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Kerr Black Hole in Canonically Deformed Space-Time

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Abstract

We investigate the Kerr black hole defined on canonically deformed space-time. Particularly, we find the corresponding event horizon, the ergosphere, the temperature and the entropy of such deformed object.

1 Introduction

The idea to use noncommutative coordinates is quite old - it goes back to Heisenberg and was firstly formalized by Snyder in [1]. Recently, however, there were found new formal arguments based mainly on Quantum Gravity [2], [3] and String Theory models [4], [5], indicating that space-time at Planck scale should be noncommutative, i.e. it should have a quantum nature. On the other side, the main reason for such considerations follows from many phenomenological considerations, which state that relativistic space-time symmetries should

be modified (deformed) at Planck scale, while the classical Poincare invariance still remains valid at larger distances [6], [7].

In accordance with the Hopf-algebraic classification of all deformations of relativistic and nonrelativistic symmetries (see [8], [9]) the most general form of space-time noncommutativity looks as follows

$$[x_\mu, x_\nu] = \theta_{\mu\nu}(x), \quad (1)$$

where

$$\theta_{\mu\nu}(x) = \theta_{\mu\nu}^{(0)} + \theta_{\mu\nu}^{(1)\rho} x_\rho + \theta_{\mu\nu}^{(2)\rho\tau} x_\rho x_\tau. \quad (2)$$

For the simplest, canonical noncommutativity ($\theta_{\mu\nu}(x) = \theta_{\mu\nu}^{(0)}$), the corresponding Poincare Hopf algebra has been provided in [10] and [11] with the use of twist procedure [12]-[14], while its nonrelativistic counterparts have been discovered by various contraction schemes in [15].

The Lie-algebraic ($\theta_{\mu\nu}(x) = \theta_{\mu\nu}^{(1)\rho} x_\rho$) relativistic and nonrelativistic symmetries have been proposed in [16] and [17] respectively. In the literature they are known as κ -Poincare and κ -Galilei Hopf algebra with mass-like deformation parameter κ . Besides, there were proposed the twist deformations of a Lie-type at relativistic and nonrelativistic level in [18], [19] and [15].

The quadratic deformation ($\theta_{\mu\nu}(x) = \theta_{\mu\nu}^{(2)\rho\tau} x_\rho x_\tau$) has been studied in [20] and [18].

Recently, there appeared a lot of papers dealing with classical ([21]-[27]) and quantum ([28]-[32]) mechanics, Doubly Special Relativity frameworks ([33], [34]), statistical physics ([35], [36]) and field theoretical models (see e.g. [37]), defined on the canonically and Lie-algebraically deformed space-times¹. It should be noted, however, that the especially interesting studies have been performed in articles [40]-[44] in the context of so-called black hole physics, i.e. there have been investigated (with use of different methods and techniques) the basic types of noncommutative black hole metrics.

In this article we study the effect of canonical noncommutativity on the Kerr solution of General Relativity equation with use of treatment proposed in [40]-[42]. Particular, we investigate the impact of quantum space on the event horizon as well as on the shape of ergosphere. Besides, as in the case of basic nonrotating black holes described by Schwarzschild and Reisner-Nordstrom metric tensors respectively, we find the temperature and entropy of such canonically deformed object.

In our investigation we proceed in accordance with the following algorithm (see [40]-[42])². Firstly, we assume that particle-like gravitational source remains point-like. Next, we exchange the commutative variables in classical

¹For earlier studies see [38] and [39].

²It should be noted, however, that the results obtained by us make full solutions in deformation parameter θ of singularity equations, contrary to the results of articles [40]-[42].

Kerr solution of field equation by noncommutative ones. Further, we rewrite the obtained in such a way metric tensor in terms of commutative phase space variables and deformation parameter θ (see formula (28)). Finally, we perform the basic analysis of such a prepared mathematical object.

It should be noted that the classically noncommutative Kerr black hole has been already analyzed in article [44]. However, the used therein techniques are completely different than the methods preferred by us. For example, there is assumed that due to the space-time noncommutativity the mass density particle-like gravitational source is smeared. Such an assumption leads to the new (Kerr) solution of General Relativity equations for which the corresponding event horizon can be found only numerically.

The paper is organized as follows. In Sect. 2 we recall basic facts concerning the canonical deformed Poincare Hopf algebra and the corresponding quantum space-time provided in article [11]. The third section is devoted to the short review of Kerr black hole defined in commutative (classical) space. In Sect. 4 we analyze the effect of space-time noncommutativity (7) on the Kerr metric. The final remarks are presented in the last section.

2 Canonically deformed Minkowski space-time

In this section we recall basic facts associated with θ -deformed Poincare Hopf algebra $\mathcal{U}_\theta(P)$ and with the corresponding canonically deformed quantum space-time [11]. Firstly, it should be noted that such objects can be get by so-called twist procedure, in which the algebraic sector of Hopf structure $\mathcal{U}_\theta(P)$ remains undeformed, i.e. it takes the form

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma}) , \\ [M_{\mu\nu}, P_\rho] &= i(\eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu) \quad , \quad [P_\mu, P_\nu] = 0 . \end{aligned} \quad (3)$$

Besides, the coproduct of considered algebra is given by

$$\Delta_0(a) \rightarrow \Delta_\theta(a) = \mathcal{F}_\theta \circ \Delta_0(a) \circ \mathcal{F}_\theta^{-1} , \quad (4)$$

where

$$\mathcal{F}_\theta = \exp \left[\frac{i}{2} \theta^{\mu\nu} P_\mu \otimes P_\nu \right] , \quad (5)$$

denotes the canonical twist factor, while $\Delta_0(a) = a \otimes 1 + 1 \otimes a$. Consequently, using (3)-(5) we get

$$\begin{aligned} \Delta_\theta(P_\mu) &= P_\mu \otimes 1 + 1 \otimes P_\mu , \\ \Delta_\theta(M_{\mu\nu}) &= M_{\mu\nu} \otimes 1 + 1 \otimes M_{\mu\nu} - \frac{1}{2}(\theta_\mu{}^\rho P_\nu - \theta_\nu{}^\rho P_\mu) \otimes P_\rho + \\ &+ \frac{1}{2}P_\rho \otimes (\theta_\mu{}^\rho P_\nu - \theta_\nu{}^\rho P_\mu) . \end{aligned} \quad (6)$$

The corresponding quantum Minkowski space-time is defined as the representation space (Hopf module) for Poincare Hopf algebra $\mathcal{U}_\theta(P)$. It is given by the following commutation relations

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}, \quad (7)$$

and it can be extended to the whole algebra of momentum and position operators as follows

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}, \quad [\hat{p}_\mu, \hat{p}_\nu] = 0, \quad [\hat{x}_\mu, \hat{p}_\nu] = i\eta_{\mu\nu}. \quad (8)$$

Of course, for deformation parameter θ running to zero the all above objects become classical.

3 Kerr black hole

The Kerr metric describes the solution of field equation for rotating (with angular momentum L), uncharged and axially-symmetric massive (with mass M) object in empty space-time. In spherical coordinate system (r, φ, ϕ) it can be written as follows [45]

$$\begin{aligned} c^2 d\tau^2 = & \left(1 - \frac{r_s r}{\rho^2}\right) c^2 dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\varphi^2 + \\ & - \left(r^2 + \alpha^2 + \frac{r_s r \alpha^2}{\rho^2} \sin^2 \varphi\right) \sin^2 \varphi d\phi^2 + \frac{2r_s r \alpha \sin^2 \varphi}{\rho^2} c dt d\phi, \end{aligned} \quad (9)$$

with r_s denoting the Schwarzschild radius

$$r_s = \frac{2GM}{c^2}, \quad (10)$$

and with the length-scales α , ρ and Δ introduced for brevity

$$\alpha = \frac{L}{Mc}, \quad (11)$$

$$\rho^2 = r^2 + \alpha^2 \cos^2 \varphi, \quad (12)$$

$$\Delta = r^2 - r_s r + \alpha^2. \quad (13)$$

Of course, for parameter α approaching zero the above metric passes into well-known solution for Schwarzschild black hole [45]. It should be noted that the Kerr metric (9) has four (only two of them are physically relevant) surfaces on which it appears to be singular. First pair occurs when the purely radial component of metric goes to infinity. Then the solutions of corresponding quadratic equation

$$\frac{1}{g_{rr}} = \frac{\Delta}{\rho^2} = 0, \quad (14)$$

look as follows

$$r_{\text{inner}} := r_{i+} = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2}}{2}, \quad (15)$$

and

$$r_{i-} = \frac{r_s - \sqrt{r_s^2 - 4\alpha^2}}{2}, \quad (16)$$

respectively. The first of them (physical one) describes the event horizon while the second solution (due to the fact that radius r_{i+} is bigger than r_{i-}) has just unphysical properties.

The second pair of singularities occurs when the purely temporal component of the Kerr metric changes sign. Again, solving a quadratic equation of the form

$$g_{tt} = \left(1 - \frac{r_s r}{\rho^2}\right) = 0, \quad (17)$$

we get

$$r_{\text{outer}} := r_{o+} = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2 \cos^2 \varphi}}{2}, \quad (18)$$

$$r_{o-} = \frac{r_s - \sqrt{r_s^2 - 4\alpha^2 \cos^2 \theta}}{2}, \quad (19)$$

where only radius r_{o+} remains physical ($r_{o+} > r_{i+} > r_{o-}$).

One can observe that due to the $\cos^2 \varphi$ term in (18) the outer sphere touches the inner one (see (15)) at the poles of rotation axis, where φ equals 0 or π ; the space between these two surfaces is called the ergosphere. Obviously, for parameter α running to zero we have

$$\lim_{\alpha \rightarrow 0} r_{\text{inner}} = \lim_{\alpha \rightarrow 0} r_{\text{outer}} = r_s, \quad (20)$$

and the ergosphere disappears.

Finally, it should be mentioned that in accordance with article [46], [47] the temperature and entropy of Kerr black hole are given by

$$T = \frac{\hbar}{4\pi k c} \frac{(r_{i+} - r_{i-})}{(r_{i+}^2 + \alpha^2)}, \quad (21)$$

and

$$S = \frac{kc}{4\hbar} (r_{i+}^2 + \alpha^2), \quad (22)$$

respectively, with symbol k denoting the Boltzman's constant.

4 Noncommutative Kerr black hole

Following the treatment proposed in papers [40]-[42] we define the metric for Kerr black hole in θ -deformed space-time with $\theta_{0i} = 0$ by

$$\begin{aligned} c^2 d\tau^2 = & \left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{\rho}\hat{\rho}}\right) c^2 dt^2 - \frac{\hat{\rho}\hat{\rho}}{\hat{\Delta}} d\hat{r}d\hat{r} - \hat{\rho}\hat{\rho} d\varphi^2 + \\ & - \left(\hat{r}\hat{r} + \alpha^2 + \frac{r_s \sqrt{\hat{r}\hat{r}} \alpha^2}{\hat{\rho}\hat{\rho}} \sin^2 \varphi\right) \sin^2 \varphi d\phi^2 + \frac{2r_s \sqrt{\hat{r}\hat{r}} \alpha \sin^2 \varphi}{\hat{\rho}\hat{\rho}} c dt d\phi, \end{aligned} \quad (23)$$

where

$$\hat{\rho}\hat{\rho} = \hat{r}\hat{r} + \alpha^2 \cos^2 \varphi, \quad (24)$$

$$\hat{\Delta} = \hat{r}\hat{r} - r_s \sqrt{\hat{r}\hat{r}} + \alpha^2, \quad (25)$$

and where the components of $\hat{r} = \sqrt{\hat{x}_1^2 + \hat{x}_2^2 + \hat{x}_3^2}$ satisfy the commutation relations (7). The solutions of

$$\frac{\hat{\Delta}}{\hat{\rho}\hat{\rho}} = \frac{\hat{r}\hat{r} - r_s \sqrt{\hat{r}\hat{r}} + \alpha^2}{\hat{r}\hat{r} + \alpha^2 \cos^2 \varphi} = 0, \quad (26)$$

$$\left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{\rho}\hat{\rho}}\right) = \left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{r}\hat{r} + \alpha^2 \cos^2 \varphi}\right) = 0, \quad (27)$$

are the singularities of the metric (23).

In order to analyze the above system we represent the noncommutative variables $(\hat{x}_i, \hat{p}_i)$ in terms of classical phase space (x_i, p_i) as (see e.g. [48], [39])

$$\hat{x}_i = x_i - \frac{1}{2} \theta_{ij} p_j, \quad \hat{p}_i = p_i, \quad (28)$$

where

$$[x_i, x_j] = 0 = [p_i, p_j], \quad [x_i, p_j] = i\delta_{ij}. \quad (29)$$

Then, the equations (26) and (27) take the form

$$\frac{\hat{\Delta}}{\hat{\rho}\hat{\rho}} = \frac{(x_i - \frac{1}{2}\theta_{ij}p_j)(x_i - \frac{1}{2}\theta_{ik}p_k) - r_s \sqrt{(x_i - \frac{1}{2}\theta_{ij}p_j)(x_i - \frac{1}{2}\theta_{ik}p_k)} + \alpha^2}{(x_i - \frac{1}{2}\theta_{ij}p_j)(x_i - \frac{1}{2}\theta_{ik}p_k) + \alpha^2 \cos^2 \varphi} = 0, \quad (30)$$

and

$$\left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{\rho}\hat{\rho}}\right) = \left(1 - \frac{r_s \sqrt{(x_i - \frac{1}{2}\theta_{ij}p_j)(x_i - \frac{1}{2}\theta_{ik}p_k)}}{(x_i - \frac{1}{2}\theta_{ij}p_j)(x_i - \frac{1}{2}\theta_{ik}p_k) + \alpha^2 \cos^2 \varphi}\right) = 0, \quad (31)$$

respectively. First of all, one should notice that for α running to zero the above conditions become the same, and we reproduce the noncommutative Schwarzschild black hole singularity equation given by [40], [41]

$$\frac{\hat{\Delta}}{\hat{\rho}\hat{\rho}} = \left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{\rho}\hat{\rho}}\right) = 1 - \frac{r_s}{\sqrt{\hat{r}\hat{r}}} = 0. \quad (32)$$

Further, we rewrite the formulas (30) and (31) as follows

$$\frac{\hat{\Delta}}{\hat{\rho}\hat{\rho}} = \frac{\tilde{r}^2 - r_s \tilde{r} + \alpha^2}{\tilde{r}^2 + \alpha^2 \cos^2 \varphi} = 0, \quad (33)$$

$$\left(1 - \frac{r_s \sqrt{\hat{r}\hat{r}}}{\hat{\rho}\hat{\rho}}\right) = 1 - \frac{r_s \tilde{r}}{\tilde{r}^2 + \alpha^2 \cos^2 \varphi} = 0, \quad (34)$$

where

$$\tilde{r} = \sqrt{r^2 - \frac{1}{2}\vec{\theta}\vec{L} + \frac{1}{16}\left(\vec{p} \times \vec{\theta}\right)^2}, \quad \vec{L} = \vec{x} \times \vec{p} \quad \text{and} \quad \theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta_k, \quad (35)$$

as well as we find the corresponding solutions in the form

$$\tilde{r}_{\pm}(a) = \frac{r_s \pm \sqrt{r_s^2 - 4a}}{2}, \quad (36)$$

with $a = \alpha^2$ and $a = \alpha^2 \cos^2 \varphi$ in the case of equations (33) and (34) respectively.

Hence, in accordance with the formula (35) we get

$$r_{\pm}(a) = \frac{1}{2}\sqrt{\left(r_s \pm \sqrt{r_s^2 - 4a}\right)^2 + 2\vec{\theta}\vec{L} - \frac{1}{4}\left(\vec{p} \times \vec{\theta}\right)^2}, \quad (37)$$

while in the limit of commutative space, one obtains

$$r_-(\alpha^2) = r_{i-}, \quad (38)$$

$$r_+(\alpha^2) = r_{i+} = r_{\text{inner}}, \quad (39)$$

$$r_-(\alpha^2 \cos^2 \varphi) = r_{o-}, \quad (40)$$

$$r_+(\alpha^2 \cos^2 \varphi) = r_{o+} = r_{\text{outer}}. \quad (41)$$

Consequently, due to the above limits we define the θ -deformed ergosphere as the space occurring between radiuses $r_+(\alpha^2)$ and $r_+(\alpha^2 \cos^2 \varphi)$, i.e. as the region existing between deformed outer radius and "noncommutative" event horizon. Besides, one can observe that for α and θ_{ij} approaching zero, we reproduce the Schwarzschild event horizon

$$\lim_{\alpha, \theta \rightarrow 0} r_+(\alpha^2) = \lim_{\alpha, \theta \rightarrow 0} r_+(\alpha^2 \cos^2 \varphi) = r_s, \quad (42)$$

with remaining radii vanishing.

Finally, it should be mentioned that in accordance with formulas (21) and (22) as well as due to the limits (38)-(41), the temperature and entropy of noncommutative Kerr black hole are given by

$$T = \frac{\hbar}{4\pi k c} \frac{(r_+(\alpha^2) - r_-(\alpha^2))}{(r_+^2(\alpha^2) + \alpha^2)}, \quad (43)$$

and

$$S = \frac{kc}{4\hbar} (r_+^2(\alpha^2) + \alpha^2), \quad (44)$$

respectively.

5 Final remarks

In this article we investigate the Kerr black hole defined on canonically deformed space-time. Particularly, we find the corresponding event horizon, the proper ergosphere, the temperature and the entropy of such deformed object. Besides, for parameters α and θ_{ij} approaching zero we reproduce the classical Schwarzschild black hole solution, while in the limit of commutative space we arrive to the case of undeformed Kerr black hole metric tensor; the presented studies has been performed with use of methods and techniques proposed in articles [40], [41] and [42].

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