sqrt{1 + \delta_{\lambda_1 \lambda_2}}} (d_{\lambda_1}^+ d_{\lambda_2}^+)_{IM} |0 \rangle$$

For spherical nuclei $v^{(\lambda)}(r)$ has the form.

$$v^{(\lambda)}(r) = 4\pi \int_0^1 \frac{-(v + iw)}{1 + \exp\left[(r - R_0(1 + \sum \beta_{\lambda} \varphi_{\lambda\mu}(\theta)/a)\right]} + \frac{-4iW_D \exp\left[(r - R_0(1 + \sum \beta_{\lambda} \varphi_{\lambda\mu}(\theta)/a)\right]}{[1 + \exp\left[(r - R_0(1 + \sum \beta_{\lambda} \varphi_{\lambda\mu}(\theta)/a)\right]]} \times Y_{\lambda\mu}(\theta) d \cos(\theta) \quad (12)$$

Solution of the system of coupled differential equations for the radial functions $R_{l_n j_n J}(\bar{r})$ with certain boundary conditions makes it possible to find elements S - matrix and construct cross sections of elastic and inelastic scattering, which are expressed in the form:

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2}{h^4} \mu^2 \frac{k'}{k} \frac{1}{2S+1} \sum_L B_L P_L(\cos \theta) \quad (13)$$

$$B_L = \sum_{l'l'j'l'_1j'_1j_1j'_1} (-)^{l'-l} (2J+1)(2J_1+1) Z \left(l j l_1 j_1 \mid \frac{1}{2} L \right) Z \left(l' j' j'_1 I'_1 \mid \frac{1}{2} L \right) W(j j_1 J_1 \mid IL) W(j' j'_1 J_1 \mid I'L) \cdot \\ \cdot \left| \langle J M l' j' I' | T | J_1 M_1 l'_1 j'_1 I'_1 \rangle \langle J_1 M_1 l'_1 j'_1 I'_1 | T | J M l j I \rangle \right|^2$$

here Z – coefficients Blagga-Videnharna, W – Racah coefficients, and

$$\langle \vec{K}' S \mu'_S I' M'_A | T | \vec{K} S \mu_S I M_A \rangle = i^{l-l'} Y_{lM-M_A-\mu_S}^* (\vec{K}') Y_{l'M-M'_A-\mu'_S} (\vec{K}') \langle lM - M_A - \mu_S S \mu_S \mid jM - M_A \rangle \cdot \\ \cdot \langle lM_A jM - M_A \mid JM \rangle \langle l'M - M'_A - \mu'_S S \mu'_S \mid j'M - M'_A \rangle \langle l'M'_A j'M - M'_A \mid JM \rangle \cdot \\ \cdot \langle l' j' I' JM | T | l j I JM \rangle \quad (14)$$

Note that the differential cross section (13) is general and independent of the order in which model elements are computed T or S-matrices.

III. Applications of the theory to the scattering of α -particles by light vibration nuclei and their comparison with experimental data

The theory of scattering α particles, taking into account the binding of scattering channels, is applied to the processes occurring at the lowest states of spherical light nuclei ^{22}Ne and ^{24}Mg . We have considered the scattering α particles with energies of to 50 MeV, close to the ground states of these nuclei. To facilitate the computer account and to analyze the possibilities of the theory consider the influence of only the first excited states with the $J = 2^+$ on the ground state of the nuclei.

Such consideration communication channels in these nuclei can be considered acceptable, since the remaining excited states are quite high, about 3 MeV above the first excited state. Therefore, we consider the solution of two interrelated equations (6). It is known that the scattering of α - particles on the ground states of nuclei is described, in general, the optical model, using an complex Wood-Saxons potential. Usually obtained in this case, a reasonable correspondence of the calculated cross sections with the experimental data [5].

In order to analyze the inelastic scattering - particles at large angles in some studies [6,7] had to summarize the Wood-Saxons potential with the addition of another member to it containing a high order.

Recently Michel and colleagues [6] conducted a systematic optical-model analysis of scattering particles on α -spherical nuclei. They gave a new parameterization for the real part of the potential.

In [8] it was an experimental study of the elastic - scattering on some light nuclei with $A = 11-24$ at energies α - particles $E = 48.7$ and 54.1 MeV. Data were analyzed by different types of optical potential and including Michel type. Particular attention was drawn to the concept of the double-folding potential.

According to the method described by our strong channel coupling we performed theoretical calculations of differential cross sections for the scattering of particles α on nuclei ^{22}Ne and ^{24}Mg on the entire angular range of 12° to 172° . In determining the theoretical values of $d\sigma/d\theta$ total optical potential is excluded part corresponding spin-orbit interaction V_{st} .

Figure 1. Scattering ${}^4\text{He}$ to the ground state ${}^{22}\text{Ne}$ Figure 2. Scattering ${}^4\text{He}$ to the ground state ${}^{24}\text{Mg}$

Figures 1 and 2 show the angular distribution of the differential cross sections at scattering angles. They are represented by solid lines the experimental scattering pattern ${}^4\text{He}$ particles on the core ${}^{20}\text{Ne}$ $E = 48,8$ $0,6$ MeV and at the nucleus ${}^{24}\text{Mg}$ $E = 50,5$ $0,5$ MeV. Dotted-calculated values.

Experimental data on the cross sections are taken from the process of experimental data of the Institute of Nuclear Physics of the Republic of Kazakhstan (INP RK) [3,5,9] performed at the isochronous cyclotron of the Institute. The parameters of the potentials are taken compile phenomenological optical-model parameters, the group performed Perey [4], and data from the INP RK [5, 9]. Table 1 gives the optimal parameters of interaction processes at the corresponding energy of the incident particle.

Table 1. Optimal parameters of interaction α - particles with nuclei

Nuclei	E(Mev)	V_0 (Mev)	r_0 (fm)	a_0 (fm)	Wr (Mev)	r_v (fm)	a_v (fm)
^{20}Ne	48,8	115,0	1,25	0,8	30,0	1,45	0,85
^{24}Mg	50,5	100,0	1,30	0,75	35,0	1,40	0,75

As can be seen from Figures 1 and 2 in the comparison of the calculated scattering cross sections with their experimental values obtained quite consistent picture. Especially there is a good agreement in the range of angles from the origin to the corners within $125^\circ - 130^\circ$ errors of experimental measurement $\sigma(\theta)$. However, the theoretical cross sections differ significantly from their experimental values of the rear scattering angles in the range of large angles from 125° to 175° . From the outset, one would assume that the general agreement over the entire range of angles obtained if somewhat change the parameters of the optical potentials. However, as shown in similar studies that gets comfortable consent to the rear corners of the scattering in the front corners of the picture changes somewhat. In some studies to obtain an overall consistent picture recommends adding an additional term in the optical potential. this case, increasing the number of adjustable parameters. We emphasize that the theoretical angular distribution of the scattering cross sections are obtained by us in the same parameter values that were used in the analyzes of other processes on other approaches, especially on the optical model. At the same time, we should remember that in the scattering at the rear corners of the known physical cause is related to the absorption α - particles. However, this phenomenon is apparently lies beyond the scope of strongly-connected channels.

IV. Conclusion

Thus, calculating the matrix elements of the operators V_{int} various communication channels scattering of incident particles with the collective states of the target nuclei and found solutions of Shredinger equations considering the matrix elements of communication channels. Found differential and integral cross sections for interaction of the incident particle with nuclear collective states of nuclei, to develop a theory of the scattering of strongly coupled channels applied to the study of their interaction ^4He particles from the ground state of light nuclei $^{22}\text{Ne}(^4\text{He}, ^4\text{He})^{22}\text{Ne}$ at the energy of the incident particle $E=48,8$ MeV and $^{24}\text{Mg}(^4\text{He}, ^4\text{He})^{24}\text{Mg}$ at the energy of the incident α - particles $E=50,5$ MeV. This solves the system is not homogeneous equations relating the first excited state 2_1^+ nuclei from the ground level 0_1^+ . Weak influence of other excited levels, which are at a high altitude of about 3 MeV, the efficiency of the optical parameters of the correlation of nuclear capability. At the same time, these parameters are taken from the systematics SM Perey and Nuclear Physics Institute of the Republic of Kazakhstan. Theoretically found angular distributions of the cross sections of

processes within the angles of 10^0 by 175^0 compared with their experimental values.

The proposed variant of the theory SCMC generally satisfactorily explains the properties of the scattering of nuclear particles to collective bosonic states of spherical nuclei.

Acknowledgements. This work is supported by the grant of the Ministry of Education and Science of the Republic Kazakhstan «101Grantfunding for scientific research 1504/GF3».

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Received: November 18, 2014; Published: December 30, 2014