The Influence of Diffusion on Generalized Magneto-Thermo-Viscoelastic Problem of a Homogenous Isotropic Material

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Abstract

The present paper is aimed at studying the effects of viscosity and diffusion on generalized magneto- thermoelastic interactions in an isotropic with a spherical cavity. A harmonic function techniques are used. The thermal stresses, temperature and displacement have been obtained. These expressions are calculated numerically for a Copper material and depicted graphically.

Keywords: Viscoelastic; Generalized magneto thermo; Isotropic; Diffusion; Spherical

1-Introduction

The diffusion can be defined as the spontaneous migration of substances from regions of high concentration to regions of low concentration. Thermodiffusion in the solids is one of the transport processes that has great practical importance. Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Nowacki[1-3] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used which implies infinite speeds of propagation of thermoelastic waves. Sherief et al.[4] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Sherief and Saleh [5] worked on a problem of a thermoelastic half-space with a permeating substance, in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion. Recently, Xia et al.[6] studied the dynamic response of an infinite body with a cylindrical cavity whose surface
suffers thermal shock using finite element method. Effect of rotation in a
generalized magneto - thermo - viscoelastic media discussed by Abd-Alla et. [7].
Lord and Shulman [8,9], Green and Lindsay [10] explained two important
generalized theories of thermoelasticity that have become the center of the
interest of recent research in this area. Some problems of thermoelasticity or
generalized thermoelasticity are investigated by Abd-Alla [11,12] Abd-Alla et
al.[13,14] and Elnagar and Abd-Alla [15] respectively.
Roychoudhuri and Mukhopadhyay [16] studied the effect of rotation and relaxation times on plane
waves in generalized thermo-viscoelasticity. Roychoudhuri and Banerjee [17]
investigated the magneto-thermoelastic interactions in an infinite viscoelastic
cylinder of temperature rate dependent material subjected to a periodic loading.
Spherically symmetric thermo-viscoelastic waves in a viscoelastic medium with a
spherical cavity discussed by Banerjee, et al. [18].

2- Formulation of the problem

Let us consider the medium is a perfect electric conductor and the
linearized Maxwell equations governing the electromagnetic field, in the absence
of the displacement current (SI) in the form as in Roychoudhuri and
Mukhopadhyay [16] and Kraus [19]:

\[
\begin{align*}
\vec{J} &= \text{curl} \ \vec{h}, \\
- \mu \frac{\partial \vec{h}}{\partial t} &= \text{curl} \ \vec{E}, \\
div \vec{h} &= 0, \\
div \vec{E} &= 0 \\
\vec{E} &= -\mu_e \left( \frac{\partial \vec{U}}{\partial t} \times \vec{H} \right)
\end{align*}
\]

(2.1)

where \(\vec{h}\) is the perturbed magnetic field over the primary magnetic field, \(\vec{E}\) is
the electric intensity, \(\vec{J}\) is the electric current density, \(\mu_e\) is the magnetic
permeability, \(\vec{H}\) is the constant primary magnetic field and \(\vec{U}\) is the
displacement vector.

Applying an initial magnetic field vector \(\vec{H}(0,0,H_\phi)\) in spherical
coordinate \((r,\theta,\phi)\) to Eq.(2.1) we have

\[
\vec{U} = \vec{U}(u(r,t),0,0), \ \vec{H} = \vec{H}(0,0,H_\phi), \ \vec{E} = -\mu_e(0,-H_\phi \frac{\partial u}{\partial t},0),
\]

(2.2a)
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\[ \vec{J} = \left( 0, -\frac{\partial h}{\partial r}, 0 \right), \quad h = -H_s \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right), \]  
\[ \vec{E} = (0, E_z, 0). \]  

The elastic medium is rotating uniformly with an angular velocity \( \Omega = \Omega n \) where \( n \) is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame has two additional terms: centripetal acceleration, \( \Omega \times (\Omega \times \vec{u}) \) is the centripetal acceleration due to time-varying motion only and \( 2\vec{\Omega} \times \vec{u} \) is the Coriolis acceleration, and \( \vec{\Omega} = (0, \Omega, 0) \).

Following [4,5], the governing equations for an isotropic, homogeneous elastic solid with generalized magneto-thermo-viscoelastic diffusion under effect of rotation are:

(i) The Maxwell's electro-magnetic stress tensor \( \tau_{ij} \) is given by

\[ \tau_{ij} = \mu \left( H_j h_i + H_i h_j - (H \cdot H) \delta_{ij} \right), \quad i, j = 1, 2, 3. \]  

(ii) The equation of motion:

\[ \tau_{n} \mu u_{i,j} + \tau_{n} (\lambda + \mu) u_{j,i} - \tau_{n} \beta C_j + f_r = \rho \left( \vec{J} \times \vec{H} \right) = \rho \mu H_s \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right), \]  

where \( f_r \) is defined as Lorentz's force, Kraus [19], which may be written as

\[ f_r = \mu_s (\vec{J} \times \vec{H}) = \mu_s H_s \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right). \]

(iii) The equation of heat conduction:

\[ K T = (\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}) \left( \rho \alpha T + T_0 \tau \beta C + c T \right), \]  

(iv) The equation of mass diffusion:

\[ D \tau_{n} \beta e_{ik} + D C T + \left( \frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C = Dbc C, \]

(v) The constitutive equations:

\[ \sigma_{ij} = \tau_{m} \left[ 2\mu e_{ij} + \delta_{ij} (\lambda e_{kk} - \beta(T - T_0) - \beta_C) \right], \]

\[ P = -\tau_{m} \beta e_{kk} + b C - c(T - T_0), \]

where \( \rho \) is the mass density of the material, \( K \) is thermal conductivity, \( \beta_1 = (3\lambda + 2\mu)\alpha \) and \( \beta_2 = (3\lambda + 2\mu)\alpha_c \), \( \alpha \) and \( \alpha_c \) are, respectively, the coefficients of linear thermal and diffusion expansion, \( C \) is specific heat at constant strain, \( \lambda \), \( \mu \) are Lame' elastic constants, \( T \) is the absolute temperature of the medium, \( T_0 \) is the reference uniform temperature of the body chosen such that \( |(T - T_0)|/T_0 << 1 \), \( \theta = T - T_0 \), \( \tau_0 \) is the mechanical relaxation time due to the viscosity, \( \tau_1 = (1 + \tau_0 \frac{\partial}{\partial t}) \), \( \tau_1 \) is the thermal relaxation time will ensure that
the heat conduction equation will predict finite speed of heat propagation, and \( \tau \) is the diffusion relaxation time, which will ensure that the equation satisfied by the concentration \( C \) will also predict finite speed of propagation of matter from one medium to the other, and \( D \) is the diffusive coefficient. The constants \( c \) and \( b \) are the measures of thermodiffusion effects and diffusive effects, respectively, \( \sigma_{ij} \) are the components of the stress tensor, \( e_{ij} \) are the components of the strain tensor, and \( \rho \) is the chemical potential.

The non-vanishing displacement component is the radial one \( u_r = u(r,t) \), is

\[
e_j = \frac{1}{2}(u_{ij} + u_{ji}) , \quad e_{ij} = \Delta, \quad i,j = 1,2,3 .
\]

Then, the strain tensor has the following components

\[
e_{rr} = \frac{\partial u}{\partial r} , \quad e_{\theta\theta} = e_{\phi\phi} = \frac{u}{r} .
\]

The cubical dilatation \( e \) is thus given by:

\[
e = \frac{\partial u}{\partial r} + \frac{2u}{r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 u \right) .
\]

Due to spherical symmetry, Eqs. (2.4)-(2.8), take the form

\[
\begin{align*}
\frac{\partial e}{\partial t} - \tau_m \beta_2 \frac{\partial C}{\partial r} - \tau_m \beta_1 \frac{\partial \theta}{\partial r} &= \rho \left[ \frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial u}{\partial t} \right], \\
KV^2 T &= (\frac{\partial}{\partial t} + \tau_1 \frac{\partial^2}{\partial t^2}) \left( \rho c_v \theta + T_0 \tau_m \beta_1 \epsilon_{kk} + cT_0 C \right), \\
D\tau_m \beta_2 \nabla^2 e + Dc \nabla^2 T + \left( \frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C &= Db \nabla^2 C, \\
\sigma_{rr} &= \tau_w \left( 2 \mu \frac{\partial u}{\partial r} + \lambda e - \beta_\lambda \theta - \beta_2 C \right), \\
\sigma_{\theta\theta} = \sigma_{\phi\phi} &= \tau_w \left( 2 \mu \frac{u}{r} + \lambda e - \beta_\lambda \theta - \beta_2 C \right), \\
P &= -\tau_m \beta_2 e + bC - c\theta,
\end{align*}
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} , \quad f_r = \mu_c H_\phi \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right) , \quad \alpha_o = \tau_m (\lambda + 2\mu) + 2\mu c^2 .
\]

3- Dimensionless quantities

Now we introduce the following non-dimensional variables in Eqs. (2.12)-(2.17):
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\[ r^* = c_i \eta_i r, \quad u^* = c_i \eta_i u, \quad \theta = \frac{\tau_m \beta_i (T - T_0)}{\alpha_o}, \quad C^* = \frac{\tau_m \beta_i C}{\alpha_o}, \quad \Omega^* = \frac{\Omega}{c_i \eta_i}, \]

\[ \sigma_{ij}^* = \frac{\sigma_{ij}}{\alpha_o}, \quad p^* = \frac{p}{\tau_m \beta_i}, \quad e^* = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 u^* \right), \]

\[ t^* = c_i^2 \eta_i \tau_i, \quad \tau_0^* = c_i^2 \eta_i \tau_0 , \tau_i^* = c_i \eta_i \tau_i , \quad \tau^* = c_i \eta_i \tau , \]

where \( c_i^2 = \frac{\alpha_i}{\rho} \) and \( \eta_0 = \rho c_v / K \), we have

\[ \frac{\partial e^*}{\partial r} - \frac{\partial C^*}{\partial r} - \frac{\partial \theta}{\partial r} = \frac{c_i^2 u^*}{\tau_m^2} - \Omega^* u^* - 2 \Omega^* \frac{\partial u^*}{\partial t}, \]

\[ \nabla^2 \theta = \left( \frac{\partial}{\partial t} + \tau_i \frac{\partial^2}{\partial t^2} \right) \left( \theta + \tau_i \eta_i e^* + \alpha_o \epsilon e_x C^* \right), \]

\[ \tau_m^2 \nabla^2 e^* + \alpha_o \alpha_1 \nabla^2 \theta + \alpha_o \alpha_2 \left( \frac{\partial}{\partial t} + \tau_i \frac{\partial^2}{\partial t^2} \right) C^* = \alpha_o \alpha_3 \tau_m \nabla^2 C^*, \]

\[ \sigma_{rr} = \frac{2 \mu \tau_m u^*}{\alpha_o} + \tau_i \frac{\lambda}{\alpha_o} e^* - \theta - C^*, \]

\[ \sigma_{\theta \theta} = \sigma_{\phi \phi} = \frac{2 \mu \tau_m u^*}{\alpha_o} + \tau_i \frac{\lambda}{\alpha_o} e^* - \theta - C^*, \]

\[ P = -e^* + \frac{b \alpha}{\tau_m^2 \beta_i} C^* - \frac{c \alpha_o}{\tau_i^2} \theta , \]

where

\[ \epsilon_1 = \frac{\beta_i T_0}{\rho c_v}, \quad \epsilon_2 = \frac{\beta_i c T_0}{\beta_i}, \quad \alpha_1 = \frac{c}{\beta_i \beta_2}, \quad \alpha_2 = \frac{1}{\beta_i^2 D \eta_0}, \quad \alpha_3 = \frac{D \beta}{\beta_2} . \]

4- Boundary conditions

The homogeneous initial conditions are supplemented by the following boundary conditions:

1. The cavity surface is traction free

\[ \sigma_{rr}(a, t) + \tau_i = 0. \]

2. The cavity surface is subjected to a thermal shock

\[ \theta(a, t) = \theta_0 H(t). \]

3. The chemical potential is also assumed to be a known function of time at the cavity surface

\[ p(a, t) = p_0 H(t). \]

where \( \theta_0 \) and \( p_0 \) are constants and \( H(t) \) is the Heaviside unit step function.

4. The displacement function
5- Solution of the problem

Assuming a simple harmonic time dependent factor for all the quantities and omitting the factor $e^{i\omega t}$ as following:

\[ u^*(r^*,t^*) = U(r^*) \ e^{i\omega t^*}, \quad \theta(r^*,t^*) = \Theta(r^*) \ e^{i\omega t^*}, \]

\[ e^*(r^*,t^*) = E(r^*) \ e^{i\omega t^*}, \quad C^*(r^*,t^*) = G(r^*) \ e^{i\omega t^*}. \]

Eqs. (2.12)-(2.14) become

\[ \frac{\partial \Theta}{\partial r^*} - \frac{\partial G}{\partial r^*} - \frac{\partial \Theta}{\partial t^*} = \beta U \]

\[ \nabla^2 \Theta = \ell_1 (\Theta + \epsilon_3 E + \epsilon_4 G), \]

\[ \nabla^2 E + \ell_2 \nabla^2 \Theta + \ell_4 G = \ell_5 \nabla^2 G. \]

By applying the operator $\nabla^2 = \frac{\partial^2}{\partial r^*^2} + \frac{2}{r} \frac{\partial}{\partial r}$ to Eq.(5.2), we obtain

\[ (\nabla^2 - \beta) E = \nabla^2 G + \nabla^2 \Theta, \]

From Eqs. (5.3)-(5.5), we obtain

\[ (\nabla^6 + a_1 \nabla^4 + a_2 \nabla^2 + a_3) (E, \Theta, G) = 0, \]

where

\[ a_1 = \frac{-1}{(\ell_5 - 1)} \left\{ \ell_1 [\ell_5 + \ell_3 (\epsilon_3 + \epsilon_4) + \ell_5 \epsilon_3 + \epsilon_4 - \ell_1] + \ell_5 + \beta \ell_5 \right\}, \]

\[ a_2 = \frac{1}{(\ell_5 - 1)} \left\{ \beta (\ell_4 + \ell_1 (\ell_5 + \epsilon_4 \ell_3)) + \ell_1 \ell_4 (1 + \epsilon_5) \right\}, \]

\[ a_3 = \frac{-\beta \ell_1 \ell_4}{(\ell_5 - 1)}, \]

\[ \ell_1 = i\omega (1 + i\omega \tau^*_1), \ell_2 = i\omega (1 + i\omega \tau^*_2), \ell_3 = \frac{\alpha_0 \alpha_1}{\tau^*_m}, \ell_4 = \frac{\alpha_0 \alpha_2 \ell_2}{\tau^*_m}, \ell_5 = \frac{\alpha_0 \alpha_3}{\tau^*_m}, \]

\[ \epsilon_3 = \frac{\tau^*_m}{\alpha_0} \epsilon_1, \epsilon_4 = \alpha_0 \epsilon_2, \alpha_0 = \tau^*_m (\lambda + 2\mu + \mu \lambda H^2 \tau^*_m - 1 + i\omega \tau^*_0), \]

\[ \beta = -(\omega^2 + \Omega^2 + 2i\omega \Omega^*). \]

The above system of equations can be factorized as

\[ (\nabla^2 + k_1^2)(\nabla^2 + k_2^2)(\nabla^2 + k_3^2) (E, \Theta, G) = 0, \]

where $k_1, k_2$ and $k_3$ are the roots of the characteristic equation

\[ k^6 + a_1 k^4 + a_2 k^2 + a_3 = 0. \]
The solution of Eq. (5.11), which is bounded at infinity, is given by

\[
\Theta(r^*, \omega) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A_j(\omega) K_{1/2}(k_j r^*),
\]

\[
E(r^*, \omega) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A'_j(\omega) K_{1/2}(k_j r^*),
\]

\[
G(r^*, \omega) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A''_j(\omega) K_{1/2}(k_j r^*),
\]

where \( A_j, A'_j \) and \( A''_j \) are parameters depending only on \( \omega \), and \( K_{1/2} \) is the modified Bessel function of the second kind of order 1/2. Compatibility between Eqs. (5.13)-(5.15) along with (5.3) and (5.4) will give rise to

\[
A'_j(\omega) = \frac{(k_j^2 - \ell_1^2)(\ell_5^2 k_j^2 - \ell_4^2) - \epsilon_4 \ell_1 \ell_3 k_j^2}{\ell_1 \epsilon_4 k_j^2 + \ell_1 \epsilon_3 (\ell_5^2 k_j^2 - \ell_4^2)} A_j(\omega),
\]

\[
A''_j(\omega) = \frac{\epsilon_1 \ell_1 \ell_3 k_j^2 + k_j^2 (k_j^2 - \ell_4)}{k_j^2 \epsilon_4 \ell_1 + \ell_1 \epsilon_3 (\ell_5^2 k_j^2 - \ell_4)} A_j(\omega).
\]

Substituting from Eqs. (5.13)-(5.15) into (5.1), we obtain

\[
\theta(r^*, t^*) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A_j(\omega) K_{1/2}(k_j r^*) e^{i \omega t^*},
\]

\[
e(r^*, t^*) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A'_j(\omega) K_{1/2}(k_j r^*) e^{i \omega t^*},
\]

\[
C(r^*, t^*) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A''_j(\omega) K_{1/2}(k_j r^*) e^{i \omega t^*}.
\]

Integrating both sides of Eqs. (5.19) in the relation to the \( r^* \), we obtain

\[
u(r^*, t^*) = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} \left( \frac{k_j^2 - \ell_1^2)(\ell_5^2 k_j^2 - \ell_4^2) - \epsilon_4 \ell_1 \ell_3 k_j^2}{\ell_1 \epsilon_4 k_j^2 + \ell_1 \epsilon_3 (\ell_5^2 k_j^2 - \ell_4^2)} A_j(\omega) K_{3/2}(k_j r^*) e^{i \omega t^*}. \]

Also, from (3.5)-(3.7), we get

\[
\sigma_{n} = \frac{1}{\sqrt{r^*}} \sum_{j=1}^{3} A_j(\omega) \left( \frac{k_j^2 - \ell_1^2)(\ell_5^2 k_j^2 - \ell_4^2) - \epsilon_4 \ell_1 \ell_3 k_j^2}{\ell_1 \epsilon_4 k_j^2 + \ell_1 \epsilon_3 (\ell_5^2 k_j^2 - \ell_4^2)} \right) \left( \frac{1}{(m_i + m_j) - \frac{\ell_1 \epsilon_4 k_j^2 + \ell_1 \epsilon_3 (\ell_5^2 k_j^2 - \ell_4^2)}{(k_j^2 - \ell_1^2)(\ell_5^2 k_j^2 - \ell_4^2) - \epsilon_4 \ell_1 \ell_3 k_j^2}} \right) K_{1/2}(k_j r^*) \frac{\alpha}{r} K_{1/2}(k_j r^*) e^{i \omega t^*}.
\]
\[ \sigma_{ww} = \sigma_{ww} = \frac{1}{\sqrt{r'}} \sum_{j=1}^{3} A_j(\omega) \left\{ \frac{m_1 (k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2}{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} K_{1/2} (k, r') \right\} \]

\[ + \left[ \frac{m_2 (k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2}{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} - 1 - \frac{\varepsilon_\ell \varepsilon_k k_j^2}{k_j^2 \varepsilon_\ell + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} \right] K_{1/2} (k, r') e^{i\omega r} \]

\[ P = \frac{1}{\sqrt{r'}} \sum_{j=1}^{3} A_j(\omega) \left\{ -\frac{(k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2}{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} \right\} K_{1/2} (k, r') e^{i\omega r} \]

where

\[ m_1 = \frac{2\mu \tau^*}{\alpha^*}, \quad m_2 = \frac{\lambda \tau^*}{\alpha^*}, \quad m_3 = \frac{b \alpha^*}{\tau^* \beta^*}, \quad m_4 = \frac{c \alpha^*}{\tau^* \beta^* \beta^*}. \]

By using the boundary conditions, we get

\[ A_1(\omega) = -N_2 \sqrt{\alpha_1 W_3 - K_{1/2} (ak_1) p_0} + N_1 \sqrt{\alpha_1 W_2 - K_{1/2} (ak_2) p_0}, \]

\[ A_4(\omega) = \sqrt{\alpha_4 N_1 [\theta_0 W_2 - K_{1/2} (ak_1) p_0] + \sqrt{\alpha_4 N_1 [K_{1/2} (ak_1) p_0 - \theta_0 W_1]},} \]

\[ A_5(\omega) = \sqrt{\alpha_5 N_1 [K_{1/2} (ak_3) p_0 - \theta_0 W_1] - \sqrt{\alpha_5 N_1 [K_{1/2} (ak_2) p_0 - \theta_0 W_1]},} \]

\[ N = \frac{(k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2}{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} \]

\[ \left\{ \left[ m_1 + m_2 \frac{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)}{(k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2} \right] K_{1/2} (k, \alpha) + \frac{m_1 + \mu H (3 - \frac{3}{a^2})}{\alpha^*} K_{1/2} (k, \alpha) \right\} \]

\[ W_j = \left\{ -\frac{(k_j^2 - \ell_j) (\ell, k_j^2 - \ell_j) - \varepsilon_\ell \varepsilon_k k_j^2}{\ell, \varepsilon_\ell k_j^2 + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} \right\} K_{1/2} (k, \alpha) \]

\[ + m_3 \frac{\varepsilon_\ell \varepsilon_k k_j^2 + \varepsilon_\ell \varepsilon_k (k_j^2 - \ell_j)}{k_j^2 \varepsilon_\ell + \ell, \varepsilon_k (\ell, k_j^2 - \ell_j)} - m_4 \right\} K_{1/2} (k, \alpha) \]
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\[ M = \left\{ -N_1[W_3 K_{1/2} (akt)] - W_2 K_{1/2} (akt) \right\} - N_2[W_3 K_{1/2} (akt) - W_1 K_{1/2} (akt)] \\
+ N_3[W_2 K_{1/2} (akt) - W_1 K_{1/2} (akt)] \right\} \\
\] (5.28)

The discussion in thermoelastic diffusion medium is clear up from Figs. (1-6) and in thermoelastic medium is clear up from Figs. (7-10):

Fig. (1): The displacement \( u(r,t) \) in thermo viscoelastic diffusion medium
Fig.(2): The temperature $\theta(r,t)$ in thermo viscoelastic diffusion medium.
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Fig. (3): The concentration $C(r,t)$ in thermo viscoelastic diffusion medium
Fig. (4): The stress $\sigma_{rr}$ in thermo viscoelastic diffusion medium
Fig. (5): The stress $\sigma_{\phi\phi}$ in thermo viscoelastic diffusion medium
Fig. (6): The chemical potential $p$ in thermo viscoelastic diffusion medium
6- Particular case:

i- If we neglect the diffusion effect by eliminating Eqs. (2.6) and (2.8), and by putting \( \beta = c = 0 \) in Eqs. (2.4), (2.5) and (2.8), we have

\[
(\theta, \phi)(r, t) = A(\omega)e^{-\lambda r^2 \tau} + B(\omega)e^{-\lambda r^2 \tau},
\]

where

\[
\eta_1 = -\ell (\ell + 1), \eta_2 = \ell (\beta - \ell), \quad (\lambda^1, \lambda^2) = \left[ \frac{1}{2} \eta_1 \pm \sqrt{\eta_1^2 - \eta_2^0} \right],
\]

\[
u_{rr} = \left[ \frac{r^2 \lambda^2_1 + 2r \lambda_1 + 2}{r^2 \lambda_1} A(\omega) e^{-\lambda r^2 \tau} + \frac{r^2 \lambda^2_2 + 2r \lambda_2 + 2}{r^2 \lambda_2} B(\omega) e^{-\lambda r^2 \tau} + C(\omega) \right] e^{i\omega t}, \tag{6.3}
\]

\[
\sigma_{\nu} = m_i \left( \frac{4 + 4r \lambda_1 + 2r^2 \lambda^2_1 + r^3 \lambda^3_1}{r^3 \lambda_1^3} + (m_{21} - 1) \right) A(\omega)e^{-\lambda r^2 \tau} + m_i \left( \frac{4 + 4r \lambda_2 + 2r^2 \lambda^2_2 + r^3 \lambda^3_2}{r^3 \lambda_2^3} + (m_{21} - 1) \right) B(\omega)e^{-\lambda r^2 \tau} \tag{6.4}
\]

\[
\sigma_{\omega} = \sigma_{\phi} = m_i \left( \frac{r^2 \lambda^2_1 + 2r \lambda_1 + 2}{r^2 \lambda_1} \right) - (m_{21} - 1) A(\omega)e^{-\lambda r^2 \tau} + \left( \frac{r^2 \lambda^2_2 + 2r \lambda_2 + 2}{r^2 \lambda_2} \right) - (m_{21} - 1) B(\omega)e^{-\lambda r^2 \tau} \tag{6.4}
\]

By using the boundary conditions, we have

\[
A(\omega) = \frac{h_0}{h \theta} e^{-\lambda r^2 \tau} - \frac{h_0}{h \theta} e^{-\lambda r^2 \tau}, \quad B(\omega) = \frac{h_0}{h \theta} e^{-\lambda r^2 \tau} - \frac{h_0}{h \theta} e^{-\lambda r^2 \tau}, \tag{6.5}
\]

\[
C(\omega) = \frac{(h_0 h_0 h_2) \theta_0}{h \theta} e^{-\lambda r^2 \tau} - \frac{(h_0 h_0 h_2) \theta_0}{h \theta} e^{-\lambda r^2 \tau},
\]

\[
h_1 = m_i \left( \frac{4 + 4a \lambda_1 + 2a^2 \lambda^2_1 + a^3 \lambda^3_1}{a^3 \lambda^3_1} + (m_{21} - 1 - \mu \lambda^2 \lambda_1) \right) e^{-\lambda r^2 \tau}, \tag{6.6}
\]

\[
h_2 = m_i \left( \frac{4 + 4a \lambda_2 + 2a^2 \lambda^2_2 + a^3 \lambda^3_2}{a^3 \lambda^3_2} + (m_{21} - 1 - \mu \lambda^2 \lambda_2) \right) e^{-\lambda r^2 \tau},
\]

\[
h_3 = \left( \frac{a^2 \lambda^2_1 + 2a \lambda_1 + 2}{a^2 \lambda^2_1} \right) e^{-\lambda r^2 \tau}, \quad h_4 = \left( \frac{a^2 \lambda^2_2 + 2a \lambda_2 + 2}{a^2 \lambda^2_2} \right) e^{-\lambda r^2 \tau}. \tag{6.7}
\]
Fig. (7): The displacement $u(r, t)$ in thermo viscoelastic medium.
Influence of diffusion

Fig.(8): The temperature $\theta(r,t)$ in thermo viscoelastic medium
Fig. (9): The stress $\sigma_{rr}$ in thermo viscoelastic medium
Influence of diffusion

Fig.(10): The stress $\sigma_{\phi\phi}$ in thermo viscoelastic medium
7- Numerical results and discussion

The Copper material was used chosen for purposes of numerical evaluations. The constants of the problem given by [20] and [21], are

\[ \mu = 3.86 \times 10^{10} \, \text{kg} / \text{m}^3, \quad \lambda = 7.76 \times 10^{10} \, \text{kg} / \text{m}^3, \quad \rho = 8954 \, \text{kg} / \text{m}^3, \]
\[ c_v = 383.1 \, \text{J} / \text{kg} \, \text{K}, \quad \alpha_r = 1.78 \times 10^{-5} \, \text{K}^{-1}, \quad \alpha_c = 1.98 \times 10^{-4} \, \text{m}^3 / \text{kg}, \]
\[ k = 386 \, \text{W} / \text{mK}, \quad D = 0.85 \times 10^{4} \, \text{kg} / \text{s} / \text{m}^3, \quad T_0 = 293 \, \text{K}, \]
\[ c = 1.2 \times 10^4 \text{m}^2 / \text{s}^2 \text{K}, \quad b = 0.9 \times 10^4 \text{m}^3 / \text{s}^2 \text{kg}, \quad \eta_0 = 8886.73 \text{s} / \text{m}^2 \]

Using these values, it was found that:
\[ \mu_v = 0.2, \quad \theta_0 = 1, \quad p_0 = 1, \quad a = 2, \quad \omega = 9.5, \quad H_f = 0.7 \times 10^{-5}, \tau_0 = 0.01, \tau = 0.2, \tau_1 = 0.2. \]

The numerical technique outlined above was used to obtain the temperature, radial displacement, radial stress and concentration as well as the chemical potential distribution inside the sphere. These distributions are shown in Figs. 1-10 respectively. For the sake of brevity some computational results are not being presented here.

- In thermo-viscoelastic diffusion medium

Fig. (1) shows the variation of the radial displacement \( u(r,t) \) under effects the change of time \( t \), rotation \( \Omega \), mechanical relaxation time \( \tau_0 \) and the radius \( r \). It was found that the values of \( u(r,t) \) decreased with an increased of time, rotation and radius, while the values of \( u(r,t) \) are take the same values (a slight change) at different values of mechanical relaxation time.

Fig. (2) shows the temperature distribution \( \theta(r,t) \) under effects the change of time \( t \), rotation \( \Omega \), mechanical relaxation time \( \tau_0 \) and the radius \( r \). It was found that the values of \( \theta(r,t) \) decreased and increased with an increased of time and radius respectively, while the values of \( \theta(r,t) \) are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (3) shows the concentration distribution \( C(r,t) \) under effects the change of time \( t \), rotation \( \Omega \), mechanical relaxation time \( \tau_0 \) and the radius \( r \). It was found that the values of \( C(r,t) \) decreased with an increased of time, radius and mechanical relaxation time, while the values of \( C(r,t) \) are take the same values (a slight change) at different values of rotation.

Fig. (4) shows the radial stress stress \( \sigma_r \) under effects the change of time \( t \), rotation \( \Omega \), mechanical relaxation time \( \tau_0 \) and the radius \( r \). It was found that the values of \( \sigma_r \) decreased and increased with an increased of (time and rotation) and radius respectively, while the values of \( \sigma_r \) are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (5) shows the tangential stress \( \sigma_\theta \) under effects the change of time \( t \), rotation \( \Omega \), mechanical relaxation time \( \tau_0 \) and the radius \( r \). It was found that
the values of $\sigma_{xx}$ decreased with an increased of time, rotation and radius, while the values of $\sigma_{yy}$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (6) shows the chemical potential distribution $p$ under effects the change of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$. It was found that the values of $p$ decreased with an increased of time, rotation and radius, while the values of $p$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

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Fig. (7) shows the radial displacement $u(r,t)$ under effects the change of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$. It was found that the values of $u(r,t)$ decreased with an increased of time, rotation and radius, while the values of $u(r,t)$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (8) shows the temperature distribution $\theta(r,t)$ under effects the change of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$. It was found that the values of $\theta(r,t)$ decreased and increased with an increased of time, (rotation and radius) respectively, while the values of $\theta(r,t)$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (9) shows the radial stress $\sigma_{rr}$ under effects the change of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$. It was found that the values of $\sigma_{rr}$ decreased and increased with an increased of (time and radius) respectively, while the values of $\sigma_{rr}$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

Fig. (10) shows the tangential stress $\sigma_{\phi \phi}$ under effects the change of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$. It was found that the values of $\sigma_{\phi \phi}$ decreased and increased with an increased of (time and rotation) and radius respectively, while the values of $\sigma_{\phi \phi}$ are take the same values (a slight change) at different values of mechanical relaxation time and rotation.

8- Conclusions

From the previous discussion in thermo viscoelastic diffusion medium, we have the values of the displacement $u(r,t)$, the concentration distribution $C(r,t)$, the stress $\sigma_{rr}$ and the chemical potential distribution $p$ decreased with increased values of time $t$, rotation $\Omega$ and the radius $r$, and take the same values (a slight
change) at different values of mechanical relaxation time $\tau_0$. While the values of the temperature distribution $\theta(r,t)$, the stress $\sigma_r$, take changes values increased, decreased or slight change with increased values of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$.

From the previous discussion in thermo viscoelastic medium, we have the values of the displacement $u(r,t)$ decreased with increased values of time $t$, rotation $\Omega$ and the radius $r$, and take the same values (a slight change) at different values of mechanical relaxation time $\tau_0$. While the values of the temperature distribution $\theta(r,t)$, the stress $\sigma_r$, and the stress $\sigma_\phi$ take changes values increased, decreased or slight change with increased values of time $t$, rotation $\Omega$, mechanical relaxation time $\tau_0$ and the radius $r$.

Finally, the results obtained in this chapter should prove useful for research in material science, designers of new materials, low-temperature physicists, as well as for those working on the development of a theory of hyperbolic thermo diffusion. Cross effects of heat, magnetic field, rotation and mass diffusion exchange with the environment arising from and inside nuclear reactors influence their design and operations. Study of the phenomenon of diffusion is also used to improve the conditions of oil extraction.

References


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