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Some Analytical Results of the Theory of Equivalence Measures and Stochastic Theory of Turbulence for Non-Isothermal Flows

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Abstract

Fundamentals of the new theory for the processes of transition from deterministic state to the chaos state (turbulence) for non-isothermal flows are presented. According to this theory, systems of stochastic equations of energy, momentum and mass are applied for non-isothermal flows. Then the analytical dependence for estimation of the value of the critical Reynolds number and the value of the critical point of regime change for non-isothermal and compressible flows are written. As an example, consider the classic flow of a Newtonian medium in the circular smooth tube. The values of the critical point of the beginning of the transition from laminar (deterministic) to turbulent motion for non-isothermal flows in the pipe are predicated. It is shown, that predicated values of the critical Reynolds numbers are in satisfactory agreement with the classical data.

Keywords: equivalent measures, stochastic equations, turbulence

1 Introduction

As known, the impact of initial fluctuations on the final solution of the equations, which are used to describe turbulent processes, is significantly. However, numerous private numerical solutions do not allow define the essence of the phenomenon of turbulence till today. Also numerous private numerical solutions do not allow to provide the answer, which will propose the physical regularity for the turbulence onset and present the mathematical method to describe the phenomenon on the basis of this physical regular pattern.

In the theory, which was developed in works [1 - 3], along with the new stochastic equations for a continuous medium also was obtained the new physical regularity for the studied phenomenon of turbulence. This regularity was installed theoretically and was called as an equivalence of measures between determi - nistic movement and random movement. The new results for an incompressible fluid were obtained for critical Reynolds number [1], for profiles of the velocity and for correlation of the second order in function of initial turbulence for the flow in the tube[2, 3], on the flat plate, in the plane jet, around a circular cylinder, near the rotating disk [3]. Here the regular pattern of the equivalence of measures and new systems of the stochastic equations for non - isothermal flow are presented. The obtained solutions show the possibilities of an application of the proposed theory in a wide range of the velocity and the temperature of flows taking into account an influence of an initial turbulence of each fields.

2 The Formulation of the Problem

In according to [1-3] considered the physical process is represented as a nonequilibrium thermodynamic system with i - subsets, which is characterized by the values of energy $\bigcup_{i}^{i=n}(E_i)_{g_{st}}$, momentum $\bigcup_{i}^{i=n}(MU_i)_{g_{st}}$ and mass $\bigcup_{i}^{i=n}(M)_{g_{st}}$.

The phenomenological law of conservation and transformation of energy during the evolution of a random system — the first law of thermodynamics — is written as

$$d\bigcup_{i}^{i=n}(E_i)_{col\ st} = (\delta Q + \delta L)_{(col)\mathrm{int},ext} + \Big[(\delta(Q+L)\cdot \delta(r(\vec{x}) - r(\vec{x}_{cr}))\cdot \delta(\tau - \tau_{cr}))_{(st)\mathrm{int},ext} -$$

$$- \{ d \bigcup_{j=0}^{j=k} \bigcup_{i=0}^{i=n} ((E_i)_{st} + (E_i)_{cor\ st})\} \cdot \delta(r(\vec{x}) - r(\vec{x}_{cr}))\cdot \delta(\tau - \tau_{cr}) \Big]$$

The same equations of conservation of momentum and mass were written [1, 3].

Here U_i is the speed. $\bigcup_{i}^{i=n}(E_i)_{g_{st}}$ -the energy stochastic field (index g_{st});

$\bigcup_{i}^{i=n}(E_i)_{col\ st}$ -the part of the energy field, its deterministic component (index col $_{st}$), having the stochastic component measures null; $\bigcup_{i}^{i=n}(E_i)_{st}$ -part of the energy field, in fact stochastic component of the field (index st).
Similarly, identify the components of the momentum and mass (ρ - density).
$(\delta Q + \delta L)_{(col\,)\mathrm{int}}$ and $(\delta Q+\delta L)_{(st)\mathrm{int}}$ - deterministic and stochastic components of the internal (int) heat and work. Also here deterministic and stochastic components of the external (ext) heat and work $(\delta Q+\delta L)_{(col)ext}=0$ $(\delta Q + \delta L)_{(st)ext}=0$.

Also in [1-3], subject to analysis [4-20], the equivalence of measures and the correlation function for an interaction between deterministic (laminar) and chaotic (turbulent) movement were obtained. This correlation function in the critical point of space-time $r_i \to r_c; \Delta\tau_i \to \tau_c$ for the parameter $m_i \to m_c$ can be written in form [1 - 3]:

$$\sum_j D_{M,N}(r_c;m_{cj};\tau_c)=\sum_j \sum_i \operatorname*{Lim}_{m_i\to m_{cj}} \operatorname*{Lim}_{r_i\to r_c} \operatorname*{Lim}_{\Delta\tau_i\to\tau_c}\{\ m\left(T^M Z^* \bigcap T^N Y^*\right)-R_{T^M Z^* T^N Y^*}\cdot m\left(T^M Z^*\right)\ \}=0$$

Index j is determined parameters m_{cj} (j=3 means: mass, momentum, energy). In the case of the binary intersections X = Y + Z + W. Here a subset of Y, Z, W are called extended to X, if the measures $\mathrm{m}(\mathrm{Y}), \mathrm{m}(Z), m(W)$ have the property [1, 3]:

$m(Y)=m(Y^*)=m(T^nY)+\bigcup_{k=0}^{k=n-1} m(T^k(G_1^{n-k}))$ and wandering subsets $\bigcup_{k=0}^{k=n-1}(T^k(G_1^{n-k}))\subset Y$;
$m(Z)=m\left(Z^*\right)=m\left(T^nZ^*\right)+\bigcup_{k=0}^{k=n-1} m\left(T^k\left(G^{n-k}\right)\right)$ and wandering subset $\bigcup_{k=0}^{k=n-1}(T^k(G_2^{n-k}))\subset Z$;
$m(W)=m(W^*)=m(T^nW)+\bigcup_{k=0}^{k=n-1} m(T^k(G_3^{n-k}))$ and wandering subset $\bigcup_{k=0}^{k=n-1} m(T^k(G_3^{n-k})\subset W$.
This correlation function produces the system of equations of equivalent measures

$$\left|m\left(T^M Z\right)\right|=\left(R_{T^M ZT^N Y}\right)_{n,m}\left|m\left(T^N Y\right)\right|,\ 0\le\left(R_{T^M ZT^N Y}\right)\le 1\ .$$

Here $R_{T^M ZT^N Y}$ is a fractal correlation function, and then we assume it is equal to the unit to obtain analytical solutions. Therefore for the pair (N, M)=(1, 0) $\left|m(Z)\right|=\left(R_{ZTY}\right)_{n,m}\left|m(TY)\right|$, and for (N, M)=(1, 1) $\left|m(TZ)\right|=\left(R_{TZTY}\right)_{n,m}\left|m(TY)\right|$.
Here T^n is a conservative transformation of X for all n, then there exists n> n_d, such that there T^n is dissipation and transformation for $\mathrm{Y}^*\subset X$ and $Z^*\subset X$.
Then corresponds to the set X value the total energy of the stochastic field

$\bigcup_{i}^{i=n}(E_i)_{g\ st}$ .Corresponds to a subset $\Upsilon \subset X$ energy $\bigcup_{i}^{i=n}(E_i)_{col\ st}$ -deterministic component of the stochastic field, wandering subset $G_1^{\ n}$ extended subset $\Upsilon^* \subset X$ placed respectively $(\delta Q+\delta L)_{(col)\mathrm{int}}$; subset $Z \subset X$ of the measure m (Z)> 0 is placed under the actual value of the energy $\bigcup_{i}^{i=n}(E_i)_{st}$ -stochastic component; respectively wandering subset $G_2^{\ n}$ of the extended subset $Z \subset X$ is put under $(\delta Q+\delta L)_{(st)\mathrm{int}}$; a subset W $\subset X$ and wandering subset $G_3^{\ n}$ put the value of the $\bigcup_{i}^{i=n}(E_i)_{cor\ st}$. Also corresponds to the transformation T^n the set of differential operators $\left\{d;\frac{d}{d\tau}\right\}$. In [1], for the transfer of substantial values F (the mass (the density-ρ), the momentum ($\rho\vec{U}$), the energy (E)) of deterministic (laminar) motion into random (turbulent) motion (for the area 1) the beginning of the generation of turbulence, the pair (N, M) = (1,0), equivalence of measures was written as $\left(d\Phi_{col,st}\right)_{1,0} = -R_{1,0}\left(\Phi_{st}\right)$ *and* $\left(\frac{d(\Phi)_{col,st}}{d\tau}\right)_{1,0} = -R_{1,0}\left(\frac{\Phi_{st}}{\tau_{cor}}\right)$.Also in [1], for «correlator» $D_{N,M}(r_c;m_{ci};\tau_c)=D_{1,1}(r_c;m_{ci};\tau_c)$, the pair (N,M)=(1,1) it was written $\left(d\Phi_{col,st}\right)_{1,1}=-R_{1,1}\left(d\Phi_{st}\right)$, $\left(\frac{d(\Phi)_{col,st}}{d\tau}\right)_{1,1}=-R_{1,1}\left(\frac{d\Phi_{st}}{d\tau}\right)$.

$R_{1,0}$,$R_{1,1}$- fractal coefficients. For example, to obtain the new analytical dependen--ces, these coefficients are taken equal unit. Here indexes “cr” or “c” refer to critical point - r ($\mathbf{x}_{cr}$, τ_{cr}) or r_c: the point of the space-time of the beginning of the interaction between of deterministic field and random field which leads to turbulence.

3 The System of Equations

Stochastic equations of conservation, defined in [1 - 3] for an isothermal and non-conducting medium, in the absence of external forces, radiation, chemical reactions, baro - and thermal diffusion [4, 10, 16, 21], for non-isothermal condition take the form [1 - 3]: equation of mass (continuity)

$$\left(\frac{d(\rho)_{col\ st}}{d\tau}\right) = -\left(\frac{\rho_{st}}{\tau_{cor}}\right) - \left(\frac{d\rho_{st}}{d\tau}\right), \tag{1}$$

the momentum equation

$$\frac{d\rho u_{i\,col\,st}}{d\tau} = div(\tau_{i,j})_{col\,st} + div(\tau_{i,j})_{st} - \frac{(\rho U)_{st}}{\tau_{cor}} - \frac{d(\rho U)_{st}}{d\tau}, \quad (2)$$

energy equation

$$\frac{dE_{col\,st}}{d\tau} = div(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j})_{col\,st} + div(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j})_{st} - \left(\frac{E_{st}}{\tau_{cor}}\right) - \left(\frac{dE_{st}}{d\tau}\right). \quad (3)$$

$\rho, \vec{U}, u_i, u_j, u_l, \mu, \tau, \tau_{i,j}$ -density, velocity vector, the velocity component in the direction x_i, x_j, x_l (i, j, l = 1, 2, 3), the dynamic viscosity, time and stress tensor $\tau_{i,j} = P + \sigma_{i,j}$, $\sigma_{i,j} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\left(\xi - \frac{2}{3}\mu\right)\frac{\partial u_l}{\partial x_l}$ respectively. Also $\delta_{ij}=1$, if i=j, $\delta_{ij}=0$ for $i \neq j$. P - is the pressure of liquid or gas, λ- thermal conductivity, c_p и c_v specific heat at constant pressure and volume. For example, equations of equivalent measures for mass transfer, in the case $\left(R_{T^M ZT^N Y}\right) = 1$, can be written for the pair (N, M) = (1, 0) as $(d\rho_{col,st})_{1,0} = -(\rho_{st})$ and $\left(\frac{d(\rho)_{col,st}}{d\tau}\right)_{1,0} = -\left(\frac{\rho_{st}}{\tau_{cor}}\right)$. Equations of equivalent measures for mass transfer for the Pair (N, M) = (1, 1) are: $d(\rho_{col,st})_{1,1} = -d(\rho_{st})$ and $\left(\frac{d(\rho)_{col,st}}{d\tau}\right)_{1,1} = -\left(\frac{d(\rho)_{st}}{d\tau}\right)$.

Then for non-isothermal motion of the medium, using the definition of measures equivalency between deterministic and random process [1 - 3] in the critical point, the system of stochastic equations of energy, momentum and mass are defined for the next space-time areas: 1) the beginning of the generation; 2) generation; 3) diffusion and 4) the dissipation of the turbulent fields. So for the pair (N, M) = (1, 0) we have the system of equations of mass, momentum and energy for region of the beginning of turbulence generation $r_{c0}(x_c + \Delta x_0, \tau_c + \Delta\tau_0) \to r_c$ (system 4) :

$$\left(\frac{d(\rho)_{col,st}}{d\tau}\right)_{1,0} = -\left(\frac{\rho_{st}}{\tau_{cor}}\right)$$

$$\left(\frac{d(\rho\vec{U})_{col,st}}{d\tau}\right)_{1,0} = -\left(\frac{(\rho\vec{U})_{st}}{\tau_{cor}}\right); \; div(\tau_{i,j})_{col,st\,1} = \frac{(\rho\vec{U})_{st}}{\tau_{cor}}$$

$$\frac{d\left(E_{col\,st}\right)_{1,0}}{d\tau} = -\left(\frac{E_{st}}{\tau^0_{cor}}\right)_{1,0}, \; div(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j})_{col\,st\,1} = \left(\frac{E_{st}}{\tau^0_{cor}}\right)_{1,0}. \quad (4)$$

Here $\tau_{cor} = \tau^0_{cor}$. Index (colst1) refers to the pair (N, M) = (1, 0). For the pair (N, M) = (1, 1), the turbulence generation region $r_{c1}(x_c + \Delta x_0 + \Delta x_1, \tau_c + \Delta\tau_0 + \Delta\tau_1) \to r_{c0}$ we have a system

$$\left\{\begin{array}{l}\left(\dfrac{d(\rho)_{col,st}}{d\tau}\right)_{1,1}=-\left(\dfrac{d(\rho)_{st}}{d\tau}\right)\\ \left(\dfrac{d(\rho\vec{U})_{col,st}}{d\tau}\right)_{1,1}=-\left(\dfrac{d(\rho\vec{U})_{st}}{d\tau}\right);\ div\ (\tau_{i,j})_{col,st\,2}=\dfrac{d(\rho\vec{U})_{st}}{d\tau}\\ \dfrac{d\left(E_{col\,st}\right)_{1,1}}{d\tau}=-\left(\dfrac{dE_{st}}{d\tau}\right)_{1,1},\ div\ \left(\lambda\dfrac{\partial T}{\partial x_j}+u_i\tau_{i,j}\right)_{col\,st\,2}=\left(\dfrac{dE_{st}}{d\tau}\right)_{1,1}\end{array}\right. \quad (5)$$

Index (colst2) refers to the pair (N, M) = (1, 1). For the pair (N=p, M=k, l) = (1, 1, 0), $r_{c2}(x_c+\Delta x_0+\Delta x_1+\Delta x_2,\tau_c+\Delta\tau_0+\Delta\tau_1+\Delta\tau_2)-r_{c1}$ is the turbulence diffusion region. So we have a system of equations (6)

$$\left(\frac{d(\rho)_{st}}{d\tau}\right)=-\left(\frac{\rho_{st}}{\tau_{cor}}\right),\quad \left(\frac{d(\rho\vec{U})_{st}}{d\tau}\right)=-\left(\frac{(\rho\vec{U})_{st}}{\tau_{cor}}\right);$$

$$\left|\frac{dE_{st}}{d\tau}\right|=(R_{zTz})_{1,1,0}\left|\frac{E_{st}}{\delta\tau}\right|;\qquad (R_{zTz})_{1,1,0}=1\,. \quad (6)$$

For the region $r_{c3}(x_c+\Delta x_1+\Delta x_2+\Delta x_{3,}\tau_c+\Delta\tau_0+\Delta\tau_1+\Delta\tau_2+\Delta\tau_3)-r_{c2}$ of the turbulence dissipation we have the system of equations (7):

$$\frac{d(\rho\vec{U})_{st}}{d\tau}=div(\tau_{i,j})_{st};\quad div\ \left(\lambda\frac{\partial T}{\partial x_j}+u_i\tau_{i,j}\right)_{st}=\frac{dE_{st}}{d\tau}\,. \quad (7)$$

4 The critical Reynolds Number for Non-Isothermal Flow in Pipe

Now, on the basis of equivalence of measures [1 - 3] define the expressions for the critical Reynolds number for the non-isothermal processes. For the generating region $r_{c0}(x_c+\Delta x_0,\tau_c+\Delta\tau_0)-r_c$, refers a pair (N, M) = (1, 0), we have a system of equations of mass, momentum and energy (4). According to [10, 16, 21] the motion is determined by a quadratic equation of the velocity profile and the temperature profile as a function of the fourth power of the vertical coordinate assuming constant physical properties of the medium $u_1=U_0\left(\frac{x_2}{R}\right)^2$,

$\left[\frac{T-T_w}{T_0-T_w}=\left(\frac{x_2}{R}\right)^4\right]$. Here T_0 and T_w - the temperature on the axis and the radius on the tube wall, and R, U_0 , u_1 - velocity on the axis and along, the x_1, x_1 and x_2- longitudinal and transverse coordinates. Then we find that

$$div\ (u_i\tau_{i,j})_{col\ st\ 1} = \frac{\partial}{\partial x_2}\left\{\left[U_0\left(\frac{x_2}{R}\right)^2\right]\frac{\partial}{\partial x_2}\mu\left[U_0\left(\frac{x_2}{R}\right)^2\right]\right\} = 6\mu\left(\frac{U_0}{R}\right)^2\left(\frac{x_2}{R}\right)^2,$$

$$div\ \left(\lambda\frac{\partial T}{\partial x_i}\right)_{col\ st\ 1} = 12\lambda\left(\frac{(T_0-T_w)}{R^2}\right)\left(\frac{x_2}{R}\right)^2,$$

$$\frac{\partial}{\partial x_i}\left(u_i\tau_{ij}+\lambda\frac{\partial T}{\partial x_i}\right)_{col\ st\ 1} = \left(\frac{E_{st}}{\tau_{cor}}\right)_{1,0} = \left|\frac{\left(\rho u^2 i/2\right)_{st}}{\tau^0_{cor\ (U,P)}}\right| + \left|\frac{\rho c_p T_{st}}{\tau^0_{corT}}\right|.$$

Then

$$6\mu\left(\frac{U_0}{R}\right)^2\left(\frac{x_2}{R}\right)^2+12\lambda\left(\frac{(T_0-T_w)}{R^2}\right)\left(\frac{x_2}{R}\right)^2 = \left|\frac{\left(\rho u^2 i/2\right)_{st}}{\tau^0_{cor\,(U,P)}}\right| + \left|\frac{\rho c_P T_{st}}{\tau^0_{corT}}\right| = \left|\frac{\left(\rho u_i^2\right)_{st}(u_i)_{st}}{L_{(U,P)}}\right| + \left|\frac{\rho c_P\left(u_j\right)_{st}T_{st}}{L_T}\right|.$$

Now it is possible to determine the dependence of the Reynolds number in the tube at non-isothermal flow with constant thermal properties

$$\mathrm{Re}_{d(T,U)} = \left[12\left(\frac{\rho U_0^2}{(E_{st})_{U,P}}\right)\left(\frac{x_2}{R}\right)^2\left(\frac{\tau^0_{corU,P}}{R/U_0}\right)+24\frac{1}{\mathrm{Pr}}\left(\frac{\rho c_P(T_0-T_w)}{(E_{st})_{U,P}}\right)\left(\frac{x_2}{R}\right)^2\left(\frac{\tau^0_{corU,P}}{R/U_0}\right)\right]\left[\frac{1}{\left[1+\frac{\tau^0_{corU,P}}{\tau^0_{corT}}\frac{(E_{st})_T}{(E_{st})_{U,P}}\right]}\right]. \quad (8)$$

In (8), it is necessary to determine the correlation time for which we have the following representation for the case of non-isothermal flow:

$$\left(\tau^0_{cor}\right)_{1U,P}=\frac{L}{\left((E_{st})_{U,P}/\rho\right)^{1\backslash 2}},\quad \left(\tau^0_{cor}\right)_{2U,P}=\frac{L^2}{(\nu)},\quad \left(\tau^0_{cor}\right)_{3U,P}=\frac{\nu}{\left((E_{st})_{U,P}/\rho\right)}, \quad (9)$$

$$\left(\tau^0_{cor}\right)_{1T}=\frac{L_T}{\left((E_{st})_{u_j}/\rho\right)^{1\backslash 2}},\quad \left(\tau^0_{cor}\right)_{2T}=\frac{L^2_T}{\left(\lambda/\rho\cdot c_p\right)},\quad \left(\tau^0_{cor}\right)_{3T}=\frac{\lambda/\rho\ c_p}{\left((E_{st})_{u_j}\right)}. \quad (10)$$

$$\left(\tau^0_{cor}\right)_{1T}=\left(\frac{L_T}{\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}\right)=\left(\frac{L}{\left(\left(u_i^2\right)_{st}/2\right)^{1\backslash 2}}\right)\left(\frac{\left(\left(u_i^2\right)_{st}/2\right)^{1\backslash 2}}{\mathrm{Pr}\cdot\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}\right)=\left(\tau^0_{cor}\right)_{1U,P}\left(\frac{\left(\left(u_i^2\right)_{st}/2\right)^{1\backslash 2}}{\mathrm{Pr}\cdot\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}\right).$$

Further $L = L_{U,P} = L_U$ - linear of the perturbation -the scale of turbulence. Indexes ($_{U,P}$), ($_U$) refers to the velocity field, index ($_T$) refers to the temperature field. L_y on $x_2 = y$, or L_x, $x_1 = x$. Here x_1 and x_2 are coordinates along and normal to the wall.

Here $L_T = \frac{L}{\Pr}$, $\Pr = \frac{\rho \cdot \nu \cdot C_p}{\lambda}$ - Prandtl number. Then determine that

$$\left(E_{st}\right)_T / \left(E_{st}\right)_{U,P} = \frac{\left(c_P\ T_{st}\right)}{\left(\left(E_{st}\right)_{U,P}\right)/\rho} = 2\left(\frac{T_{st}}{(T_0 - T_w)}\right)\cdot\left(\frac{U_0^2}{u_{st}^2}\right)\frac{c_P(T - T_w)}{U_0^2} = 2\frac{T_T}{Ec \cdot Tu^2}.$$

$Ec = \frac{U_0^2}{c_p(T_0 - T_w)}$ - Eckert number. $T_T = \left|T_{st}\right| / \left(T_0 - T_w\right)$, $T_u = \sqrt{\frac{\Sigma\ \left(u_i^2\right)_{st}}{U_0^2}}$.

Now determine the value of the critical point $\left(\frac{x_2}{R}\right)^2_{critic}$ for non-isothermal process.

As for the isothermal process [1-3], we find the expression for the critical point by applying the relation for the equivalent measures: $d\left(E_{col_{st}}\right)_{1;0} = -E_{st}$.Here left side is and the right side is : $-E_{st} = -\left((E_{st})_{UP} + (E_{st})_T\right) = \left|\left(\rho u_i^2 / 2\right)_{st} + \rho c_p T_{st}\right|$.

In order to integrate the left-hand side, define the limits of integration as $[(x_2)_{cr} - L\backslash 2]$ and $[(x_2)_{cr} + L\backslash 2]$, whereas for isothermal write

$$\int_{-L|2}^{+L|2} d\left(\left(E_{UP}\right)_{col_{st}}\right)_{1;0} == \rho\frac{U_0^2}{R^4}\left[4Lx_2^3 + L^3x_2\right] \cong \rho\frac{U_0^2}{R^4}\left[4Lx_2^3\right] \cong 8\frac{\rho U_0^2}{2}\left(\frac{x_2}{R}\right)^3\frac{L}{R},$$

$$\int_{-L_T|2}^{+L_T|2} d\left(\left(E_T\right)_{col_{st}}\right)_{1;0} = \rho c_p\frac{\left[(T_0 - T_w)\right]}{R^4}\left[4L_Tx_2^3 + L_T^3x_2\right] \cong 4\rho c_p\left[(T_0 - T_w)\right]\left(\frac{x_2}{R}\right)^3\frac{L_T}{R}.$$

Then we have

$$4\rho U_0^2\left(\frac{x_2}{R}\right)^3\frac{L_U}{R} + 4\rho c_P\left[(T_0 - T_w)\right]\left(\frac{x_2}{R}\right)^3\frac{L_U}{\Pr^*\ R} = \left|\left(\rho u_i^2 / 2\right)_{st} + \rho c_p T_{st}\right|.$$

Thus, the critical point is determined by the expression

$$\left(\frac{x_2}{R}\right) = \left[\frac{1}{4}\frac{\left(E_{st}\right)_{UP}}{U_0^2}\left(\frac{R}{L_U}\right)\frac{Ec\cdot\Pr}{1 + Ec\cdot\Pr}\left(1 + \frac{\left(E_{st}\right)_T}{\left(E_{st}\right)_{UP}}\right)\right]^{\frac{1}{3}} = \left[\frac{1}{4}\frac{\left(E_{st}\right)_{UP}}{U_0^2}\left(\frac{R}{L_U}\right)\frac{Ec\cdot\Pr}{1 + Ec\cdot\Pr}\left(1 + \frac{2\cdot T_T}{Tu^2\cdot Ec}\right)\right]^{\frac{1}{3}}. \quad (11)$$

The value of the first critical Reynolds number (8) using (9)-(11) is determined as:

$$\mathrm{Re}_{d(T,U)} = 12\left(\frac{\rho U_0^2}{\left(E_{st}\right)_{U,P}}\right)\left(\frac{\tau^0{}_{corU,P}}{R/U_0}\right)\left[\left[\frac{1}{4}\frac{\left(E_{st}\right)_{UP}}{U_0^2}\left(\frac{R}{L_U}\right)\frac{Ec\cdot\Pr}{1 + Ec\cdot\Pr}\left(1 + \frac{\left(E_{st}\right)_T}{\left(E_{st}\right)_{UP}}\right)\right]\right]^{\frac{2}{3}}\left(1 + 2\frac{1}{\Pr}\left(\frac{\rho c_P(T_0 - T_w)}{\rho U_0^2}\right)\left[\frac{1}{\left[1 + \frac{\tau^0{}_{corU,P}}{\tau^0{}_{corT}}\frac{\left(E_{st}\right)_T}{\left(E_{st}\right)_{U,P}}\right]}\right]\right). \quad (12)$$

Since $\left(E_{st}\right)_T / \left(E_{st}\right)_{U,P} = \dfrac{2T_T}{Ec \cdot Tu^2}$, $\dfrac{\left(\tau^0_{cor}\right)_{1U,P}}{\left(\tau^0_{cor}\right)_{1T}} = \dfrac{\Pr \cdot \left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}/2\right)^{1\backslash 2}}$, then we write

$$\mathrm{Re}_{d(T,U)} = \left(6\left(\frac{1}{2}\right)^{1/3}\left(\frac{U_0}{\sqrt{E_{st}/\rho}}\right)^{5/3}\left(\frac{L_U}{R}\right)^{1/3}\right) \cdot \left(\frac{\left[1+\dfrac{2T_T}{Ec \cdot Tu^2}\right]^{\frac{2}{3}}}{\left[1+\left(\dfrac{\Pr \cdot \left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}\right)^{1\backslash 2}}\right)\dfrac{2T_T}{Ec \cdot Tu^2}\right]}\left(\frac{\left(1+2\dfrac{1}{\Pr \cdot Ec}\right)}{\left(1+\dfrac{1}{\Pr \cdot Ec}\right)^{2/3}}\right)\right). \quad (13)$$

Index (T, U) refers to the non-isothermal flow. The first bracket in (13) is an expression of the critical Reynolds number for an isothermal process. The second bracket determines the effects of temperature field (Pr, Ec), the turbulence intensities (Tu, T_T) and also (u_i/u_j) on the critical Reynolds number. It is seen that decreasing T_{st} and increasing the cooling of the wall lead to increasing of the critical Reynolds number. So, if Eckert number $Ec^{-1} = c_p\left(T_w - T_0\right)/U_0^2 = -0.01$, and $T_T = T_U = 0.01 \div 0.03$, Pr = 0.72, $\dfrac{\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}\right)^{1\backslash 2}} \approx 0.3 \div 0{,}5$, then the critical Reynolds number is increasing by ~$1.5 \div 1.9$ times. It is in satisfactory agreement with the experimental data which are presented in [10, 16, 21].

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