Heat and Mass Transfer in
Anisotropic Porous Media

Safia Safi
Climatic Engineering Department. Faculty of engineering
University Mentouri – Constantine - Algeria
s.safisafia@gmail.com

Smail Benissaad
Mechanical Engineering Department. Faculty of engineering
University Mentouri – Constantine - Algeria
Benissaad.smail@gmail.com

Abstract
This paper reports a numerical study of heat and mass transfer in an homogeneous and anisotropic porous media. The left and right vertical walls of a rectangular enclosure filled with a porous media, are submitted to horizontal thermal and concentration gradients. The horizontal walls of the enclosure are adiabatic and impermeable. The formulation of Darcy-Brinkman-Forchheimer is used to describe the flow in the enclosure. The finite volume method is used to resolve the differential equations governing the flow. Obtained results have shown the anisotropic properties effect on the heat and mass transfer.

Keywords: Anisotropic porous media, natural convection, model of Darcy-Brinkman-Forchheimer, heat and mass transfer

1. INTRODUCTION
The thermal convection in porous media interests many researchers during the last decades. Review of the extensive work that has gone into this subject is available in the books published by Nield and Bejan [14], Vafai [21] and Pop and Ingham [17].

Castinel and Combarnous [6] conducted the first study on the convective flow in anisotropic porous media. Their work was supplemented by Epherre [9] and
Tyvand [19] which took into account respectively the anisotropy of the thermal diffusivity and the effect of dispersion in the case of an uniform flow, while Burns et al. [5] examined the natural convection in a vertical slot filled with an anisotropic porous media. The effects of anisotropy on the convective stability of the porous layer are reported by McKibbin [12]. Tyvand and Storesletten [20] and Chen and Hsu [8] were interested by the convective threshold in the case of an anisotropic porous media.

Many analytical and numerical studies on the validity of Darcy and Darcy-Brinkman-Forchheimer models were conducted by Prasad and Kulacki [18] and Lauriat and Prasad [10]. Ni and Beckermann [13] presented a numerical study on a natural convection heat. The porous media is considered hydraulically and thermally anisotropic. Chang and Hsiao [7] investigated the effect of anisotropy in permeability and thermal conductivity on convective flow in a rectangular porous layer. It was shown that the Nusselt number decreases continuously with the increase of the permeability ratio. An infinite horizontal layer heated from below and cooled from above was investigated by Tyvand and Storesletten [20], Zhang et al. [22] and Mamou et al. [11] for various thermal boundary conditions. However, a numerical study was conducted by Bera et al. [3] on thermosolutal convection within a rectangular enclosure. It was found that anisotropy causes significant changes in the Nusselt and Sherwood numbers.

Recently, Bera and Khalili [4] investigated the natural convection by combined heat and mass transfer with opposing horizontal heat and solute gradients in an anisotropic porous cavity using the Darcy model. Bennacer et al. [1] studied the double-diffusive natural convection within a multilayer anisotropic porous media numerically and analytically. Darcy model with classical Boussinesq approximation was used in formulating the mathematical model. Pakdee and Rattanadecho [15] in their investigation have found that the heat transfer coefficient, Rayleigh number and Darcy number considerably influenced characteristics of flow and heat transfer mechanisms.

The objective of the present study is to obtain an improved understanding of natural convection heat and mass transfer through rectangular cavity filled with an anisotropic porous media and to analyze the effects of various parameters, such as the Darcy and Lewis numbers and the conductivity ratio.

2. FORMULATION OF THE PROBLEM

The studied physical system is shown in Figure 1. It consists of rectangular cavity of height \( H \) and width \( L \) filled with a porous media saturated by a binary fluid. The conductivity is considered anisotropic and the aspect ratio \( A \) is equal to 4. Horizontal boundaries are adiabatic and impermeable. Vertical left and right boundaries are exposed to a constant and uniform temperatures and concentrations \( T_1 \) and \( T_2 (C_1 and C_2) \) respectively (Figure 1).
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The flow in the cavity is considered laminar, two dimensional, and incompressible. The thermo physical properties are considered to be constant. The Boussinesq approximation whose density variation is expressed as:

$$\rho = \rho_0 [1 - \beta_t (T - T_0) - \beta_s (C - C_0)]$$  \hspace{1cm} (1)

Where \(\beta_t\) and \(\beta_s\) are the thermal and concentration expansion coefficients respectively, \(\rho\) is the density), was used to model solution that saturates the porous media. Soret and Dufour effects on heat and mass diffusion are neglected.

The conductivity in principal directions \(\lambda_x\) and \(\lambda_y\) are assumed to coincide, respectively, with the horizontal and vertical coordinate axes. The anisotropy in flow conductivities of the porous media is characterized by the following relation:

$$\lambda = \frac{\lambda_x}{\lambda_y}$$  \hspace{1cm} (2)

The problem is modeled by the equations, of continuity, momentum (along X and Y axis), energy and mass conservation, respectively given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (3)

$$\frac{1}{\varphi} \frac{\partial u}{\partial t} + \frac{1}{\varphi^2} \frac{\partial (\varphi u)}{\partial x} + \frac{1}{\varphi^2} \frac{\partial (\varphi v)}{\partial y} = - \frac{\partial P}{\partial x} - \frac{Pr}{\varphi} \frac{\partial}{\partial x} \sqrt{U^2 + V^2} U + \frac{Pr}{\varphi} \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right)$$  \hspace{1cm} (4)

$$\frac{1}{\varphi} \frac{\partial v}{\partial t} + \frac{1}{\varphi^2} \frac{\partial (\varphi u)}{\partial x} + \frac{1}{\varphi^2} \frac{\partial (\varphi v)}{\partial y} = - \frac{\partial P}{\partial y} - \frac{Pr}{\varphi} \frac{\partial}{\partial y} \sqrt{U^2 + V^2} V + \frac{Pr}{\varphi} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + \frac{\partial T}{\partial t} + \frac{\partial (\varphi T)}{\partial x} + \frac{\partial (\varphi T)}{\partial y} = \left[ \left( \varphi \left( \frac{\partial}{\partial x} \cdot \frac{\partial T}{\partial x} \right) \right) + \left( \varphi \left( \frac{\partial}{\partial y} \cdot \frac{\partial T}{\partial y} \right) \right) \right]$$  \hspace{1cm} (5)

$$\frac{\partial c}{\partial t} + \frac{1}{\varphi} \left( \frac{\partial (Uc)}{\partial x} + \frac{\partial (Vc)}{\partial y} \right) = \frac{1}{Le} \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)$$  \hspace{1cm} (6)

The initial and boundary conditions corresponding to the problem are of the following forms:
To put the preceding system of equations, (3) - (9), in the dimensionless form, the following non dimensional variables are then used:

\[
\begin{align*}
X &= \frac{x}{H} \\
Y &= \frac{y}{H} \\
t &= \frac{\alpha t}{H^2} \\
U &= \frac{u}{H} \\
V &= \frac{v}{H} \\
P &= \frac{(P-P_0)}{\rho_f (\frac{H^4}{\alpha_l^2})} \\
T &= \frac{T_1-T_2}{\Delta T} \quad \text{with} \quad \Delta T = T_{\text{max}} - T_{\text{min}} \\
C &= \frac{C_1-C_2}{\Delta C} \quad \text{with} \quad \Delta C = C_{\text{max}} - C_{\text{min}}
\end{align*}
\]

Where \( \varphi \) is the porosity, \( C_f \) is the Forchheimer coefficient, \( U \) and \( V \) are the horizontal and vertical dimensionless velocities, \( X \) and \( Y \) are the horizontal and vertical dimensionless coordinates, \( T \) and \( C \) are the dimensionless temperature and concentration, \( P \) is the dimensionless pressure and \( t \) is the dimensionless time. \( \sigma = \frac{(pc)_m}{(pc)_f} \) is heat capacity ratio; where \( (pc)_m \) is the heat capacity of porous medium and \( (pc)_f \) is the heat capacity of the fluid. \( \alpha = \frac{k_f}{(pc)_f} \) is the effective thermal diffusivity in porous medium; where \( k_f \) is the thermal conductivity of the saturated porous medium.

The dimensionless parameters as buoyancy ratio \( N \), Darcy number \( Da \), Prandtl number \( Pr \), Lewis number \( Le \), thermal Rayleigh number \( Rt \) and aspect ratio \( A \) are then defined as:
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\[ N = \left( \frac{\beta_2 \Delta C}{\beta_1 \Delta T} \right) \]
\[ Da = \frac{K}{H^2} \]
\[ Le = \frac{\alpha}{D} \]
\[ Pr = \frac{v}{D} \]
\[ A = \frac{H}{L} \]
\[ Rt = \frac{2\beta_1 \Delta T H^3}{\alpha \delta} \]

Where \( g \) is the gravity acceleration, \( K \) the permeability of the porous media, \( D \) the mass diffusivity coefficient, \( v \) the kinematic viscosity of the fluid, \( \Delta T \) and \( \Delta S \) are the differences of temperature and concentration.

The Nusselt and Sherwood numbers are defined respectively as:

\[ Nu = -\int_0^1 \left( \frac{dT}{dx} \right)_{x=0} \, dY \]  
\[ Sh = -\int_0^1 \left( \frac{DC}{dx} \right)_{x=0} \, dY \]

The anisotropy in thermal conductivity is expressed by the dimensionless second order tensor as:

\[ \lambda = \begin{bmatrix} x & y \\ y & 1 \end{bmatrix} \]

3. NUMERICAL SOLUTION

Numerical solutions of the governing dimensionless equations (3) to (7) subject to their corresponding initial and boundary conditions given in equations (8) and (9) were obtained using the control-volume procedure described in detail by Patankar [16]. It ensures the conservation of mass and momentum at each control volume and across the computational domain. The domain is divided into rectangular control volumes with one grid located at the centre of the control volume that forms a basic cell. Non-uniform grids are used to have fine grid spacing near the two vertical walls. The system of linear equations is solved by a tridiagonal matrix algorithm (TDMA). The pressure-velocity coupling is ensured by the SIMPLER algorithm (Patankar [16]). The temperature and pressure are scalar quantities calculated at a point in the volume, while the velocity is related to a stream, considered constant across a surface. The results are obtained with a mesh size 132x68. The convergence criterion is based on both gender balance of energy and mass on the active walls and the control the values of different magnitudes, for which the maximum relative error must be less than 10^{-6}.

To validate the performed code, the obtained results for isotropic and anisotropic cases showed good agreement with those of Bennacer et al. [2],

Table 1. Comparison of Nusselt numbers for different Darcy numbers (isotropic case)

<table>
<thead>
<tr>
<th>Darcy number</th>
<th>$10^{-7}$</th>
<th>$10^{-6}$</th>
<th>$10^{-5}$</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>8.83</td>
<td>8.81</td>
<td>8.63</td>
<td>7.61</td>
<td>5.46</td>
<td>3.26</td>
</tr>
<tr>
<td>Bennacer et al. [2]</td>
<td>8.80</td>
<td>8.68</td>
<td>8.37</td>
<td>7.30</td>
<td>5.38</td>
<td>3.26</td>
</tr>
<tr>
<td>Lauriat and Prasad [10]</td>
<td>8.84</td>
<td>8.72</td>
<td>8.41</td>
<td>7.35</td>
<td>5.42</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Table 2. Comparison of Nusselt numbers for different conductivity ratios ($\lambda$) (anisotropic case)

<table>
<thead>
<tr>
<th>$\lambda = \lambda_y / \lambda_x$</th>
<th>$10^{-3}$</th>
<th>$10^{-1}$</th>
<th>1</th>
<th>10</th>
<th>$10^{2}$</th>
<th>$10^{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>13.83</td>
<td>13.79</td>
<td>13.09</td>
<td>9.41</td>
<td>4.77</td>
<td>2.00</td>
</tr>
</tbody>
</table>

From the above tables, it is clearly showed that the elaborated code in this study gives good results. The Nusselt numbers obtained in the present investigation are very acceptable in comparison with those presented by Bennacer et al. [2] and Lauriat and Prasad [10] for different values of Darcy number in isotropic case. In the anisotropic case, the same observations can be concluded for different conductivities ratios.

4. RESULTS AND DISCUSSION

Figures 2 and 3 show the effect of Darcy number on heat and mass transfer respectively. Different values of the Darcy number $Da$ ranging from $10^{-2}$ to $10^{-7}$ have been used when the Lewis number $Le$, the Buoyancy ratio $N$ and Rayleigh number $Ra$ are taken constant and equal respectively to $Le = 10$, $N = 0$, $\lambda = 1$ and $Ra = 10^6$ and $10^8$.

It has been found that the convective solutions can be obtained whatever the value of $Ra$. Note that for a very low Darcy and with $Ra = 10^6$ convection exists but has a negligible effect on heat transfer. We also note that increasing the number of Darcy, the Sherwood number approaches an asymptotic value which depends on the Rayleigh number. The results also show that the mass transfer is enhanced faster when $Ra$ increases.
Figures 4 and 5 show the effect of the thermal anisotropy ratio $\lambda$ on the heat and mass transfer for different values of Darcy number $Da$, varied in the range $10^{-7}$ to $10^{-2}$, $Le = 10$, $N = 0$, $Pr = 0.71$.

![Figure 2](image1.png)

Figure 2. Influence of the Darcy number $Da$ on Nusselt number $Nu$ for different values of Rayleigh $Ra$ for $Le = 10$, $N = 0$, $Pr = 0.71$, $A = 4$.

![Figure 3](image2.png)

Figure 3. Influence of the Darcy number $Da$ on Sherwood number $Sh$ for different values of Rayleigh $Ra$ for $Le = 10$, $N = 0$, $Pr = 0.71$, $A = 4$

It is observed that the Nusselt and Sherwood numbers increase with the effective horizontal thermal conductivity $\lambda_x$ for a fixed value of Darcy number. $Nu$ approaches the pure diffusive regime ($Nu$ tends to 1) when $\lambda$ is made small enough. As expected, $Nu$ and $Sh$ increase with increasing $\lambda$. It is also seen from these figures that $Nu$ and $Sh$ reach their maximum values for $\lambda$ greater than 10. It should be noted that $Nu$ may be less than 1 (case of low $\lambda r$), which is consequence of the choice of the reference diffusive flux based on $\lambda y$ and not on $\lambda x$. 
The evolution of Nusselt and Sherwood numbers is more significant with low values of Darcy number. Their evolution between the two extremes thermal conductivity ratio is quite different. Thus, upon increasing progressively the value of $\lambda$ it is seen that $Sh$ increases monotonously specially for lower Darcy number. However, the evolution of the heat transfers starts first to remain constant and not influenced by the $\lambda$ ratio. Both $Nu$ and $Sh$ increase continuously and pass through a maximum at about $\lambda = 10$. 
The above results show the existence of three regimes for the evolution of the heat and mass transfer with the conductivity ratio. A diffusive regime for low values of \( \lambda \), an intermediate regime for intermediate values of \( \lambda \) in which both \( Nu \) and \( Sh \) increase with increasing \( \lambda \) and a convective regime for high values of \( \lambda \). This evolution of \( Sh \) and \( Nu \) can be explained by the fact that an increase in \( \lambda \) can be interpreted as an increase in the conductivity \( \lambda x \), resulting in a stronger convective flow. The reverse is also true when the value of \( \lambda \) is made very small. These results were also reported by Bennacer et al. [2].

Figure 6 displays the variation of mass transfer with the thermal anisotropy ratio \( \lambda \) for different values of the Lewis number \( Le \) while a Darcy number \( Da \) is kept constant equal to \( Da = 10^{-7} \). The presence of three different regimes is observed and similar to what has been discussed previously in figures 2 and 3. For low anisotropy ratio (10^{-4} to 10), increased mass transfer appears to be linear (log-log scale) until it reaches a critical value beyond which the Sherwood number decreases slightly and then remains constant. It can be also seen that for a given value of \( \lambda \), the mass transfer increases with the Lewis number. This is due to an increase in \( Le \) which corresponds to a decrease in the mass diffusivity. The reduction in the solutal boundary layer thickness leads to a higher Sherwood number.

Anisotropy that has interested us so far was in thermal conductivity with buoyancy forces essentially thermal origin. The transfer was due to the thermal effects. The change in the anisotropy modifies at the same time the flow and the transfers. However, where the flow would solutal origin, changing the thermal anisotropy ratio does not flow and mass transfer which remain unchanged.
Figure 7 presents the variation of $Sh$ as a function of buoyancy ratio $N$ for different values of $\lambda$. It appears clearly that for values of $N$ less than 10 (the solutal buoyancy forces are the source of flow) the effect of the anisotropy rate on the mass transfer is very small and becomes negligible beyond this value. The flow is mainly due to solutal buoyancy forces original in the equation of motion. These results confirm well those presented by Bennacer et al. [2]

![Figure 7. Influence of the buoyancy ratio ratio on Sherwood number for different values of conductivities ratio $\lambda$ for $Le = 10, Da = 10^{-7}, Pr = 0.71, A = 4$.](image)

![Figure 8. Influence of the conductivity ratio on Nusselt number for different values of Rayleigh number $Ra$ for $Da = 10^{-7}, N = 0, Pr = 0.71, A = 4, Le = 10$.](image)
Figure 8 show that the slopes of the Nusselt numbers $Nu$ versus Rayleigh numbers $Ra$. The average Nusselt number is an increasing function of $Ra$. Curves are quite similar for relatively large ranges of the thermal conductivity ratios. These correlations are observed to predict accurately the three different regimes discussed before. The limits between these regimes depend on the Rayleigh value. It is seen from the figure 6 that for $Ra = 10^2$ the second regime becomes visible from $\lambda \geq 1$. However for $Ra = 10^3$ and $Ra = 10^4$ it becomes visible for respectively $\lambda \geq 10^{-2}$ and $\lambda \geq 10^{-3}$.

5. CONCLUSION

Thermosolutal natural convection in a rectangular cavity filled with a fluid and inserted with anisotropic porous media has been studied by numerical method. The effect of the thermal anisotropic ratio, buoyancy ratio and Lewis and Darcy numbers on the Nusselt and Sherwood numbers were predicted and discussed. The validation of the numerical code was performed over a large range of parameters. The obtained results, although enjoying many similarities with those attained at by many other authors.

It is found that the heat and mass transfer are weak functions of the Darcy number for high and low conductivity regimes. For a certain range of the parameters, the heat transfer decreases when the flow penetrates into the porous media. Hence, there is an optimum (minimum) value of Nusselt number, which is a function of the anisotropy parameter. It is also seen the presence of three regimes depend on the $\lambda$ value. Finally, it could be mentioned that the natural convection thermosolutal be somewhat controlled by the influence of different parameters characterizing this kind of problem.

References


