Electron Acceleration Gains Electron Mass

Whether it Radiates or Not

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Abstract

For an accelerated electron, electrodynamics yields two self-forces on the electron, a radiation reaction force and a previously unappreciated retardation force. These forces are examined and compared. The traditional radiation reaction force exists only when there is radiation that produces a time rate of change of acceleration. A retardation force results whenever the electron's self-field changes due to acceleration, and is present even if there is no radiation and the acceleration is constant. It is found that the retardation force on the electron is $\gggg$ the radiation reaction force. In a new way, this circumvents problems of pre-acceleration, and energy run-away solutions. It is predicted that because the retardation force F_{retard} manifests itself inertially, F_{retard} makes the dynamic mass of any accelerated subatomic singly charged particle about 1/137 higher than its rest mass.

Keywords: Radiation reaction force, Retardation force, Electromagnetic accelerated mass increase, Absence of pre-acceleration, Absence of energy run-away solutions, Accelerated charged particles

1 Introduction

The traditional view is that an accelerated charged particle experiences a back-reaction force in the process of losing energy by radiation. By simple analogy with projectiles fired from a gun, there is a back-reaction force on the particle, which is called the radiation reaction force. However, what is overlooked is that whether or not the particle radiates, there is a retarding force simply because the

charged particle is accelerated. As we shall see, whether and when an accelerating charged particle radiates is a subtle matter. And even more profoundly troubling questions relate to the effects of the radiation reaction force.

Richard Feynman's [2] remarks on the importance of the radiation reaction force F_{rad} [at times he calls it radiation resistance] in his Nobel prize speech are noteworthy: "When you accelerate an electron it radiates energy and you have to do extra work to account for that energy.... So, the force of radiation resistance, which is absolutely necessary for the conservation of energy would disappear if I said that a charge could not act on itself." His ideas evolve to: "Thus, it became clear that there was the possibility that if we assume all actions are via half-advanced and half-retarded solutions of Maxwell's equations and assume that all sources are surrounded by material absorbing all the light which is emitted, then we could account for radiation resistance as a direct action of the charges of the absorber acting back by advanced waves on the source." Independently in the more limited context of just F_{rad} , Dirac also used advanced potentials. As we shall see, charge can act on itself even when it is not radiating. Feynman's many insights had far-reaching consequences in developing his version of quantum electrodynamics. However, problems related to the radiation reaction force are still with us both classically and quantum mechanically. In the quantum theory of electrodynamics the question of radiation reaction for an electron leads to sizable difficulties. To date these have only been resolved in a somewhat arbitrary manner by means of re-normalization.

2 Self Retarding Force in Creating Induced Electric and Magnetic Fields

Let us consider the self retarding force on an electron of mass m , radius b , and charge e . Acceleration of the electron creates a time varying magnetic field B , which consequently induces an electric field E which opposes the acceleration. In differential form the Biot-Savart law is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{ed\vec{v} \times \vec{r}}{r^3}, \quad (1)$$

where μ_0 is the permeability of free space, and v is the electron velocity.

In the context of Jefimenko's time dependent generalization [6] of the Coulomb and Biot-Savart Laws, Griffiths and Heald [4] discuss the limitations of these laws, and their possible modification. Even though the eq. (1) form of the Biot-Savart law for a very small accelerated charge e is only approximately correct, Jackson [5] points out that an exact result may be obtained by integration. "...How can ...[eq. (1) here, which is only approximate] yield exact results ...? then the sum of the exact relativistic fields, *including acceleration effects* [and

hence time retardation], gives a magnetostatic field equal to the field obtained by integrating [eq. (1) here]” Also the Larmor equation for radiated power can be derived without Lienard-Wiechert fields as done for example by Purcell [14]. Unexpected solutions can be found to long standing problems [15]. The analysis in the present paper is well suited for the electron because it is structureless and point-like, but should be a good approximation for structured particles like the proton.

In the context of the present paper, the author thinks that inclusion of time retardation effects due to the finite speed of light (electromagnetic propagation) will result in only a small correction to the self retardation force which is an integrated result from the creation of the electric and magnetic fields. For the purposes of this paper the standard form of Maxwell's equations should be adequate, and esoteric questions can be dealt with in a more detailed paper.

We start with Maxwell's Equation,

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2)$$

relating the time rate of change of flux density B to the induced electric field E . For simplicity, we'll consider one dimensional motion in the z direction. In cylindrical coordinates eq. (2) implies

$$\frac{\partial E_z}{\partial r} = \frac{\partial B}{\partial t} \Rightarrow eE_z \delta t = e \int_b^\infty \delta B \partial r \quad (3)$$

$$\text{Substituting eq. (1) in (3)} \Rightarrow eE_z = e \int_b^\infty \frac{\mu_0}{4\pi} \frac{e(\delta v_z / \delta t)}{r^2} \partial r . \quad (4)$$

So in creating the induced E and B fields, a force F_{retard} opposes the electron's motion in the z direction at velocity $v \ll c$:

$$F_{retard} = eE_z = \int_b^\infty \frac{e^2}{4\pi\epsilon_0 c^2} \frac{\ddot{z}}{r^2} \partial r = \frac{-e^2}{4\pi\epsilon_0 c^2} \frac{\ddot{z}}{b} , \quad (5)$$

where $\mu_0 = 1/\epsilon_0 c^2$, ϵ_0 is the permittivity of free space, c is the speed of light; and the electron acceleration $\delta v_z / \delta t \equiv \ddot{z}$.

$$F_{ext} + F_{retard} = m\ddot{z} \Rightarrow F_{ext} = -F_{retard} + m_e \ddot{z} = \left(\frac{e^2}{4\pi\epsilon_0 b c^2} + m_e \right) \ddot{z} \quad (6)$$

Where F_{ext} is the applied external force. So effectively, F_{retard} acts to add an electromagnetic accelerated mass

$$m_{acc} = \frac{e^2}{4\pi\epsilon_0 bc^2} \quad (7)$$

to the mass of the electron. For a proton

$$M_{acc} = \frac{e^2}{4\pi\epsilon_0 Rc^2} \quad (8)$$

where $R = \frac{\hbar}{M_{0p}c} = \frac{\hbar}{1836m_e c}$ is the Compton radius of the proton. Taking the ratio of eq. (8) to eq. (7)

$$\frac{M_{acc}}{m_{acc}} = \frac{b}{R} = \frac{M_{0p}}{m_{0e}}. \quad (9)$$

Because the Compton radius of the proton is smaller than that of the electron, both the proton and electron have proportionately as much electromagnetic accelerated mass added to them relative to their rest mass.

Only charge acceleration is required for a retardation force. For F_{retard} , it makes no difference if energy resides in the near field, far field, or if it is radiated away or not. There is no requirement for time varying acceleration, nor even bounded motion.

3 Classical Charge Radiation and the Radiation Reaction Force

The power radiated from an accelerated charge is customarily derived from the condition that the Poynting vector $E \times H$ (electromagnetic energy flow per unit area per unit time) continues to exist as $r \rightarrow \infty$ when $E \times H$ is integrated over a sphere of radius r . Since the area of the sphere goes as r^2 , this implies that $E \times H$ must fall off no faster than r^{-2} . [However, a $\leq 1/r$ dependence for E and H may not always lead to radiation. Perhaps the ability to excite a receiver like an atom should also be considered.] This results in the familiar Larmor equation [8] for the radiated power from an accelerated charge e :

$$P_{rad} = \frac{-2e^2}{(4\pi\epsilon_0)3c^3} \ddot{z}^2 \equiv \beta \ddot{z}^2. \quad (10)$$

We may think of the radiated electric and magnetic fields E and H (or magnetic flux density B) that carry this power as having an independent detached existence from the accelerating charge that created them. Thereafter the time changing magnetic field creates the electric field, and vice versa.

From eq. (10), the energy expended by the radiation reaction force F_{rad} is:

$$\int_t^T F_{rad} \dot{z} dt = \int_t^T P_{rad} dt = \int_t^T \beta \ddot{z}^2 \dot{z} dt. \quad (11)$$

Integrating eq. (11) by parts:

$\frac{d}{dt}(\beta \dot{z} \ddot{z}) = \beta \ddot{z}^2 + \beta \ddot{z} \dddot{z} \Rightarrow [\beta \dot{z} \ddot{z}]_t^T = \int_t^T \beta \ddot{z}^2 dt + \int_t^T \beta \ddot{z} \dddot{z} dt = 0$. If the motion is bounded such that $\dot{z}$ and/or $\ddot{z} = 0$ at the end points t and T :

$$\Rightarrow \int_t^T \beta \ddot{z}^2 dt = - \int_t^T \beta \ddot{z} \dddot{z} dt. \quad (12)$$

Thus for bounded motion between the times t and T , substituting eq. (12) in (11)

$$\Rightarrow \int_t^T F_{rad} \dot{z} dt = - \int_t^T \beta \ddot{z} \dddot{z} dt \Rightarrow \int_t^T (F_{rad} + \beta \ddot{z}) \dot{z} dt = 0. \quad (13)$$

One solution for eq. (11) is for the integrand = 0: $\Rightarrow F_{rad} + \beta \ddot{z} = 0$

$$\Rightarrow F_{rad} = -\beta \ddot{z} = \frac{2e^2 \ddot{z}}{(4\pi\epsilon_0)3c^3} = \frac{e^2}{4\pi\epsilon_0} \left[\frac{2\ddot{z}}{3c^3} \right]. \quad (14)$$

As given by eq. (14), F_{rad} is the conventional radiation reaction force that yields pre-acceleration (acceleration before a force is applied); and energy run-away solutions (self-acceleration), which violate conservation of energy by ending up with more energy than the system had originally. Rohrlich [17] takes the position that sophisticated ways have been found to avoid the problems associated with F_{rad} . In a more recent paper, O'Connell [11] reviews some of the previous work on the problem of runaway solutions of the Abraham-Lorentz equation for a radiating electron, and concludes that his new approach is needed to really solve the problem. The present paper finds it unnecessary to side with either view since as will soon be shown $F_{retard} \gg \gg F_{rad}$ for an electron, thus it is F_{retard} that matters rather than F_{rad} .

We should bear in mind that F_{rad} of eq. (14) yields only one of possibly a number of other solutions of eq. (13), which originated from the Larmor formula for P_{rad} , given by eq. (10). Feynman [3] said "... we have inherited a prejudice that an accelerating charge should radiate the power radiated by an accelerating charge [the Larmor formula] has led us astray, it does not suffice to tell us 'when' the energy is radiated."

Feynman's first point [3] that it is possible for an accelerating charge not to radiate, is not so much related to non-radiating accelerated electrons in atoms as it is to the equivalence principle. The combination of eqs.(10) and (14) seem to be in contradiction to the equivalence principle of general relativity which argues that a body in free fall is not slowed down even if it were to radiate. A possible reconciliation in the spirit of Rohrlich [17] is that extra energy is stored in the electrostatic field due to being acted on by the gravitational field. Rohrlich argues that a charged particle falls with the same acceleration as a neutral particle, even though it radiates. For him, the extra energy in the electrostatic field accounts for the radiated energy.

Feynman's second point [3] that the Larmor formula for P_{rad} does not "tell us 'when' the energy is radiated," can easily be illustrated using eq.(10). For a periodically oscillating charge, when $\ddot{z} = 0$, then $P_{rad} = 0$. For sinusoidal variation this is at $z = 0$. However $\dot{z}$ is a maximum at $z = 0$. Now $P_{input} = F_{ext} \cdot \dot{z}$. So P_{input} is a maximum when $P_{rad} = 0$. The end points are equally troubling where $\dot{z} = 0 \Rightarrow P_{input} = 0$, but P_{rad} is a maximum at the end points where $\ddot{z}$ is a maximum. Using the radiation reaction force F_{rad} instead of F_{ext} to examine P_{rad} does not ameliorate these complexities and introduces troubling problems of its own.

The radiation reaction force F_{rad} was introduced by M. Abraham [1] in 1903 in the third of three papers with the same title. In 1904 H. A. Lorentz [9] generalized what is known as the Lorentz force to include radiation reaction:

$$\vec{F}_{Lorentz} = e[\vec{E} + \vec{v} \times \vec{B}] + F_{rad} = e[\vec{E} + \vec{v} \times \vec{B}] + \frac{e^2}{4\pi\epsilon_0} \left[\frac{2\ddot{\vec{v}}}{3c^3} \right]. \quad (15)$$

Rohrlich [17] gives a brief review of the history of the classical equations of motion of accelerated charged particles including relevant references.

4 Relative Magnitudes of the Electron Retardation and Radiation Reaction Forces

From eqs. (14) and (5) we have the ratio of the magnitudes of the retardation force F_{retard} and the radiation reaction force F_{rad} :

$$\left| \frac{F_{retard}}{F_{rad}} \right| = \left| \frac{\frac{e^2}{4\pi\epsilon_0} \left[\frac{\ddot{z}}{bc^2} \right]}{\frac{e^2}{4\pi\epsilon_0} \left[\frac{2\ddot{z}}{3c^3} \right]} \right| = \left| \frac{3c\ddot{z}}{2b\ddot{z}} \right|. \quad (16)$$

So for the retardation force to be much greater than the radiation reaction force, we need

$$\left| \frac{\ddot{z}}{\dot{z}} \right| \gg \frac{(2b/c)}{3}. \quad (17)$$

The ratio $(2b/c)$ is the time for light to cross the electron of diameter $2b$. Using the reduced Compton wavelength for b in eq. (17)

$$\left| \frac{\ddot{z}}{\dot{z}} \right|_e \gg \frac{(2b/c)}{3} = \frac{(2\lambda_c)}{3c} = \frac{(2\hbar/m_e c)}{3c} = \frac{2\hbar}{3m_e c^2} \approx 7 \times 10^{-22} \text{ sec}. \quad (18)$$

Hence for $|F_{\text{retard}}| \gg |F_{\text{rad}}|$ all that is required is for $\left| \frac{\ddot{z}}{\dot{z}} \right| \gg 10^{-21} \text{ sec}$. In terms of frequency in periodic motion, eq. (18) $\Rightarrow$ that as long as $\left| \frac{\ddot{z}}{\dot{z}} \right| \ll 10^{21} \text{ Hz}$, F_{retard} dominates easily. To put this in perspective, typical gamma ray frequencies are $\sim 10^{19} \text{ Hz}$. Therefore

$$F_{\text{ext}} + F_{\text{retard}} + F_{\text{rad}} = m\ddot{z} \Rightarrow F_{\text{ext}} = m\ddot{z} - F_{\text{retard}} - F_{\text{rad}} \approx m\ddot{z} - F_{\text{retard}}. \quad (19)$$

5 Relative Magnitudes of the Proton Retardation and Radiation Reaction Forces

From eq. (16), we have

$$\left| \frac{F_{\text{retard}}}{F_{\text{rad}}} \right| = \left| \frac{3c\ddot{z}}{2R\ddot{z}} \right|, \quad (20)$$

where $R = \frac{\hbar}{M_p c} = \frac{\hbar}{1836m_e c} \approx 2 \times 10^{-16} \text{ m}$ is the reduced Compton radius of the proton. So for a proton $|F_{\text{retard}}| \gg |F_{\text{rad}}|$ when

$$\left| \frac{\ddot{z}}{\dot{z}} \right| \gg \frac{(2R/c)}{3} \approx \frac{2(2 \times 10^{-16} \text{ m})}{3(3 \times 10^8 \text{ m/sec})} \approx 4 \times 10^{-25} \text{ sec}. \quad (21)$$

So for a proton as for an electron

$$F_{ext} + F_{retard} + F_{rad} = m\ddot{z} \Rightarrow F_{ext} = m\ddot{z} - F_{retard} - F_{rad} \approx m\ddot{z} - F_{retard}. \quad (22)$$

6 Electromagnetic Accelerated Mass of Singly Charged Subatomic Particles

The (reduced by 2π) Compton wavelength $\tilde{\lambda}_c = \frac{\hbar}{m_0 c}$ is quantum mechanically considered to be the effective radius of interaction for a fundamental particle. Writing eq.(7) for the electromagnetic mass of any fundamental particle, we have

$$m_{acc} = \frac{e^2}{4\pi\epsilon_0 b c^2} = \frac{e^2}{4\pi\epsilon_0 \tilde{\lambda}_c c^2} = \frac{e^2}{4\pi\epsilon_0 c^2} \frac{m_0 c}{\hbar} = m_0 \frac{e^2}{4\pi\epsilon_0 c \hbar} = \alpha m_0 \approx m_0 / 137, \quad (23)$$

$\alpha \equiv e^2 / 4\pi\epsilon_0 \hbar c \approx 1/137$. Sec.7 discusses whether other electromagnetic or electrostatic masses should be included.

Therefore for all singly charged accelerating subatomic particles with $v \ll c$, the electromagnetic accelerated mass which gives rise to the retardation force is about 1/137 higher than the rest mass. This mass increase equals and is in addition to a relativistic mass increase at $v \approx 0.12c$. So experimental measurements at $v \ll 0.12c$ appear easiest.

Equation (7), $m_{acc} = \frac{e^2}{4\pi\epsilon_0 b c^2}$ came naturally in the derivation of F_{retard} without a troublesome 4/3 factor. Using eq. (23) and Einstein's length contraction for the radius b_0 in the rest frame, $b = b_0 [1 - v^2/c^2]^{1/2}$, in eq. (7), gives the relativistic mass increase

$$m_{acc} = \frac{e^2}{4\pi\epsilon_0 c^2 (b_0 [1 - v^2/c^2]^{1/2})} = \frac{\alpha m_0}{[1 - v^2/c^2]^{1/2}}. \quad (24)$$

When an electron is accelerated, the dominant relativistic energy change is due to F_{retard} :

$$\begin{aligned} \frac{dE_{energy}}{dt} &= F_{retard} \cdot v \Rightarrow \frac{d(mc^2)}{dt} = \left[\frac{-e^2}{4\pi\epsilon_0 c^2} \frac{\ddot{z}}{b_0} \right] \cdot \dot{z} \Rightarrow m^2 c^2 = m \left[\frac{-e^2}{4\pi\epsilon_0 c^2} \frac{\dot{z}^2}{b_0} \right] + m_0^2 c^2 \\ &\Rightarrow (m_0^2 - m^2) c^2 = m m_{acc} \dot{z}^2. \end{aligned} \quad (25)$$

Eqs. (24) and (25) hint that the physics underlying relativistic mass and energy change with velocity $\dot{z}$ may be fundamentally electromagnetic in origin. In relativity,

eq. (24) applies to both charged and neutral matter suggests that most neutral matter may ultimately be composed of equal and opposite charge. If we model the H atom as two concentric spheres of + and – charge, acceleration of an H atom yields $m_{Hacc} \approx \frac{e^2}{4\pi\epsilon_0 c^2} \left[\frac{b_- - b_+}{b_+ b_-} \right]$. So, neutral atoms also increase mass upon acceleration.

7 Discussion

It is important to make distinctions here to avoid conceptual problems. The electrostatic mass of an electron (or any fundamental charged particle) is distinct from the electromagnetic accelerated mass. We have an electron that when stationary, has a mass $m_e \approx m_0 + \frac{e^2}{4\pi\epsilon_0 b c^2}$ where its additional electrostatic mass

$m_{es} \approx \frac{e^2}{4\pi\epsilon_0 b c^2}$ just from assembling charge on or in it. Equating the electrostatic

energy to $m_{es} c^2 \approx \frac{e^2}{4\pi\epsilon_0 b}$. $\Rightarrow m_{es} \approx \frac{e^2}{4\pi\epsilon_0 b c^2}$. For $v \ll c$, it is sufficient to deal with just m as was done in Sec. 2. A retarding force F_{retard} enters in because the charge e is accelerated. This retarding force can be written in the form

$F_{retard} = m_{acc} \ddot{z} = \frac{e^2}{4\pi\epsilon_0 b c^2} \ddot{z}$ wherein the additional electromagnetic accelerated

mass $m_{acc} = \frac{e^2}{4\pi\epsilon_0 b c^2} \sim$ electrostatic mass $m_{es} \approx \frac{e^2}{4\pi\epsilon_0 b c^2}$. What is significant is that for an electron, the magnitude of the retarding force $\gg \gg$ the conventional radiation reaction force as shown by eqs. (16) and (18):

$$|F_{retard}| = |m_{acc} \ddot{z}| = \left| \frac{e^2}{4\pi\epsilon_0 b c^2} \ddot{z} \right| \gg \gg |F_{rad}| = \left| \frac{e^2}{4\pi\epsilon_0} \left[\frac{2\ddot{z}}{3c^3} \right] \right|. \quad (26)$$

Furthermore, the retardation force F_{retard} is present whether there is radiation or not, because it is related to the energy needed to create the fields. The electromagnetic field momentum yields a mass $m_{mom} \approx m_{acc}$, which can be transformed away in the rest frame (assuming no hidden or circulating momentum), and is distinct from m_{acc} . Since $F_{retard} = (m_{acc} + m_{es} + m_{mom}) \ddot{z} \sim m_{acc} \ddot{z}$, it is a moot point whether F_{retard} should include m_{es} and/or m_{mom} as well as questions of kinetic versus canonical momentum, since this would not change the relationship $F_{retard} \gg \gg F_{rad}$. It is clear that m_{acc} , m_{mom} , and m_{es} are inertial masses that resist acceleration. But each may not consist of energy $= mc^2$.

When the radius b is large enough that F_{retard} is not $> F_{\text{rad}}$, it is not sufficient to change eq. (26) into an equality, and solve for b . This will result in such a large value of b that it is not physically relevant. Complications of the non-uniformity of N discrete charges would enter in. If the radiation were coherent, the radiated power $\propto (Ne)^2$ could be large, but F_{rad} and $F_{\text{retard}} \propto (Ne)^2$ would also be large. The non-uniformity may change with acceleration so that the center of charge deviates as a function of time from the center of mass. This can be dealt with, but the author is not aware of this having been done.

Although Moniz and Sharp [10] consider charged spheres with $b \geq b_{\text{Compton}}$, they do not deal with questions related to the non-uniformity of discrete charges as b gets large. For a realistic representation of a large sphere containing discrete charges, the formulation of the problem would require complications that go well beyond the present calculation. Not the least of these problems would be to properly use the Lienard-Wiechert retarded fields to obtain the Larmor radiated power for a large accelerating charged sphere. To then obtain the radiation reaction force would be a much more difficult problem.

Landau and Lifshitz [7] acridly illustrate the extreme unreasonableness of the possible solutions allowed by the Abraham-Lorentz equation (14) for the radiation reaction force: "... a charge passing through any field, upon emergence from the field, would have to be infinitely 'self-accelerated'." In considering the energy source of this difficulty, they say: "When in the equation of motion we write a finite mass for the charge, then in doing this we essentially assign to it formally an infinite negative 'intrinsic mass' of nonelectromagnetic origin, which together with the electromagnetic mass should result in a finite mass for the particle." Their circumvention of these problems is by means of an approximate alternative to the Abraham-Lorentz equation.

$$\text{Peirls [13] finds fault with the Larmor eq. (10) } P_{\text{rad}} = \frac{-2e^2}{(4\pi\epsilon_0)3c^3} \ddot{z}^2 = \beta a^2,$$

and argues that it should be replaced by $P_{\text{rad}} = \beta a\ddot{a}$. He then examines the impact this has on F_{rad} . He cogently discusses the ways that the equivalence principle has been preserved in resolving the paradoxes related to the acceleration of charge by a gravitational field and the presumed need to have $F_{\text{rad}} = 0$. Peirls points out that Pauli [12] was the first to argue that hyperbolic motion (free fall) of a charged particle does not lead to the emission of radiation. And hence there would be no radiation reaction force in free fall. This view is antithetical to that of Rohrlich [16] as discussed above in Sec. 3.

8 Conclusion

The inconsistencies of the radiation reaction force related to pre-acceleration, and energy run-away solutions are shown to be negligible because the radiation reaction

force F_{rad} is exceedingly smaller than the previously unappreciated retardation force F_{retard} . It is shown that F_{rad} is inconsequential relative to F_{retard} for an electron and other charged subatomic particles due to their small radii. Because the retardation force is proportional to acceleration, it manifests itself as an inertial force. A prediction of this analysis is that for $v \ll c$, the dynamic mass of an accelerated subatomic singly charged particle like an electron or proton is about 1/137 higher than its rest mass.

What is important to bear in mind, is that even without radiation, an accelerated electron's changing magnetic and electric fields act back on the electron to retard its motion. And when the electron radiates, the radiation reaction force is so extremely negligible relative to the retardation force that F_{rad} is usually lost in the noise. The domination of F_{retard} results in an easily measurable increase in dynamic mass. Acceleration of charged particles and most neutrals always results in greater mass, appears to be a novel finding.

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