Double Diffusion Natural Convection of Binary
Fluid in a Square Enclosure with Top Active Vertical Wall

Abdennacer BELAZIZIA
System and Transport Laboratory
University of Technology, Belfort, France
nacer.belazizia@Gmail.com

Smail BENISSAAD
Mechanical Engineering Department, Faculty of engineering
University Mentouri, Constantine, Algeria
Benissaad.smail@gmail.com

Said ABOUDI
System and Transport Laboratory
University of Technology - Belfort- France.

Abstract

We consider a two-dimensional numerical study of double diffusion natural convection in a square enclosure subjected to horizontal temperature and concentration gradients. The flow is driven by opposite thermal and solutal buoyancies. The top active vertical left side wall and fully active vertical right side wall of the enclosure are maintained at two different but uniform temperatures and concentrations. The remaining boundaries are impermeable and thermally insulated. The physical problem depends on five parameters: thermal Rayleigh number, Prandtl number, Schmidt number, buoyancy forces ratio and the aspect ratio of the enclosure. The main focus of the study is on examining the effect of Rayleigh number $Ra_t$ on thermosolutal natural convection flow. The obtained results show that the increase of $Ra_t$ leads to enhance heat and mass transfer rates. The flow is steady and permanent for $Ra_t<7.10^4$. The unsteady flow appears by the formation of regular (periodic) oscillations of particles in the flow when $Ra_t=7.10^4$. 
Keywords: Thermosolutal natural convection, Top active vertical wall, Opposing buoyancies, Unsteady flow, Instability.

Nomenclature

\[ A \] aspect ratio, \( H/L \)
\[ C^* \] concentration, Kg.m\(^{-3}\)
\[ C \] dimensionless concentration, \((C-C_{min})/\Delta C\)
\[ D \] solutal diffusivity, m\(^2\).s\(^{-1}\)
\[ E \] spectral energy
\[ F \] frequency of oscillations
\[ g \] gravitational acceleration, m.s\(^{-2}\)
\[ H \] wall height, m
\[ h \] height of the active part of the wall, m
\[ L \] cavity length, m
\[ Le \] Lewis number, \( \alpha/D \)
\[ Nu \] local Nusselt number, Eq. (8)
\[ Nu \] average Nusselt number, Eq. (9)
\[ N \] buoyancy ratio, \( \beta \Delta C/\beta_T \Delta T \)
\[ p \] pressure N/m\(^2\)
\[ P \] dimensionless pressure, \( p/(\alpha/H)^2 \)
\[ Pr \] Prandtl number of the fluid, \( \nu/\alpha \)
\[ Ra \] thermal Rayleigh number, \( g\beta_T H^2 \Delta T/\alpha \)
\[ Sc \] Schmidt number, \( \nu/D \)
\[ Sh \] local Sherwood number, Eq. (8)
\[ S\bar{h} \] average Sherwood number, Eq. (9)
\[ T \] temperature, K

Greek symbols

\( \tau^* \) time, s
\( \tau \) dimensionless time, \( \tau^*/(H^2/\alpha) \)
\( u, v \) velocity components, m.s\(^{-1}\)
\( U, V \) dimensionless velocity components, \( u/(\alpha/H), v/ (\alpha/H) \)
\( V \) dimensionless velocity of the flow
\( x, y \) Cartesian coordinates, m
\( X, Y \) dimensionless Cartesian coordinates \( x/H, y/H \)

1. Introduction

Natural convection in which the buoyant forces are due both to temperature and concentration gradients is generally referred to as thermosolutal convection or double-diffusive convection. Various modes of convection are possible depending on how temperature and concentration gradients are oriented relative to each other as well as to gravity: the stratified fluid can be subjected to horizontal or vertical temperature and concentrations gradients [5].
Natural convection in enclosures is investigated by many researches due to its wide application areas: Thermal design of buildings, thermal energy storage systems, melting and solidification process, pollution dispersion in lakes and etc. Thermosolutal convection is also important in crystal growth processes. The transport process in the fluid phase during the growth of a crystal has a profound influence on the structure and quality of the solid phase.

Heat and mass transfer through an enclosure is influenced by parameters such as wall boundary conditions, inclination, aspect ratio and cavity geometry. We shall refer to a few important works that may serve as background for the present work. Double diffusive convection has been studied by many others: Bennacer et al. [1], Gobin et al. [4], Kamotani et al. [5], Kerr [6], Lee et al. [8], Ostrach [11], and Sezai et al. [13].

In the above studies thermosolutal convection is due to the imposed temperature and concentration gradients between the opposing walls of the enclosure taking the entire vertical wall to be thermally active. But in many engineering applications such as solar energy collectors it is only a part of the wall which is thermally active [10].

Natural convection in an enclosure with partially active walls is studied by: Erbay [2], Frederik [3], Kandaswany [7], Nithyadevi [9, 10], Oztürk [12], Sezai [13], Valencia [14] and Yucel [15].

In this work we present a numerical study of thermosolutal natural convection in a square enclosure filled with a binary fluid (an aqueous solution) \((Pr=7\text{ and } Sc=240)\) and imposed (submitted) to horizontal temperature and concentration gradients. The hot region is located at the top of the left vertical wall of the enclosure. The boundary conditions are taken in the way to obtain opposite thermal and solutal buoyancies. The main focus is on examining the effect of thermal Rayleigh number \((10^3 \leq Ra_t \leq 7 \times 10^4)\) on fluid flow. The results are presented in the form of streamlines, isotherms, isoconcentration and mid-height velocity profiles to show the fluid flow, heat and mass transfer phenomena in steady state. The unsteady oscillatory flow is also studied. The dominate frequency is determined by fast Fourier transformation method. The rate of heat and mass transfer in the enclosure is measured in terms of the average Nusselt and Sherwood numbers.

2. Problem geometry

The geometry of the problem is shown in figure 1. The partially heated active vertical left side wall \((h=H/2)\) and fully heated active vertical right side wall of the enclosure are maintained at two different but uniform temperatures and concentrations: \((T_{\text{max}}>T_{\text{min}})\) and \((C_{\text{max}}>C_{\text{min}})\). The remaining boundaries of the enclosure are impermeable and thermally insulated. The problem is solved for the aqueous solution with: \(Pr=7, Sc=240\). The boundary conditions are selected in which to obtain opposite thermal and solutal buoyancies.
3. Governing equations

The flow in the enclosure is assumed to be two-dimensional. All fluid properties are constant. The fluid is considered to be incompressible and Newtonian. The Boussinesq approximation is applied:

\[ \rho(T, C') = \rho_0 \left[ 1 - \beta \left( T - T_0 \right) - \beta \left( C' - C_0' \right) \right] \]  

(1)

Viscous dissipation, heat generation, radiation and Soret effects are neglected. The problem is governed by continuity, momentum, energy and concentration equations. The dimensionless form of these equations can be written as:

♠ at \( t = 0 \) : \( U = V = 0 \); \( \theta = 0 \); \( C = 0 \); \( 0 \leq X \leq 1 \), \( 0 \leq Y \leq 1 \)

♠ for \( t > 0 \):

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  

(3)

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \nu \frac{\partial^2 U}{\partial X^2} \]  

(3)

\[ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \nu \frac{\partial^2 V}{\partial Y^2} + p \cdot Ra (\theta - NC) \]  

(4)

\[ \frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \nabla^2 \theta \]  

(5)

\[ \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{p \cdot \nabla^2 C}{Sc} \]  

(6)

The boundary conditions in the dimensionless form are:

\( X=0, Y \geq 1/2 : U=V=0 ; \theta=C=1 \)

(7a)

\( X=0, Y < 1/2 : U=V=0 ; \frac{\partial \theta}{\partial X} = \frac{\partial C}{\partial X} = 0 \)

(7b)

\( X=1, 0 \leq Y \leq 1: U=V=0 ; \theta=C=0 \)

(7c)
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\begin{equation}
Y=0 \text{ or } Y=1; \ 0 \leq X \leq 1: \ U=V=0 \ ; \ \frac{\partial \theta}{\partial Y} = \frac{\partial C}{\partial Y} = 0
\end{equation}

The local Nusselt and Sherwood numbers are defined by:
\begin{equation}
Nu = -\frac{\partial \theta}{\partial X} \ ; \ \text{Sh} = -\frac{\partial C}{\partial X}
\end{equation}

The average Nusselt and Sherwood numbers are:
\begin{equation}
\text{Left wall: } \overline{Nu} = -\int_{0}^{1} \frac{\partial \theta}{\partial X} dy \ ; \ \overline{Sh} = -\int_{0}^{1} \frac{\partial C}{\partial X} dy
\end{equation}
\begin{equation}
\text{Right wall: } \overline{Nu} = -\int_{0}^{1} \frac{\partial \theta}{\partial X} dy \ ; \ \overline{Sh} = -\int_{0}^{1} \frac{\partial C}{\partial X} dy
\end{equation}

4. **Numerical method**

Equations (2) to (6) subjected to the boundary conditions (7) are integrated numerically using the finite volume method described by Patankar [16]. A uniform mesh is used in X and Y directions. A hybrid scheme and first order implicit temporally discretisation are used. Because of the nonlinearity of the momentum equations, the velocity pressure coupling, and the coupling between the flow and the energy and concentration equations, an iterative solution is necessary. The SIMPLER algorithm and Tri-Diagonal Matrix algorithm iteration procedure [16] are used to solve the algebraic equations. The iteration process is terminated under the following conditions:
\begin{equation}
\sum_{i,j}^{n} \left| \phi_{i,j}^{n} - \phi_{i,j}^{n-1} \right| \sum_{i,j}^{n} \left| \phi_{i,j}^{n} \right| \leq 10^{5}
\end{equation}

Where \( \phi \) represent: \( U, V, \theta \) and \( C \); \( n \) denotes the iteration step.
\begin{equation}
\left| \overline{Nu} \right|_{x=0} = \left| \overline{Nu} \right|_{x=1} \ ; \ \left| \overline{Sh} \right|_{x=0} = \left| \overline{Sh} \right|_{x=1}
\end{equation}

A 60x60 grid and time steps \( \Delta t = 10^{-4} \) was selected and used in all the computations. To validate our results a comparison was made with the numerical results obtained by Nithyadevi [10] for partially heated vertical left wall for \( Ra_t=10^5, Pr=11.573, Sc=5, N=1 \) and \( A=1 \). A good agreement between the obtained and reported results was observed.

5. **Results and discussion**

5.1 **Fluid flow evolution**

At time \( t=0 \), the fluid contained in the entire enclosure is homogenous at \( \theta=0 \) and \( C=0 \). For \( t>0 \), temperature and concentration are changed to \( \theta=1 \) and \( C=1 \) in the left top active wall and that of right wall are maintained at \( \theta=0 \) and \( C=0 \).
Since the temperature of the left wall is higher than that of the fluid inside the enclosure, the wall transmits heat to the fluid by conduction and raises the temperature of fluid particles adjoining the left wall.

We note in this study that thermal and solutal buoyancies are in opposite directions. The flow convection is dominated by thermal buoyancy because Lewis and buoyancy ratio numbers take respectively the values $Le=34.28$ and $N=1$.

Figures 2-4 illustrate the transient results of streamlines, isotherms and isoconcentrations for three values of thermal Rayleigh number $Ra_t=10^3, 10^4$ and $6.10^4$. In the initial stage ($t=0.001$) a small amount of fluid near the hot region is activated. A weak convection indicated by a small and single clockwise rotating cell which appears near the top heating location figures 2-4(a). At time $t=0.01$ the rotating cell grows in its size and occupies the majority of the enclosure figures 2-4(b). When $t=0.05$ the cell expands, elongates to elliptic shape (for $Ra_t=6.10^4$) and occupies the entire enclosure figures 2-4(c). As time increases from 0.1 to 0.8 the centre of the convective cell is situated almost in the middle of the enclosure (for $Ra_t=10^3$ and $10^4$), while for $Ra_t=6.10^4$ the centre cell moves to the right wall figures 2-4(d-f). Finally in the steady state the single cell keeps the same form as time evolves figures 2-4(g). The cell intensification is greater for high Rayleigh number so the convection is more important.

![Streamlines, Isotherms, Isoconcentrations](image)

Figure 2. Transient state of streamlines (top), isotherms (middle), and isoconcentrations (bottom) for $Ra_t=10^3$.

The isotherms and isoconcentrations in the initial stage ($t=0.001$) are almost parallel lines to the top active wall where $\theta=1$ and $C=1$ indicating that only diffusion mode of heat and mass transfer are done figures 2-4(a). Increasing time to ($t=0.01$) convection is more important. Isotherms become parabolic, while isoconcentrations are parabolic especially for $Ra_t=6.10^4$ figures 2-4(b). For ($t=0.05$ to 0.1) isotherms and isoconcentrations lines reach the right side of the enclosure for the higher Rayleigh number figures 2-4(c-d). When time increasing ($t=0.1$ to 0.8) thermal boundary layer is well established before the concentration one figures 2-4(e-f). The steady state shows the development of the convection mode of heat and mass transfer figures 2-4(g).
5.2 Effect of thermal Rayleigh number

The effect of thermal Rayleigh number on fluid flow, thermal and solutal fields is illustrated in figure 5. The hot active region is along the half portion of the top left vertical wall. A single cell rotating in clockwise direction appears inside the enclosure. In consequence natural convection is dominated by thermal buoyancy \((N=1\) and \(Le=\alpha/D=34.28\)). By the increase of \(Ra_t\), we observe an intensification of stream lines and a translation of the centre of the cell to the right wall and consequently a deformation of the cell structure. The maximum absolute values of stream line function \(\psi\) and velocity flow \(V\) are also more important.

The fluid rise along the hot wall and falls along the right cold wall; this leads to formation of thermal and solutal boundary layers at the upper part of the cold wall. Thermal and concentration gradients are very important in these regions. The concentration in the middle of the enclosure is almost constant. When \(Ra_t\) increases, it is clear that the average Nusselt and Sherwood numbers are increasing also as mentioned in figure 5.

In consequence, we can notes that the increase of thermal Rayleigh number has
a noticeable effect on thermostolual natural convection. It leads to intensify the fluid flow and to enhance the rate of heat and mass transfer.

\[
Ra_t = 10^3 \quad \psi_{\text{max}} = 0.87, \quad V = 2.87 \quad \overline{Nu} = 0.86 \quad \overline{Sh} = 2.52
\]

\[
Ra_t = 10^4 \quad \psi_{\text{max}} = 3.62, \quad V = 14.61 \quad \overline{Nu} = 1.44 \quad \overline{Sh} = 4.69
\]

\[
Ra_t = 6.10^4 \quad \psi_{\text{max}} = 6.64, \quad V = 41.68 \quad \overline{Nu} = 2.41 \quad \overline{Sh} = 8.26
\]

Figure 5. Steady state of streamlines (left), isotherms (middle) and isoconcentrations (right) for different values of thermal Rayleigh number.

The effect of thermal Rayleigh number on mid-height horizontal and vertical velocity profiles is shown in figure 6 a-b. It is observed that the fluid particles move with greater velocity for high value of \( Ra_t \). Figure 6 justify the clockwise direction of the cell.

Figure 6. Horizontal (a) and vertical (b) velocities respectively at \( X=0.5 \) and \( Y=0.5 \), for different thermal Rayleigh number.

To know that the flow is steady or unsteady in each value of \( Ra_t \), we follow the temporary evolution of velocity components \( U \) and \( V \) at six locations arbitrary selected in the fluid \( P_i \) (\( i=1:6 \)) corresponding respectively to the points : (0.06,
0.49), (0.23, 0.83), (0.31, 0.14), (0.49, 0.49), (0.66, 0.83) and (0.83, 0.66). Figures 7 and 8 show that the flow is steady in all points for \( Ra = 6.10^4 \). The same remark can be observed in figure 9, which represents temporal evolution of the average Nusselt and Sherwood numbers along the active walls. Thermal and solutal balances are reached respectively at \( t > 4 \) and \( t > 6 \).

The unsteady (transient) solution can be known by the determination of the critic thermal Rayleigh number \( Ra_{cr} \). In this case we observe the formation of regular periodic oscillations of particles in the flow. In our problem the steady solution is maintained until \( Ra = 6.10^4 \). The oscillatory unsteady one appears for \( Ra_{cr} = 7.10^4 \) as shown in figures 10 and 11.

![Figure 7](image7.png)

Figure 7. Evolution of horizontal and vertical velocities for six points \( P_i \) (i=1:6), \( Ra = 6.10^4 \).

![Figure 8](image8.png)

Figure 8. Evolution of temperature and concentration for six points \( P_i \) (i=1:6), \( Ra = 6.10^4 \).
Figure 9. Evolution of the average Nusselt (a) and Sherwood (b) numbers.
In order to avoid the numerical perturbations, we reduce the time steps from $10^{-4}$ to $2.5 \times 10^{-5}$. Figure 12 shows that the amplitude oscillations keep the same values. In consequence the obtained instability is a physical one. We note that the amplitude oscillations of each point $P_i$ depend on its position with wall enclosure.

Figure 10. Evolution of horizontal and vertical velocities for point $P_3$, $Ra=7.10^4$.

Figure 11. Evolution of temperature and concentration for point $P_3$, $Ra=7.10^4$. 
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The rate of heat and mass transfer in the enclosure are illustrated in Figure 13. The temporary evolution of the average Nusselt and Sherwood numbers along the active walls are also periodic around their average values respectively 2.51 and 8.58. To obtain the energy specter of the oscillations, we use the fast Fourier transformation (FFT) of certain number of values ($Nb=2^{11}$) corresponding to horizontal velocity. $F_{cr}$ denotes the energy pick which is the dominant frequency. Figure 14 shows the variation of energy perturbations with their frequencies in $P_3$ and $P_4$. We note that $F_{cr}=20$ which is the same in the other points. In consequence the dominant period is 1/20.
Conclusion

A numerical study was employed to analyze the flow, heat and mass transfer of a binary fluid (chemical solution) filled in a square enclosure with top active left vertical wall. The flow is driven by opposite thermal and solutal buoyancies. The following conclusions are summarized. It is found that the rate of heat and mass transfer increases and the fluid moves with greater velocity when the value of $Ra_t$ increases. The flow is steady and permanent for $Ra_t<7.10^4$. The transient unsteady flow appears by the formation of regular (periodic) oscillations of particles in the flow when $Ra_t=7.10^4$. The dominant period of oscillations calculated with fast Fourier transformation is 1/20.

References


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