Adv. Theor. Appl. Mech., Vol. 4, 2011, no. 2, 49 - 67

Vibration Suppression via Time-delay Absorber Described by Non-Linear Differential Equations

W. A. A. El-Ganaini and H. A. Elgohary

Department of Engineering Mathematics
Faculty of Electronic Engineering, Menouf 32952, Egypt
elgohary_hany@yahoo.com

Abstract

The aim of this work is to control the main system behavior represented by a beam at simultaneous primary and internal resonance condition, where the system damage is probable. In this work, we present a comprehensive investigation of the effect of the time delay on the non-linear control of a beam when subjected to multi- external excitation forces. Control is conducted via time delay absorber to suppress chaotic vibrations. Multiple scale perturbation method is applied to obtain the solution up to the second order approximations. The different resonance cases are reported and studied numerically. The stability of the steady state solution for the selected resonance case is determined and investigated applying Rung-Kutta fourth order method and frequency response equation. Time delay absorber is more effective than ordinary absorber. The delay time is an important factor in selecting the absorber. Investigating the effect of time delay on main system behavior revealed that there is a critical value where below it vibration suppression is effective. The effects of the different parameters of the absorber on the system behavior are studied numerically. The reported results are compared to the available published work.

Keywords: Vibration control, Stability, Time delay, Absorber

1. Introduction

The study of vibrating structures has been a subject of a particular interest in recent years. This is due to the fact that structures under multi-external excitation forces appear in various fields of fundamental and applied science [1-3]. Among these studies, particular attention had been paid to the dynamical behavior of beams. It was shown that when the beam is not highly loaded, its dynamics could be explained by the classical Duffing oscillator [2]. In [3] the authors used the nonlinearity of a foundation and showed that the behavior of the beam could be expressed by a ϕ^6 potential. Another important center of the interest is the study of vibrating structures under active control [4-7]. In [5], Morgan et al. proposed a semi-active piezoelectric absorber for suppressing harmonic excitations with varying frequency. Aida et al. [6] proposed a plate type dynamic vibration absorber to control the vibration of plates. In [7] the authors used the beam-type vibration absorber to control the bending vibration of a beam in this case; the beam (or plate) is assumed to be in the linear dynamics. Non-linearities necessarily introduce a whole range of phenomena that are not found in linear systems [8], including jump phenomena, occurrence of multiple solutions, modulations, shift in natural frequencies, the generation of combination resonances and chaotic motions [9–10]. In such systems the vibrations are needed to be controlled to minimize or eliminate the hazard of damage or destruction. There are two main regimes for vibration control. They are passive and active control. One of the most effective tools of passive vibration control is the dynamic absorber or the damper or the neutralizer [11]. In [12] the authors proposed an approximate condition for escape from the potential well by comparing the maximum energy of the motion with barrier of the potential. Lee et al. [13] demonstrated a dynamic vibration absorber system, which can be used to reduce speed fluctuations in rotating machinery. Eissa [14] has shown that to control the vibration of a system subjected to harmonic excitations, the fundamental or the first harmonic absorber is the most effective one. In [15], the authors considered such a problem in linear structures and showed that time-delay can even lead to the instability of the whole structure. Eissa et. al. [16-17] investigated saturation phenomena in non-linear oscillating systems subject to multi-parametric and/or external excitations. The system represents the vibration of a single-degree-of-freedom cantilever or the wing of an aircraft. They reported the occurrence of saturation phenomena at different parameters values. They applied saturation values of different parameters as optimum working conditions for vibration suppression of the cantilever. In ref [18], the authors considered such a problem in linear structures and showed that time-delay can even lead to the instability of the whole structure. Nana et al [19-21] studied the modeling and optimal active control with time delay dynamics of a strongly nonlinear beam. They investigated the control by a sandwich beam and one using piezoelectric absorber. Yaman [22] studied the primary and parametric resonances of a directly and parametrically excited nonlinear cantilever beam of varying orientation with time-delay state feedback. The perturbation method was used to obtain the bifurcation equation of the linear dynamic system. A control law based on time-delay feedback is used. Maccari [23-24] studied the vibration control for parametrically excited Lienard systems and vibration amplitude control for a van der Pol-Duffing oscillator with time delay. Using the asymptotic perturbation method, the best choices of the feedback gains and the time delay from the viewpoint of vibration control are found. In many cases the amplitudes of periodic solutions do not correspond to the technical requirements. It is demonstrated that if the vibration control terms are added, stable periodic solutions with arbitrarily chosen amplitude can be accomplished. Therefore the results obtained show that an effective vibration amplitude control is possible if appropriate time delay and feedback gains are chosen.

The objective of this work is to study the vibration of a damped beam subject to multi-external excitation forces. The model is represented by two-degree-of-freedom system consisting of the main system and the absorber. The multiple time scale perturbation technique is applied throughout to get an approximate solution up to the second order. The stability of the system is investigated numerically applying both phase-plane and frequency response functions. The effects of the different parameters of the absorber on system behavior are studied numerically. Some optimal parameters for the absorber are determined. Comparison with the available published work is reported.

2. Mathematical Modeling

The modified governing equations of the mechanical systems under control are given by equations [15].

$$\ddot{X}_1(t)+\varepsilon(\lambda_1+\alpha)\dot{X}_1(t)+\omega_1^2X_1(t)+\varepsilon cX_1^3(t)+\varepsilon dX_1^5(t)-\varepsilon\beta X_2(t)-\varepsilon\alpha\dot{X}_2(t)=\sum_{j=1}^{3}F_j\cos(\Omega_j t)$$

$$\ddot{X}_2(t)+\varepsilon(\lambda_2+\mu\alpha)\dot{X}_2(t)+\omega_2^2X_2(t)=\varepsilon\mu\beta X_1(t)+\varepsilon\mu\alpha\dot{X}_1(t) \tag{1}$$

The main system is exited by multi-external forces, $X_1, \dot{X}_1, \ddot{X}_1$ are the displacement, velocity and acceleration of the beam respectively and X_2 is the control force, λ_1 and λ_2 are the non-dimensionless damping coefficient of the two system respectively, c, d and μ are the other characteristic coefficient of the structure, α and β are the non-dimensionless control gain parameters and $F_{j\,,}\,\Omega_j$ are the external excitation forces , the natural frequencies respectively. The delay is materialized by the fact that the control system doesn't act at the same time with excited structure. Mathematically, this effect is taken into account by using the retarded functional differential equations [18, 19, 20]. Thus for a controlled system with delay, the differential equation given by (1) becomes:

$$\ddot{X}_1(t)+\varepsilon(\lambda_1+\alpha)\dot{X}_1(t)+\omega_1^2X_1(t)+\varepsilon cX_1^3(t)+\varepsilon dX_1^5(t)-\varepsilon\beta X_2(t)-\varepsilon\alpha\dot{X}_2(t)=\sum_{j=1}^{3}F_j\cos(\Omega_j t)$$

$$\ddot{X}_2(t)+\varepsilon(\lambda_2+\mu\alpha)\dot{X}_2(t)+\omega_2^2X_2(t)=\varepsilon\mu\beta X_1(t-t_q)+\varepsilon\mu\alpha\dot{X}_1(t-t_{\dot{q}}) \tag{2}$$

Where t_q and $t_{\dot{q}}$ are the time delays for the displacement and velocity feedback in the system respectively.

2.1. Perturbation analysis

Multiple scale perturbation method [25-27] is conducted to obtain an approximate solution for Eqs. (2) Assuming the solution in the form:

$$x_1(t;\varepsilon)=x_{10}(T_0,T_1)+\varepsilon x_{11}(T_0,T_1)+\varepsilon^2(0) \tag{3}$$

$$x_2(t;\varepsilon)=x_{20}(T_0,T_1)+\varepsilon x_{21}(T_0,T_1)+\varepsilon^2(0) \tag{4}$$

and the time derivatives became

$$\frac{d}{dt} = D_0 + \varepsilon D_1 \ , \ \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2 \tag{5}$$

where $T_n = \varepsilon^n t$. (n=0, 1) are the fast and slow time scales respectively. Substituting Eqs. (3), (4) and (5) into Eqs. (2) and equating the coefficients of the same power of ε in both sides, we obtain

$$(D_0^2 + \omega_1^2) x_{10} = \frac{1}{2} \sum_{j=1}^{n} F_j (e^{i\Omega_j T_0} + e^{-i\Omega_j T_0}) \tag{6}$$

$$(D_0^2 + \omega_2^2) x_{20} = 0 \tag{7}$$

$$\begin{aligned}(D_0^2 + \omega_1^2) x_{11} = {} & -2D_1 D_0 x_{10} - (\lambda_1 + \alpha) D_0 x_{10} - c x_{10}^3 - d x_{10}^5 \\ & + \beta x_{20} + \alpha D_0 x_{20}\end{aligned} \tag{8}$$

$$\begin{aligned}(D_0^2 + \omega_2^2) x_{21} = {} & -2D_0 D_1 x_{20} - (\lambda_2 + \mu\alpha) D_0 x_{20} + \mu\beta x_{10}(T_0 - t_q, T_1) \\ & + \mu\alpha D_0 x_{10}(T_0 - t_q, T_1)\end{aligned} \tag{9}$$

The solution of Eqs. (6) and (7) can be expressed in the form

$$x_{10} = A_{10} \exp(i\ \omega_1 T_0) + \frac{1}{2} \sum_{j=1}^{3} P_j \exp(i\Omega_j T_0) + cc \tag{10}$$

$$x_{20} = A_{20} \exp(i\ \omega_2 T_0) + cc \tag{11}$$

Where $P_j = \dfrac{F_j}{\omega_1^2 - \Omega_j^2}$ and A_{10}, A_{20} are complex functions in T_1 , which can be determined from eliminating the secular terms at the next approximation, and cc, stands for the conjugate of the preceding terms. Substituting Eqs. (10) and (11) into Eq. (8), eliminating the secular terms, then the first order approximation is given by:

$$\begin{aligned}x_{11} = {} & A_{11} \exp(i\ \omega_1 T_0) - \frac{i}{2}(\lambda_1 + \alpha) \sum_{j=1}^{3} E_1 \Omega_j \exp(i\Omega_j T_0) - c[\frac{-A_{10}^3}{8\omega_1^2} \exp(3i\ \omega_1 T_0) \\ & + 3A_{10}\bar{A}_{10} \sum_{j=1}^{3} E_1 \exp(i\Omega_j T_0) + \frac{3}{2} A_{10}^2 \sum_{j=1}^{3} E_2 \exp(i(2\omega_1 + \Omega_j) T_0) + \frac{3}{2} A_{10}^2 \sum_{j=1}^{3} E_3 \exp(i(2\omega_1 - \Omega_j) T_0) \\ & + \frac{3}{4} A_{10} \sum_{j=1}^{3} \sum_{k=1}^{3} E_4 \exp(i(\omega_1 + \Omega_j + \Omega_k) T_0) + \frac{3}{2} A_{10} \sum_{j=1}^{3} \sum_{k=1}^{3} E_5 \exp(i(\omega_1 + \Omega_j - \Omega_k) T_0) \\ & + \frac{3}{4} A_{10} \sum_{j=1}^{3} \sum_{k=1}^{3} E_6 \exp(i(\omega_1 - \Omega_j - \Omega_k) T_0) + \frac{1}{8} \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} E_7 \exp(i(\Omega_j + \Omega_k + \Omega_l) T_0) + \frac{3}{8} \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} E_8 \exp(i(\Omega_j + \Omega_k - \Omega_l) T_0)] \\ & + d\,[\frac{1}{24\omega_1^2} A_{10}^5 \exp(i(5\omega_1) T_0) + \frac{5}{8\omega_1^2} A_{10}^4 \bar{A}_{10} \exp(i(3\omega_1) T_0) \\ & - \frac{5}{2} \sum_{j=1}^{3} E_9 A_{10}^4 \exp(i(4\omega_1 + \Omega_j) T_0) - \frac{5}{2} \sum_{j=1}^{3} E_{10} A_{10}^4 \exp(i(4\omega_1 - \Omega_j) T_0)\end{aligned}$$

$$-15A_{10}^{2}\bar{A}_{10}^{2}\sum_{j=1}^{3}E_{1}\exp(i\Omega_{j})T_{0}-10A_{10}^{3}\bar{A}_{10}\sum_{j=1}^{3}E_{2}\exp(i(2\omega_{1}+\Omega_{j})T_{0})$$
$$-10\sum_{j=1}^{3}E_{3}A_{10}^{3}\bar{A}_{10}\exp(i(2\omega_{1}-\Omega_{j})T_{0})-\frac{5}{2}\sum_{j=1}^{3}\sum_{k=1}^{3}E_{11}A_{10}^{3}\exp(i(3\omega_{1}+\Omega_{j}+\Omega_{k})T_{0})$$
$$-5\sum_{j=1}^{3}\sum_{k=1}^{3}E_{12}A_{10}^{3}\exp(i(3\omega_{1}+\Omega_{j}-\Omega_{k})T_{0})-5\sum_{j=1}^{3}\sum_{k=1}^{3}E_{13}A_{10}^{3}\exp(i(3\omega_{1}-\Omega_{j}+\Omega_{k})T_{0})$$
$$-\frac{15}{2}\sum_{j=1}^{3}\sum_{k=1}^{3}E_{4}A_{10}^{2}\bar{A}_{10}\exp(i(\omega_{1}+\Omega_{j}+\Omega_{k})T_{0})-\frac{15}{2}\sum_{j=1}^{3}\sum_{k=1}^{3}E_{6}A_{10}^{2}\bar{A}_{10}\exp(i(\omega_{1}-\Omega_{j}-\Omega_{k})T_{0})$$
$$-15\sum_{j=1}^{3}\sum_{k=1}^{3}E_{5}A_{10}^{2}\bar{A}_{10}\exp(i(\omega_{1}+\Omega_{j}-\Omega_{k})T_{0})-15\sum_{j=1}^{3}\sum_{k=1}^{3}E_{14}A_{10}^{2}\bar{A}_{10}\exp(i(\omega_{1}-\Omega_{j}+\Omega_{k})T_{0})$$
$$-\frac{5}{4}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}E_{15}A_{10}^{2}\exp(i(2\omega_{1}+\Omega_{j}+\Omega_{k}+\Omega_{l})T_{0})-\frac{15}{4}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}E_{16}A_{10}^{2}\exp(i(2\omega_{1}+\Omega_{j}+\Omega_{k}-\Omega_{l})T_{0})$$
$$-\frac{15}{4}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}E_{17}A_{10}^{2}\exp(i(2\omega_{1}-\Omega_{j}-\Omega_{k}+\Omega_{l})T_{0})-2\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}E_{7}A_{10}\bar{A}_{10}\exp(i(\Omega_{j}+\Omega_{k}+\Omega_{l})T_{0})$$
$$-2\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}E_{8}A_{10}\bar{A}_{10}\exp(i(\Omega_{j}+\Omega_{k}-\Omega_{l})T_{0})-\frac{5}{16}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}E_{18}A_{10}\exp(i(\omega_{1}+\Omega_{j}+\Omega_{k}+\Omega_{l}+\Omega_{m})T_{0})$$
$$-\frac{5}{4}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}E_{19}A_{10}\exp(i(\omega_{1}+\Omega_{j}+\Omega_{k}+\Omega_{l}-\Omega_{m})T_{0})-\frac{5}{4}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}E_{20}A_{10}\exp(i(\omega_{1}-\Omega_{j}-\Omega_{k}-\Omega_{l}+\Omega_{m})T_{0})$$
$$-\frac{15}{8}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}E_{21}A_{10}\exp(i(\omega_{1}+\Omega_{j}+\Omega_{k}-\Omega_{l}-\Omega_{m})T_{0})-\frac{1}{32}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}\sum_{n=1}^{3}E_{22}\exp(i(\Omega_{j}+\Omega_{k}+\Omega_{l}+\Omega_{m}+\Omega_{n})T_{0})$$
$$-\frac{5}{32}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}\sum_{n=1}^{3}E_{23}\exp(i(\Omega_{j}+\Omega_{k}+\Omega_{l}+\Omega_{m}-\Omega_{n})T_{0})-\frac{5}{16}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}\sum_{n=1}^{3}E_{24}\exp(i(\Omega_{j}+\Omega_{k}+\Omega_{l}-\Omega_{m}-\Omega_{n})T_{0})$$
$$+\beta A_{20}E_{25}\exp(i\,\omega_{2}T_{0})+i\,\omega_{2}\alpha A_{20}E_{25}\exp(i\,\omega_{2}T_{0})+cc \tag{12}$$

Where $E_s\ (s=1,2,.....,25)$ are complex functions in T_1 and j= (1, 2, 3). From Eqs. (10), (11) and (12) into Eq. (9) and eliminating the secular terms the solution is given by:

$$x_{21}=A_{22}\exp(i\,\omega_{2}T_{0})+\mu\beta A_{10}H_{1}\exp(i\,\omega_{1}T_{0})+\frac{1}{2}P_{j}\mu\beta H_{2}\exp i\,\Omega_{j}(T_{0}-t_{q})$$
$$+\mu\alpha(i\,\omega_{1})A_{10}H_{1}\exp(i\,\omega_{1}T_{0})+\frac{1}{2}P_{j}\mu\alpha(i\,\Omega_{j})H_{2}\exp i\,\Omega_{j}(T_{0}-t_{q})+cc \tag{13}$$

Where $H_s\ (s=1,2)$ are complex functions in T_1 and j=(1…..3).

The reported resonance cases at this approximation order are:

(a) Trivial resonance: $\Omega_1 \cong \omega_1 \cong \omega_2 = 0$

(b) Primary resonance: $\Omega_1 \cong \omega_1$, $\Omega_1 \cong \omega_2$

(c) Sub-harmonic resonance: $\Omega_1 \cong 2\omega_1, \Omega_1 \cong 3\omega_1$, $\Omega_1 \cong 2\omega_2, \Omega_1 \cong 3\omega_2$

(d) Internal resonance: $\omega_1 \cong \omega_2$

(e) Simultaneous or incident resonance: Any combination of the above resonance cases is considered as simultaneous resonance.

2.2 Stability of the system

Introducing the detuning parameters σ_j and ρ_j for the primary and internal resonance cases to convert the small-divisor terms into the secular terms, according to:

$$\Omega_j \cong \omega_1 + \varepsilon\sigma_j \qquad , \ \Omega_j \cong \omega_2 + \varepsilon\rho_j \tag{14}$$

This case represents the system worst case (confirmed numerically) .Substituting Eq. (14) into Eqs. (8) and (9) and eliminating the secular terms, leads to the solvability conditions for the first order approximation noting that A_{10} and A_{20} are functions in T_1 we get

$$i\,\omega_1[2D_1A_{10} + (\lambda_1+\alpha)A_{10}] + \tfrac{i}{2}(\lambda_1+\alpha)P_j\Omega_j e^{i\sigma_j T_1} + 3cA_{10}^2\bar{A}_{10} + 3cA_{10}\bar{A}_{10}P_j e^{i\sigma_j T_1} + 10dA_{10}^3\bar{A}_{10}^2 + 15dA_{10}^2\bar{A}_{10}^2P_j e^{i\sigma_j T_1} = 0 \tag{15}$$

$$i\,\omega_2[2D_1A_{20} + (\lambda_2+\mu\alpha)A_{20}] - \frac{1}{2}[\mu\beta P_j e^{-i\Omega_j t_q} + i\,\mu\alpha\Omega_j P_j e^{-i\Omega_j t_q}]e^{i\rho_j T_1} = 0 \tag{16}$$

Putting

$$A_{10} = \frac{1}{2}a_1(T_1)e^{i\psi_1(T_1)},\ A_{20} = \frac{1}{2}a_2(T_1)e^{i\phi_1(T_1)} \tag{17}$$

where a_1, a_2 and ψ_1, ϕ_1 are the steady state amplitudes and the phases of the motion respectively. Substituting Eq. (17) into Eqs. (15) and (16) and separating real and imaginary parts yields,

$$a_1' = -\frac{1}{2}(\lambda_1+\alpha)a_1 - \frac{(\lambda_1+\alpha)\Omega_j P_j}{2\omega_1}\cos\theta_j - [\frac{3ca_1^2P_j}{4\omega_1} + \frac{15da_1^4P_j}{16\omega_1}]\sin\theta_j \tag{18}$$

$$a_1\psi_1' = \frac{3ca_1^3}{8\omega_1} + \frac{10da_1^5}{32\omega_1} - \frac{(\lambda_1+\alpha)P_j\Omega_j}{2\omega_1}\sin\theta_j + [\frac{3ca_1^2P_j}{4\omega_1} + \frac{15da_1^4P_j}{16\omega_1}]\cos\theta_j \tag{19}$$

$$a_2' = -\frac{(\lambda_2+\mu\alpha)a_2}{2} + \frac{\mu\alpha\Omega_j P_j}{2\omega_2}\cos(\varphi_j - \Omega_j t_q) + \frac{\mu\beta P_j}{2\omega_2}\sin(\varphi_j - \Omega_j t_q) \tag{20}$$

$$a_2\phi_1' = \frac{\mu\alpha\Omega_j P_j}{2\omega_2}\sin(\varphi_j - \Omega_j t_q) - \frac{\mu\beta P_j}{2\omega_2}\cos(\varphi_j - \Omega_j t_q) \tag{21}$$

where

$$\theta_j = \sigma_j T_1 - \psi_1, \qquad \varphi_j = \rho_j T_1 + \phi_1 - \psi_1 \tag{22}$$

For steady state solutions, $a_1' = a_2' = \theta_j' = \varphi_j' = 0$. Then from Eq. (22), we get:

$$\psi_1' = \sigma_j \quad , \ \phi_1' = \sigma_j - \rho_j \tag{23}$$

Then it follows from Eqs. (18)-(21) that the steady state solutions are given by

$$-\frac{1}{2}(\lambda_1+\alpha)a_1-\frac{(\lambda_1+\alpha)\Omega_j P_j}{2\omega_1}\cos\theta_j-[\frac{3ca_1^2P_j}{4\omega_1}+\frac{15da_1^4P_j}{16\omega_1}]\sin\theta_j=0 \quad (24)$$

$$a_1\sigma_j=\frac{3ca_1^3}{8\omega_1}+\frac{10da_1^5}{32\omega_1}-\frac{(\lambda_1+\alpha)P_j\Omega_j}{2\omega_1}\sin\theta_j+[\frac{3ca_1^2P_j}{4\omega_1}+\frac{15da_1^4P_j}{16\omega_1}]\cos\theta_j \quad (25)$$

$$-\frac{(\lambda_2+\mu\alpha)a_2}{2}+\frac{\mu\alpha\Omega_j P_j}{2\omega_2}\cos(\varphi_j-\Omega_j t_q)+\frac{\mu\beta P_j}{2\omega_2}\sin(\varphi_j-\Omega_j t_q)=0 \quad (26)$$

$$a_2\rho_j=\frac{\mu\alpha\Omega_j P_j}{2\omega_2}\sin(\varphi_j-\Omega_j t_q)+\frac{\mu\beta P_j}{2\omega_2}\cos(\varphi_j-\Omega_j t_q) \quad (27)$$

From Eqs. (24)- (27) we have the following two cases:

(1) $a_1\neq 0, a_2=0$ (2) $a_1=0, a_2\neq 0$

Table 1 gives the final results of the frequency response equations.

Table 1: Frequency response equations

No	Cases	Frequency response equations (FRE)
1	$a_1\neq 0, a_2=0$	$\sigma_j^2+k_1\sigma_j+k_2=0$
2	$a_1=0, a_2\neq 0$	$\rho_j^2+k_3=0$

Where k_1, k_2, k_3 are defined in the appendix.

The stability of the linear solution of the obtained fixed points will be determined as follows. Consider A_{10}, A_{20} in the form:

$$A_{10}=\frac{1}{2}[p_1-iq_1]e^{iv_1T_1}\ ,\ A_{20}=\frac{1}{2}[p_2-iq_2]e^{iv_2T_1} \quad (28)$$

where p_1, p_2, q_1 and q_2 are real and $v_1=\sigma_j$, $v_2=\rho_j$. Substituting Eq. (28) into the linear part of Eqs. (15) and (16) and separating real and imaginary part yields,

$$p_1'+v_1q_1+\frac{(\lambda_1+\alpha)}{2}p_1=0 \quad (29)$$

$$q_1'-v_1p_1+\frac{(\lambda_1+\alpha)}{2}q_1=0 \quad (30)$$

$$p_2'+v_2q_2+\frac{(\lambda_2+\mu\alpha)}{2}p_2=0 \quad (31)$$

$$q_2'-v_2p_2+\frac{(\lambda_2+\mu\alpha)}{2}q_2=0 \quad (32)$$

The eigen equation of the above system of equations is obtained from:

$$\begin{vmatrix} \frac{(\lambda_1+\alpha)}{2}-\lambda & v_1 & 0 & 0 \\ -v_1 & \frac{(\lambda_1+\alpha)}{2}-\lambda & 0 & 0 \\ 0 & 0 & \frac{(\lambda_2+\mu\alpha)}{2}-\lambda & v_2 \\ 0 & 0 & -v_2 & \frac{(\lambda_2+\mu\alpha)}{2}-\lambda \end{vmatrix}=0 \quad (33)$$

The eigenvalues are given by the equation

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0 \qquad (34)$$

Where, r_1, r_2, r_3 and r_4 are functions of the equations parameters and they are given in the appendix. According to the Routh-Huriwitz criterion, the necessary and sufficient conditions for all the roots of Eq. (34) to possess negative real parts is that

$$r_1 \succ 0\,, r_1 r_2 - r_3 \succ 0\,, r_3(r_1 r_2 - r_3) - r_1^2 r_4 \succ 0, \; r_4 \succ 0 \qquad (35)$$

This means that the system is stable if and only if equation (35) is verified.

3. Results and discussion

The differential equation of the main system is solved numerically (applying Rung-Kutta 4th order method) at primary resonance case without absorber as shown in Fig. 1. The steady state response is about 2100% of (the fundamental) excitation amplitude F1. The system is stable with fine limit cycle, denoting that the system is approximately free from dynamic chaos.

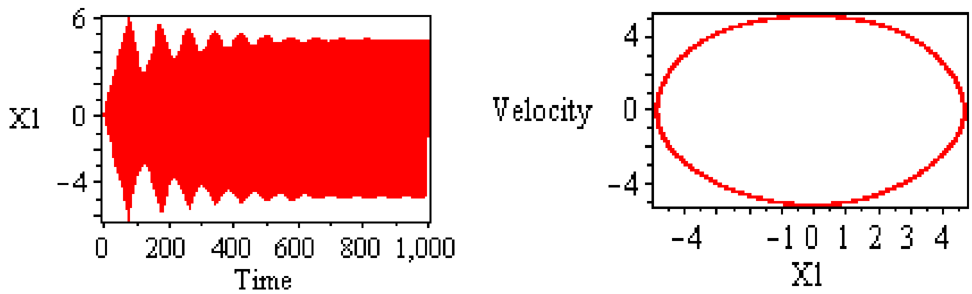
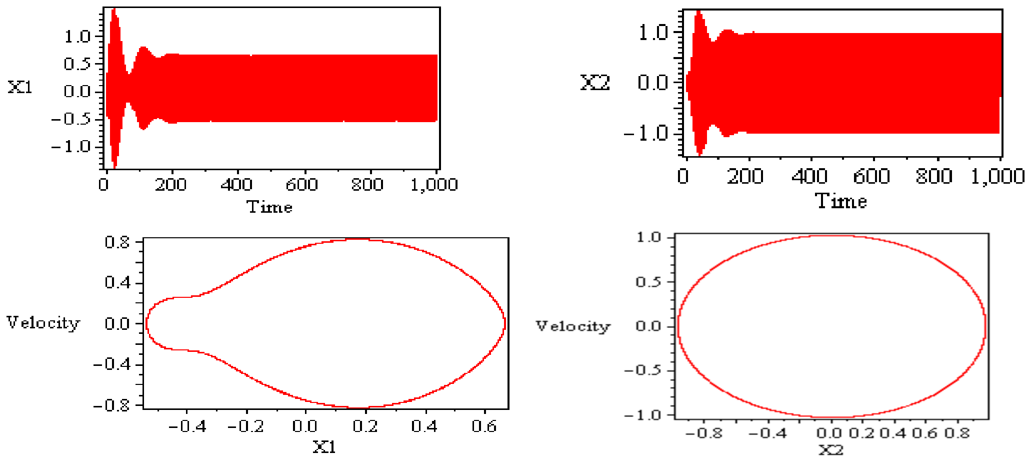


Fig.1.Response of the main system without absorber at primary resonance case $\Omega_1 \cong \omega_1$ ($\lambda_1 = 0.009, \alpha = 0.0001, \beta = 0.2, c = 0.013, d = -0.0008, \Omega_1 = 1.06$ $\omega_1 = 1.06, \Omega_2 = 2.12, \Omega_3 = 4.24, F_1 = 0.2, F_2 = 0.3, F_3 = 0.4$)

3.1. Effect of the absorber without delay time:

Fig.2, illustrates the results at simultaneous primary resonance when the absorber (without delay time) is effective, i.e., when $\Omega_1 \cong \omega_2$. It can be seen for the main system that the steady state amplitude is reduced to about 14% from its value without absorber. The absorber steady state amplitude is about 19% of the main system amplitude without absorber. This means that the effectiveness of the absorber E_a (E_a = the steady state amplitude of the main system without absorber/ the steady state amplitude of main system with absorber) is about 700.



(a) System with absorber (b) the absorber

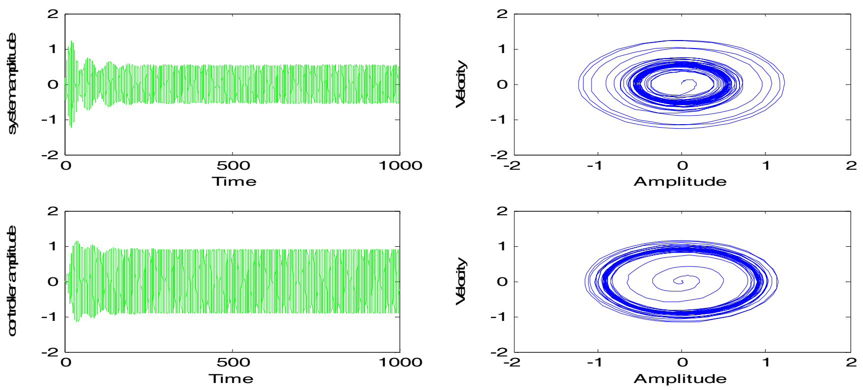
Fig. 2. Response of the main system and absorber at simultaneous Primary resonance case $\Omega_1 \cong \omega_2$, without time delay.

$\mu = 0.6, \lambda_1 = 0.009, \lambda_2 = 0.07, \alpha = 0.0001, \beta = 0.2, c = 0.013, d = -0.0008, a = 1.0036$

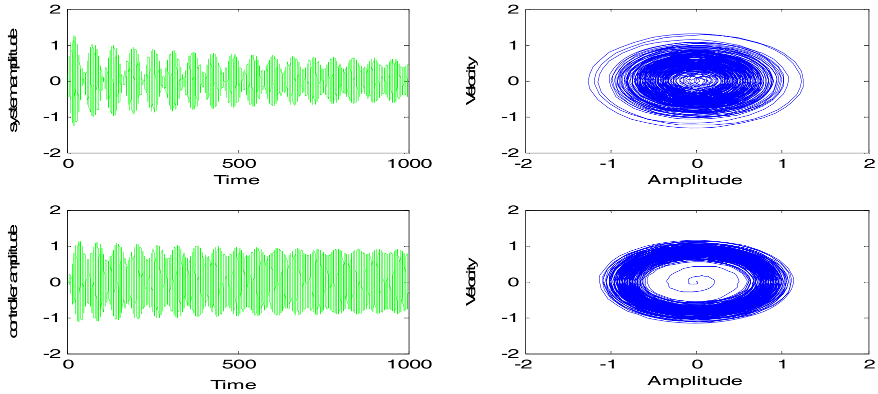
$\omega_1 = 1.06, \omega_2 = 1.06, \Omega_1 = 1.06, \Omega_2 = 2.12, \Omega_3 = 4.24, F_1 = 0.2, F_2 = 0.3, F_3 = 0.4$

3.2. Effects of delay time absorber on the system behavior:

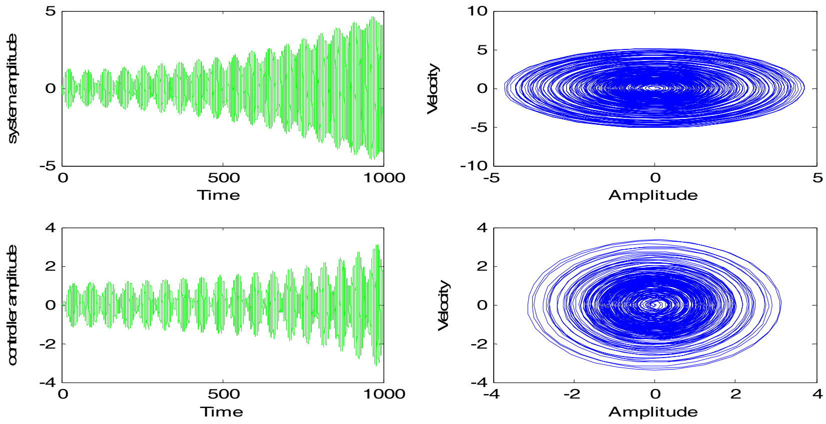
Delay time absorber is considered instead of ordinary absorber. The effect of delay time on systems response was studied numerically. It has been noticed that up to t_q =0.3 the main system has a steady state amplitude of about 12% from its value without absorber within the selected values for the parameters. When t_q was increased a jump has been occurred for the steady state amplitude to about 119% of the main system without absorber. This means that the operating range for the system is better when $t_q \leq 0.3$, as shown in Fig.3.



(a) t_q =0.01



(b) t_q =0.35



(c) t_q =0.45

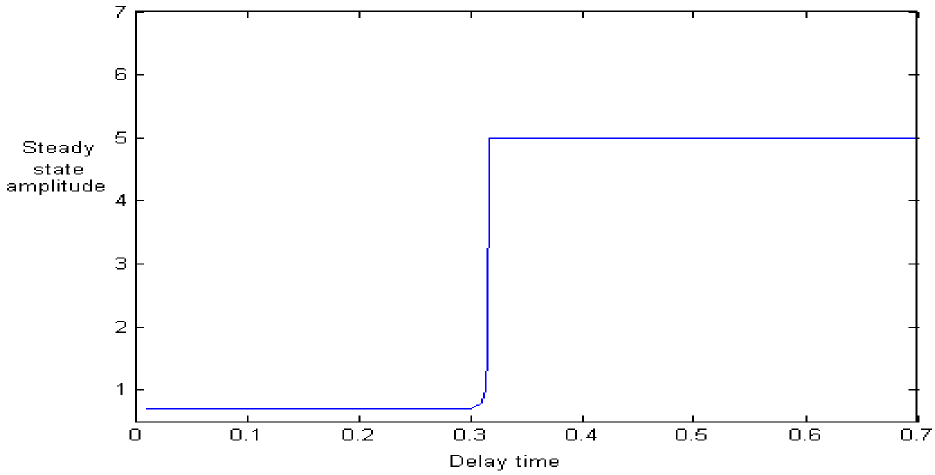


Fig. 3. Effect of delay time on system steady state amplitude.

3.3. Effects of different parameters

The effects of different parameters were investigated by solving Eqs. (24-27).The results are summarized in Tables 2 and 3. They are illustrated graphically in Figs. 5 and 6.

Table2. Effects of the different parameters on the main system stability.
Case $a_1 \neq 0, a_2 = 0$.

Parameter	Effects on steady state amplitude of the main system	Figures
	The steady state amplitude a_1 against detuning parameter σ_j	Fig.5.a
Ω_1	The steady state amplitude is a monotonic increasing function.	Fig.5.b
λ_1	The steady state amplitude is a monotonic increasing function.	Fig.5.c
ω_1	For negative and positive values of ω_1 the curve is bent to the right or the left leading to the occurrence of the jump phenomena and multi-valued amplitudes function.	Fig.5.d
c	The steady state amplitude is a monotonic increasing function.	Fig.5.e
d	For increasing d the curve is shifted and bent to the right leading to the occurrence of the jump phenomena and multi-valued amplitudes function.	Fig.5.f
α	The steady state amplitude is a monotonic increasing function.	Fig.5.g

Table 3. Effects of the different parameters on the absorber stability.
Case $a_1 = 0, a_2 \neq 0$.

Parameter	Effects on steady state amplitude of the absorber	Figures
	The steady state amplitude a_2 against detuning parameter ρ_j	Fig.6.a
μ	The steady state amplitude is a monotonic increasing function.	Fig.6.b
ω_2	The steady state amplitude is a monotonic decreasing function.	Fig.6.c
λ_2	The steady state amplitude is a monotonic decreasing function.	Fig.6.d
β	The steady state amplitude is a monotonic increasing function.	Fig.6.e
f_1	The steady state amplitude is a monotonic increasing function.	Fig.6.f

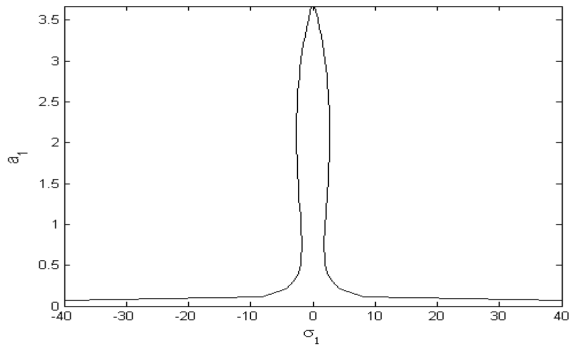


Fig.5a. Effects of the detuning parameter σ_1

$$\alpha = 0.0001, c = 0.013, d = -0.0008, \lambda_1 = 0.009, \omega_1 = 1.06, \Omega_1 = 1.06$$

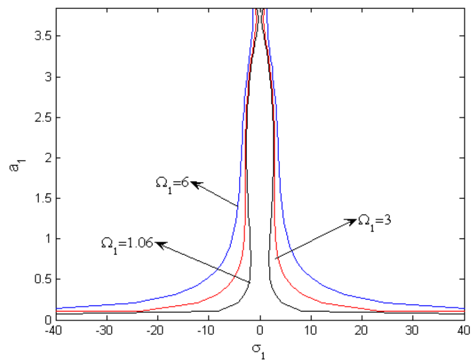


Fig 5b. Effect of the excitation amplitude Ω_1

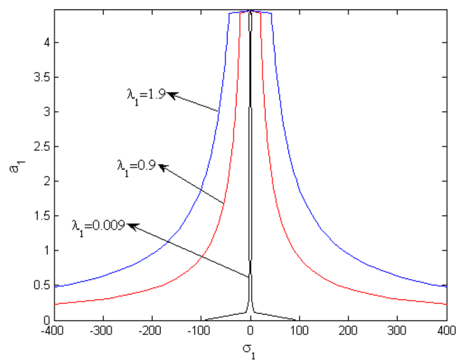


Fig 5c. Effect of the damping coefficient λ_1

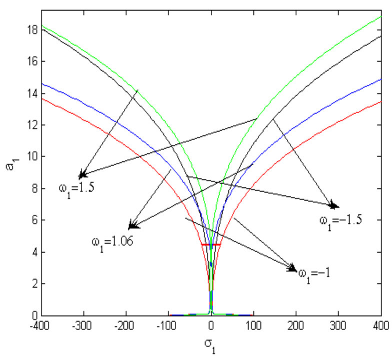


Fig.5d. Effect of the natural frequency ω_1

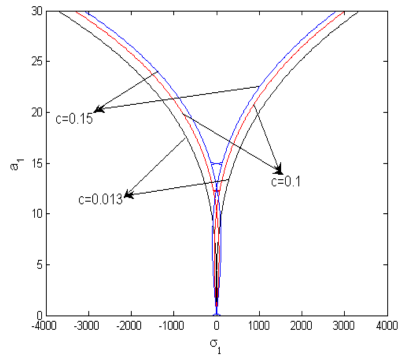


Fig.5e.Effect of the coefficient c

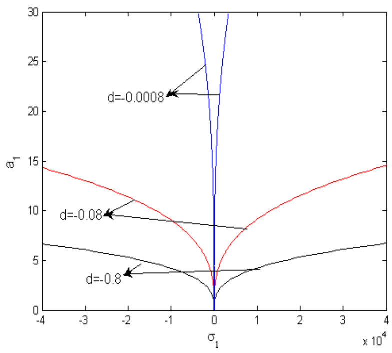


Fig.5f. Effect of the coefficient d

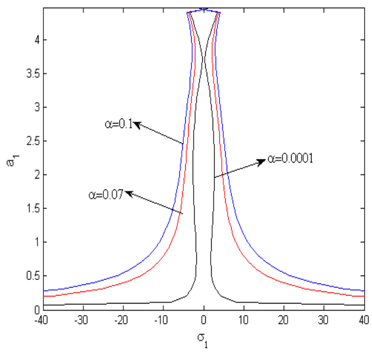


Fig.5g. Effect of the gain parametric α

Fig.5. Response curves (Different parameters against σ_j)

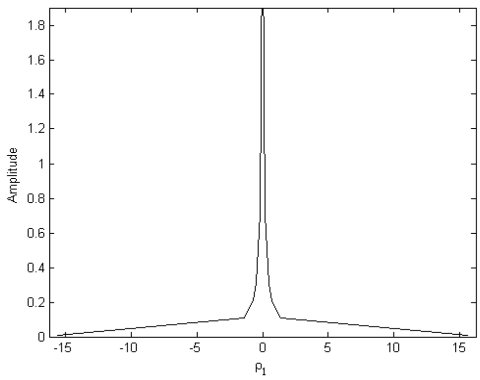


Fig.6a. Effects of the detuning parameter ρ_j (without delay time)

$$\alpha=0.0001, \beta=0.2, \mu=0.6, \lambda_2=0.07, f_1=0.2, \omega_2=1.06, \Omega_1=1.06$$

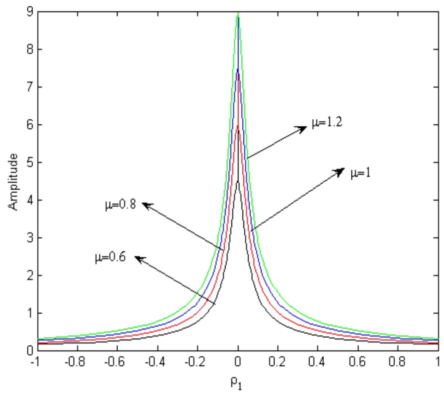


Fig 6b. Effect of the coefficient μ

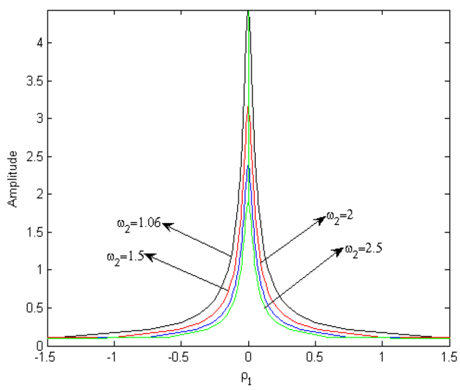


Fig 6c. Effect of the natural frequency ω_2

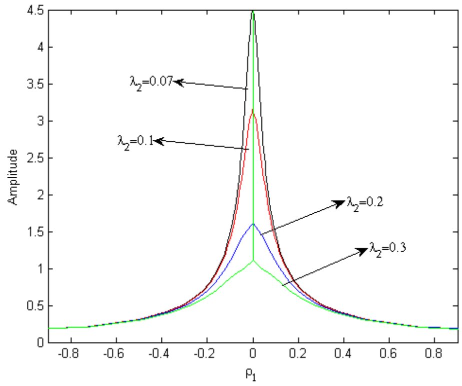


Fig 6d. Effect of the damping coefficient λ_2

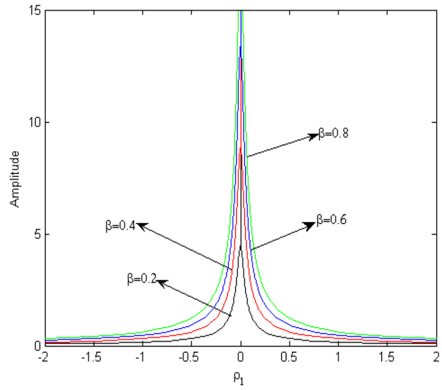


Fig.6f. Effect of the gain parametric β

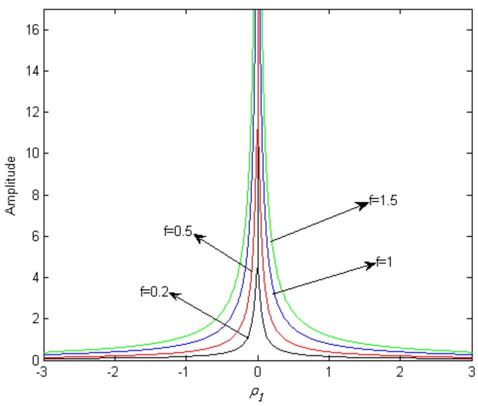


Fig.6g. Effect of the external forces f_1

Fig. 6. Response curves without delay time (Different parameters against ρ_j)

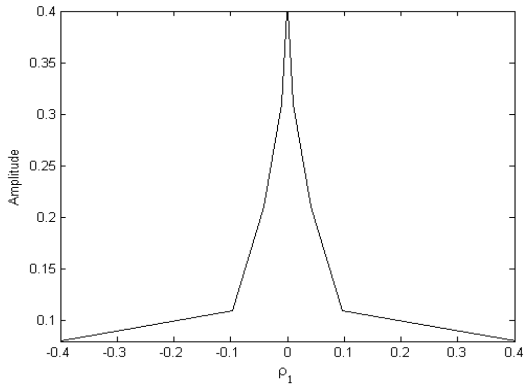


Fig.7a. Effects of the detuning parameter ρ_j (with delay time)

$\alpha = 0.0001, \beta = 0.2, \mu = 0.6, \lambda_2 = 0.07, f_1 = 0.2, \omega_2 = 1.06, \Omega = 1.06, t_q = 0, t_{\dot{q}} = 0.3$

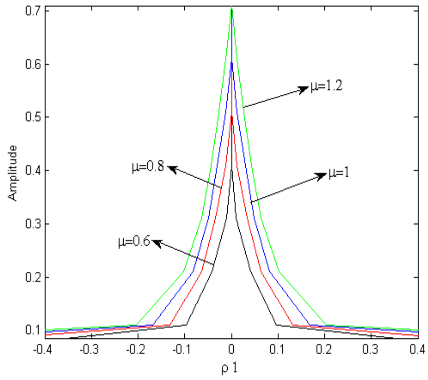


Fig 7b. Effect of the coefficient μ

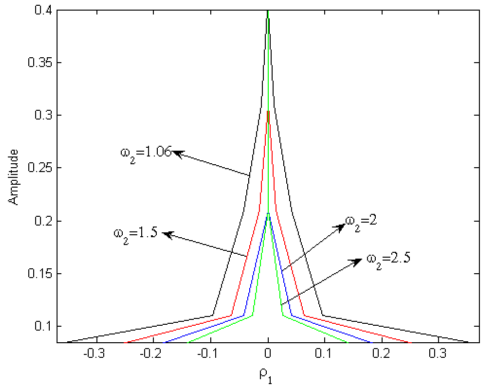


Fig 7c. Effect of the natural frequency ω_2

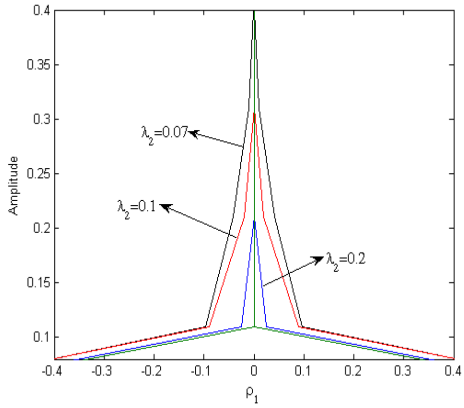


Fig 7d. Effect of the damping coefficient λ_2

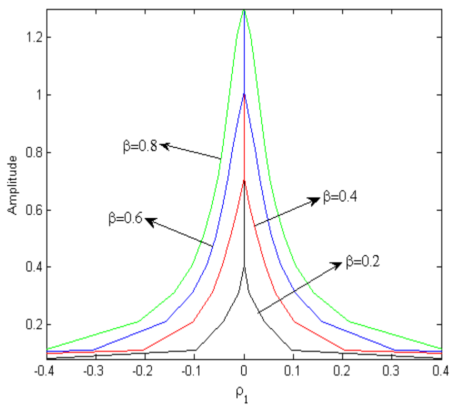


Fig.7f. Effect of the gain parametric β

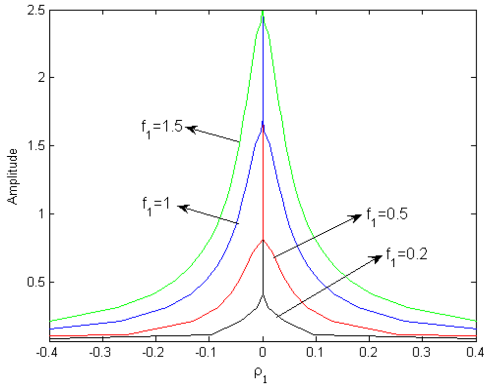


Fig.7g. Effect of the external forces f_1

Fig.7. Response curves with delay time (Different parameters against ρ_j)

4. Conclusions

The vibrations of a second order, non-linear mechanical system and time delay absorber are investigated. Multiple time scale perturbation technique is applied to determine semi-closed form solutions for the coupled differential equations up to the second order approximations. To study the stability of the system, both the frequency response equations and the phase-plane technique are applied. From the above study the following may be concluded.

1. The worst behavior of the main system occurs at primary resonance case where the steady state response is about 2100% of the fundamental excitation amplitude f_1.
2. The vibration of the main system can be reduced via an ordinary absorber and the effectiveness of the absorber is about E_a = 700, at simultaneous resonance case $\Omega_1 \cong \omega_1$, $\Omega_1 \cong \omega_2$.
3. The steady-state amplitudes of the main system are monotonic decreasing functions of ω_2, λ_2 and monotonic increasing functions of the parameters β, f_1, α, λ_1, Ω_1, μ, c.
4. For negative and positive values of ω_1 the curve is bent to the right or the left leading to the occurrence multi-valued amplitudes function.
5. Delay time absorber is more effective for vibration suppression than the ordinary one.
6. Optimum working conditions as $E_a = 1250$ are obtained when $t_q \leq 0.3$.
7. References [22-24] studied the vibration amplitude control with delay time feedback but without any absorber. Ref. [21] studied active control with delay time using piezoelectric absorber but it is limited to a single external force. The study in this paper is an expansion to illustrate the response and the stability of the two degree of freedom (x_1, x_2) of the system when subjected to multi external forces . They demonstrate the effectiveness of the delay time absorber. Unfortunately none of the above mentioned references is directly related to our work. The only point of agreement is the effectiveness of the absorber to control vibration.

References

[1] P. Holmes "A nonlinear oscillator with a strange attractor", Philos. Trans. R. soc. London ser., A282, (1979), 419-448.

[2] A. H. Nayfeh, D. T. Mook, "Nonlinear Oscillation", Wiley, New York, 1979.

[3] S. Lenci, G. Menditto, A. M. Tarantino, "Homoclinic and heteroclinic bifurcation in the nonlinear dynamics of beam resting on an elastic substrate", International Journal of Nonlinear mechanics, 34, (1999), 615-632.

[4] S. Lenci, A. M. Tarantino, "Chaotic dynamics of an elastic beam resting on a Winkler-type soil", Chaos, Solutions and Fractals 7, (1996), 1601-1614.

[5] C.R.Fuller, S. J. Eliot, P.A Nelson, "Active control of vibration", Academic Press. London, 1997.

[6] R. A. Morgan, R. W. Wang, "An active passive piezoelectric absorber for structural vibration control under harmonic excitations with time-varying frequency, part1: Algorithm development and analysis", Journal of Vibration and Acoustics, 24, (2002), 77-83.
[7] T. Aida, K. Kawazoe, S. Toda. "Vibration control of plates by plate-type dynamics vibration absorber", Journal of Vibration and Acoustics, 117, (1995), 332-338.
[8] M.S. Elnaschie, "Stress, stability and chaos", Mc Graw-Hill International Editions (1992). Singapore. Copyright (1990) Mc Graw Hill United Kingdom.
[9] M.P. Cartmell and J. Lawson, "Performance enhancement of an auto-parametric vibration absorber by means of computer control" Journal of Sound and Vibration, 177(2), (1994), 173–195.
[10] P. Woafa, H. B. Fotsin and J. C. Chedjou, "Dynamics of two nonlinearity coupled oscillators", Physica Scripta 57, (1998), 195–200.
[11] T.Y. Shen, Guo Weili and Y.C Pao, "Torsional vibration control of a shaft through active constrained layer damping treatments" Journal of Vibration and Acoustics, 119, (1997), 504–511.
[12] J. M. T. Thompson, "Chaotic phenomena triggering the escape from a potential well", Proc. R. Soc. London, 421, (1989), 195-225.
[13] Lee Cheng-Tang et al. "Sub-harmonic vibration absorber for rotating machinery", ASME Journal of Vibration and Acoustics 119, (1997), 590–605.
[14] M. Eissa, "Vibration and chaos control in I.C engines subject to harmonic torque via non-linear absorbers", ISMV-2000. In: Proc of Second International Symposium on Mechanical Vibrations. Islamabad, Pakistan, (2000).
[15] L. Zhang, C. Y. Yang, M. J. Chaves, AHD Cheng. J. Engng. Mech. ASCE. 1993, 119:1017.
[16] M. Eissa and W El-Ganaini, "Part I, Multi-absorbers for vibration control of non-linear structures to harmonic excitations", In: Proc of ISMV Conference, Islamabad, Pakistan, (2000).
[17] M. Eissa and W El-Ganaini, "Part II, Multi-absorbers for vibration control of non-linear structures to harmonic excitations", In: Proc of ISMV Conference, Islamabad, Pakistan, (2000).
[18] T. MS and W. KW, "On the damping characteristics of active piezoelectric actuator with passive shunt", Journal of Sound and Vibration, 1999, (121)(1): 1-22.
[19] B. R. Nana Nbendjo, "Dynamics and active control with delay of the dynamics of unbounded monostable mechanical structures with ϕ^6 potentials", PhD Dissertation, University of Yaoundé I. Cameroon, 2004.
[20] B. R. Nana Nbendjo, R. Tchoukuegno and P. Woafo, "Active control with delay of vibration and chaos in a double well Duffing oscillator", Chaos, Solutions and fractals, 18, (2003), 345-353.

[21] B. R. Nana, N. bendjo and P. Woafo, "Modeling and Optimal Active Control With Delay of the dynamics of a Strongly Nonlinear Beam", Journal of Advanced Research in Dynamical and control Systems, (2009), 57-74.
[22] M. Yaman, "Direct and parametric excitation of a nonlinear cantilever beam of varying orientation with time delay state feedback", Journal of Sound and Vibration 324, (2009), 892-920.
[23] A. Maccari, "Vibration control for parametrically excited Lienard systems" Int. J. of Non-linear Mechanics 41 (2006) 146-155.
[24] A. Maccari, "Vibration amplitude control for a van der Pol-Duffing oscillator With time delay", Journal of Sound and Vibration 317 (2008) 20-29.
[25] A. H. Nayfeh and B. Balachandran, "Applied non–linear mechanics", Wiley – Inter - Science, New York, 1994.
[26] J. Kevorkian and J. Cole, "Multiple scale and singular perturbation methods", Springer-Verlag, New York, 1996.
[27] A. H. Nayfeh , "Non-linear Interactions", Wiley-Interscience, New York, 2000.

Appendix

$$E_1=\frac{P_j}{\omega_1^2-\Omega_j^2}\ ,E_2=\frac{P_j}{\omega_1^2-(2\omega_1+\Omega_j)^2},E_3=\frac{P_j}{\omega_1^2-(2\omega_1-\Omega_j)^2}$$

$$E_4=\frac{P_jP_k}{\omega_1^2-(\omega_1+\Omega_j+\Omega_k)^2},E_5=\frac{P_jP_k}{\omega_1^2-(\omega_1+\Omega_j-\Omega_k)^2}$$

$$E_6=\frac{P_jP_k}{\omega_1^2-(\omega_1-\Omega_j-\Omega_k)^2},E_7=\frac{P_jP_kP_l}{\omega_1^2-(\Omega_j+\Omega_k+\Omega_l)^2}$$

$$E_8=\frac{P_jP_kP_l}{\omega_1^2-(\Omega_j+\Omega_k-\Omega_l)^2},E_9=\frac{P_j}{\omega_1^2-(4\omega_1+\Omega_j)^2}$$

$$E_{10}=\frac{P_j}{\omega_1^2-(4\omega_1-\Omega_j)^2},E_{11}=\frac{P_jP_k}{\omega_1^2-(3\omega_1+\Omega_j+\Omega_k)^2},E_{12}=\frac{P_jP_k}{\omega_1^2-(3\omega_1+\Omega_j-\Omega_k)^2}$$

$$E_{13}=\frac{P_jP_k}{\omega_1^2-(3\omega_1-\Omega_j+\Omega_k)^2},E_{14}=\frac{P_jP_k}{\omega_1^2-(\omega_1-\Omega_j+\Omega_k)^2}$$

$$E_{15}=\frac{P_jP_kP_l}{\omega_1^2-(2\omega_1+\Omega_j+\Omega_k+\Omega_l)^2},E_{16}=\frac{P_jP_kP_l}{\omega_1^2-(2\omega_1+\Omega_j+\Omega_k-\Omega_l)^2}$$

$$E_{17}=\frac{P_jP_kP_l}{\omega_1^2-(2\omega_1-\Omega_j-\Omega_k+\Omega_l)^2},E_{18}=\frac{P_jP_kP_lP_m}{\omega_1^2-(\omega_1+\Omega_j+\Omega_k+\Omega_l+\Omega_m)^2}$$

$$E_{19}=\frac{P_jP_kP_lP_m}{\omega_1^2-(\omega_1+\Omega_j+\Omega_k+\Omega_l-\Omega_m)^2},E_{20}=\frac{P_jP_kP_lP_m}{\omega_1^2-(\omega_1-\Omega_j-\Omega_k-\Omega_l+\Omega_m)^2}$$

$$E_{21}=\frac{P_jP_kP_lP_m}{\omega_1^2-(\omega_1+\Omega_j+\Omega_k-\Omega_l-\Omega_m)^2},E_{22}=\frac{P_jP_kP_lP_mP_n}{\omega_1^2-(\Omega_j+\Omega_k+\Omega_l+\Omega_m+\Omega_n)^2}$$

$$E_{23}=\frac{P_jP_kP_lP_mP_n}{\omega_1^2-(\Omega_j+\Omega_k+\Omega_l+\Omega_m-\Omega_n)^2},E_{24}=\frac{P_jP_kP_lP_mP_n}{\omega_1^2-(\Omega_j+\Omega_k+\Omega_l-\Omega_m-\Omega_n)^2}$$

$$E_{25}=\frac{1}{\omega_1^2-\omega_2^2}\quad ,H_1=\frac{1}{\omega_2^2-\omega_1^2}\quad ,\quad H_2=\frac{1}{\omega_2^2-\Omega_j^2}$$

$$k_1=-(\frac{3ca_1^2}{4\omega_1}+\frac{10da_1^4}{16\omega_1})$$

$$k_2=\frac{1}{4}(\lambda_1+\alpha)^2-\frac{(\lambda_1+\alpha)^2P_j^2\Omega_j^2}{4\omega_1^2a_1^2}+\frac{9c^2a_1^4}{64\omega_1^2}+\frac{25d^2a_1^8}{256\omega_1^2}+\frac{15dca_1^6}{64\omega_1^2}$$
$$-\frac{9a_1^2P_j^2c^2}{16\omega_1^2}-\frac{45cda_1^4P_j^2}{32\omega_1^2}-\frac{225a_1^6d^2P_j^2}{256\omega_1^2}$$

$$k_3=\frac{1}{4}(\lambda_2+\mu\alpha)^2-\frac{\mu^2\alpha^2\Omega_j^2P_j^2}{a_2^2\omega_2^2}-\frac{\mu^2\beta^2P_j^2}{a_2^2\omega_2^2}$$

Received: January, 2011