2D Dual-Tree Complex Wavelet Transform Based Image Analysis

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Abstract

Data compression is important in storage and transmission of information. Especially digital image compression is important due to the high storage and transmission requirements. By removing the redundant data, the image can be represented in a smaller number of bits and hence can be compressed. Various compression techniques have been proposed in recent years to achieve good compression. This paper investigates a proposed form of compression based on 2D Dual Tree Complex Wavelet Transform (2D DT-CWT), which results in many wavelet coefficients close to zero. Even the Thresholding can modify the coefficients to produce more zeros which allow a higher compression ratio. The wavelet analysis alone does not actually compress a signal. Therefore one of the coding techniques called Huffman coding is used along with wavelet analysis of an image in order to compress the data. Wide range of threshold values is used in the proposed form. From the results we can say that the proposed form give higher rate of compression and lower RMS error compared with that of other forms based on Discrete Wavelet Transform (DWT).

Keywords: Dual Tree Complex Wavelet Transform (2D DT-CWT), Huffman Encoder, Image Compression
1. Introduction

A typical still image contains a large amount of spatial redundancy in plain areas where adjacent picture elements (pixels) have almost the same values. It means that the pixel values are highly correlated. The redundancy can be removed to achieve compression of the image data. The basic measure for the performance of a compression algorithm is compression ratio (CR), defined as a ratio between original data size and compressed data size. In lossy compression scheme, the image compression algorithm should achieve a trade off between compression ratio and image quality. Higher compression ratios will produce lower image quality and vice versa. Quality and compression can also vary according to input image characteristics and content [11]. In recent years many studies have been made on wavelets. Image compression is one of the most visible applications of wavelets. The rapid increase in the range and use of electronic imaging justifies attention for systematic design of an image compression system and for providing the image quality needed in different applications [11]. Although the Discrete Wavelet Transform (DWT) has dominated the field of image compression over a decade, DWTs in their traditional critically sampled form are known to be some what deficient in several characteristics, lacking such properties as shift invariance and significant directional selectivity [11].

2. Complex Wavelet

The critically sampled Discrete Wavelet Transform (DWT) has been successfully applied to a wide range of signal processing tasks. However its performance is limited because of the following problems [3].

- Oscillations of the coefficients at a singularity (zero crossings).
- Shift variance when small changes in the input cause large changes in the output.
- Aliasing due to down sampling and non-ideal filtering during the analysis, which is cancelled out by the synthesis filters unless the coefficients are not altered.
- Lack of directional selectivity in higher dimensions, e.g. in ability to distinguish between \(+45^\circ\) and \(−45^\circ\) edge orientations.

To overcome the shift dependence problem, we can exploit the undecimated (over-complete) DWT, however without solving the directional selectivity problem. Another approach is inspired by the Fourier transform, whose magnitude is shift invariant and the phase offset encodes the shift. In such a wavelet transform, a large magnitude of a coefficient implies the presence of a singularity while the phase signifies its position within the support of the wavelet. The complex wavelet transform (CWT) employs analytic or quadrature wavelets.
guaranteeing magnitude phase representation, shift invariance and no aliasing [11]. Recently, complex-valued wavelet transforms CWT have been proposed to improve upon these DWT deficiencies, with the Dual-Tree CWT (DT-CWT) [3] becoming a preferred approach due to the ease of its implementation. In the DT-CWT, real valued wavelet filters produce the real and imaginary parts of the transform in parallel decomposition trees, permitting exploitation of well established real-valued wavelet implementations and methodologies. A primary advantage of the DT-CWT lies in that it results in decomposition with a much higher degree of directionality than that possessed by the traditional DWT. However both trees of the DT-CWT are orthonormal or biorthogonal decompositions, the DT-CWT taken as a whole is a redundant tight frame [3].

An analytic wavelet $\psi_r(t)$ is composed of two real wavelets $\psi_r(t)$ and $\psi_i(t)$ forming a Hilbert transform (HT) pair which means that they are orthogonal, i.e. shifted by $\pi / 2$ in the complex plane [8]

$$\psi_r(t) = \psi_r(t) + j \psi_i(t) \quad \ldots (1)$$

$$\psi_i(t) = HT[\psi_r(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\psi_r(t)}{t-t'} dt' = \psi_r(t) \ast \frac{1}{\pi t}$$

and for their Fourier transform pairs

$$H_r(\omega) = \text{real } H_i(\omega)$$

$$H_i(\omega) = FT[HT\{\psi_r(t)\}] = -j \text{sgn}(\omega) H_r(\omega)$$

The same concept of analytic or quadrature formulation is applied to the filter bank structure of standard DWT to produce complex solutions and in turn the CWT. The real-valued filter coefficients are replaced by Complex-valued coefficients by proper design methodology that satisfies the required conditions for convergence. Then the complex filter can again be decomposed into two real-valued filters. Thus, two real-valued filters that give their respective impulse responses in quadrature will form the Hilbert transform pair. The combined pair of two such filters is termed as an analytic filter. The formulation and interpretation of the analytic filter is fig (1).

![Fig (1): analytic filter](image)

![Fig (2): Filter bank structure for 2D DT-CWT](image)
2.1 2D DT-CWT

The 2D DT-CWT also more selectively discriminates features of various orientations. Whereas the critically decimated 2D DWT outputs three orientation selective sub-bands per level conveying image features oriented at the angles of 90°, ±45°, and 0°, the 2D DT-CWT produces six directional sub-bands per level to reveal the details of an image in ±15°, ±45° and ±75° directions with 4:1 redundancy [1].

The implementation of 2-D DTCWT consists of two steps. Firstly, an input image is decomposed up to a desired level by two separable 2D DWT branches, branch $a$ and branch $b$, whose filters are specifically designed to meet the Hilbert pair requirement. Then six high-pass sub-bands are generated at each level. $\text{HL}_a,\text{LH}_a,\text{HH}_a,\text{HL}_b,\text{LH}_b$, and $\text{HH}_b$. Secondly, every two corresponding sub-bands which have the same pass-bands are linearly combined by either averaging or differencing. As a result, sub-bands of 2D DT-CWT at each level are obtained as

\[
\begin{align*}
(\text{LH}_a + \text{LH}_b) / \sqrt{2}, \quad (\text{LH}_a - \text{LH}_b) / \sqrt{2}, \\
(\text{HL}_a - \text{HL}_b) / \sqrt{2}, \quad (\text{HH}_a + \text{HH}_b) / \sqrt{2}, \\
(\text{HH}_a - \text{HH}_b) / \sqrt{2},
\end{align*}
\]

The six wavelets defined by oriented shown above have the sum/difference operation is orthonormal, which constitutes a perfect reconstruction wavelet transform. The imaginary part of 2D DT-CWT has similar basis function as the real part [8]. The 2D DT-CWT structure has an extension of conjugate filtering in 2D case. The filter bank structure of 2D dual-tree is shown in figure (2).

2D structure needs four trees for analysis as well as for synthesis. The pairs of conjugate Filters are applied to two dimensions (x and y) directions, which can be expressed as:

\[
(h_x + jg_x)(h_y + jg_y) = (h_x h_y - g_x g_y) + j(h_x g_y + g_x h_y).
\]

The filter bank structure of tree $a$, similar to standard 2D DWT spanned over 3 level, is shown in figure (3).

![Filter bank structure of figure (2)](image)

All other trees-$(b, c & d)$ have similar structures with the appropriate combinations of filters for row- and column- filtering. The overall 2D dual-tree structure is 4-times redundant (expensive) than the standard 2D DWT. The tree-$a$
and \textit{tree-b} form the real pair, while the \textit{tree-c} and \textit{tree-d} form the imaginary pair of the analysis filter bank. Trees-(\textit{a-}, \textit{b-}) and trees- (\textit{c-}, \textit{d-}) are the real and imaginary pairs respectively in the synthesis filter bank similar to their corresponding analysis pairs \[4\]

\section{3. Image Compression}

Data compression is important in storage and transmission of information. Especially digital image compression is important due to the high storage and transmission requirements. By removing the redundant data, the image can be represented in a smaller number of bits and hence can be compressed. Various compression techniques have been proposed in recent years to achieve good compression. This investigation will concentrate on transform coding and then more specifically on Wavelet Transform coefficients that make only small contributions to the information contents can be omitted. Usually the image is split into blocks (sub images) of 8x8 or 16x16 pixels, and then each block in discrete cosine transform is transformed separately. However this does not take into account any correlation between blocks, and creates "blocking artifacts", which are not good if a smooth image is required.

However wavelets transform is applied to entire images, rather than sub images, so it produces no blocking artifacts. This is a major advantage of wavelet compression over other transform compression methods. For some signals, many of the wavelet coefficients are close to or equal to zero and even the Thresholding can modify the coefficients to produce more zeros. In Hard thresholding any coefficient below a threshold \(\lambda\), is set to zero which produce many consecutive zeros which can be stored in much less space, and transmitted more quickly by using entropy coding compression. An important point to note about Wavelet compression is "The use of wavelets and thresholding serves to process the original signal, but, to this point, no actual compression of data has occurred".

This explains that the wavelet analysis does not actually compress a signal. It simply provides information about the signal which allows the data to be compressed by standard entropy coding techniques, such as Huffman coding. Huffman coding is good to use with a signal processed by analysis wavelet, because it relies on the fact that the data values are small and in particular zero. It works by assigning fewer bits for large numbers and more bits for small numbers. Long strings of zeros can be encoded very efficiently using this scheme. Therefore an actual percentage compression value can only be stated in conjunction with an entropy coding technique. The number of zeros are used to compare i.e. more zeros will allow a higher compression rate. One of the popular threshold is the hard threshold function is shown in figure (4)
D. Srinivasulu Reddy, S. Varadarajan and M. N. GiriPrasad

Fig (4): Hard Thresholding function

\[ \psi_T(x) = x \{ |x| > T \} \]

which keeps the input if it is larger than the threshold, otherwise it is set to zero. The wavelet thresholding procedure removes noise by thresholding the wavelet coefficients of the detailed sub bands only, while keeping the low resolution coefficients unaltered [9].

3.1 Huffman Coding

Huffman coding is an entropy encoding algorithm used for lossless data compression. It refers to the use of a variable-length code table for encoding a source symbol (such as a character in a file) where the variable-length code table has been derived in a particular way based on the estimated probability of occurrence for each possible value of the source symbol. It uses a specific method for choosing the representation for each symbol, resulting in a prefix code that expresses the most common source symbols using shorter strings of bits than are used for less common source symbols. The Huffman algorithm is based on statistical coding, which means that the probability of a symbol has a direct bearing on the length of its representation. The more probable the occurrence of a symbol is, the shorter will be its bit-size representation. In any file, certain characters are used more than others. Using binary representation, the number of bits required to represent each character depends upon the number of characters that have to be represented. Using one bit we can represent two characters, i.e., 0 represents the first character and 1 represents the second character. Using two bits we can represent four characters, and so on [8]. Unlike ASCII code, which is a fixed-length code using seven bits per character, Huffman coding is a variable-length coding system that assigns smaller codes for more frequently used characters and larger codes for less frequently used characters in order to reduce the size of files being compressed and transferred [6].

3.2 Compression Algorithm

The basic procedure for image compression differs only in the selection of the method.

(1) Read the source image.
(2) Decompose the image into wavelet coefficients \( w \) using DT-CWT.

(3) Modify the coefficients \( w \), using Thresholding.

(4) Apply Huffman encoding to compress.

(5) Reconstruct the image using Huffman decoding and inverse DT-CWT.

(6) Calculate compression ratio, BPP, PSNR and RMS error.

![Diagram](image)

**Fig (5):** Block diagram of compression algorithm using DT CWT

### 4. Experimental Results and Discussion

Firstly, original image is applied to the 2D Dual tree Complex wavelet Transform, then the coefficient are applied to threshold function which is further compressed using Huffman Encoding. To reconstruct, the compressed image is applied to decompression program, by which Huffman decoder and Inverse Dual Tree Complex wavelet Transform is used. Then Compression Ratio (CR), RMS error, and Peak-Signal-to-Noise Ratio (PSNR) are obtained for the original and reconstructed images.

In this experiment, “Cameraman.tif” image of size 256 x 256 (65,536 Bytes) has been considered. The different statistical values of the image Cameraman.tif for Various Thresholds are summarized in the table (1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TH=6</th>
<th>TH=10</th>
<th>TH=20</th>
<th>TH=30</th>
<th>TH=40</th>
<th>TH=50</th>
<th>TH=60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original File Size (bytes)</td>
<td>65240</td>
<td>65240</td>
<td>65240</td>
<td>65240</td>
<td>65240</td>
<td>65240</td>
<td>65240</td>
</tr>
<tr>
<td>Compressed File Size (bytes)</td>
<td>10114</td>
<td>9320</td>
<td>8125</td>
<td>8505</td>
<td>8310</td>
<td>8165</td>
<td></td>
</tr>
<tr>
<td>Compression Ratio (CR)</td>
<td>6.05</td>
<td>6.45</td>
<td>7.00</td>
<td>7.43</td>
<td>7.67</td>
<td>7.85</td>
<td>7.99</td>
</tr>
<tr>
<td>Bits Per Pixel (BPP)</td>
<td>1.32</td>
<td>1.24</td>
<td>1.14</td>
<td>1.07</td>
<td>1.04</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Peak-Signal-to-Noise Ratio (PSNR)</td>
<td>40.32</td>
<td>36.25</td>
<td>31.15</td>
<td>28.99</td>
<td>27.43</td>
<td>26.23</td>
<td>25.28</td>
</tr>
<tr>
<td>RMS err</td>
<td>2.43</td>
<td>3.9</td>
<td>6.92</td>
<td>8.93</td>
<td>10.74</td>
<td>12.34</td>
<td>13.74</td>
</tr>
</tbody>
</table>
Here Bits per pixel (BPP) is calculated by the ratio of Bit depth to Compression ratio.
Thus, it can be concluded that 2D Dual Tree Complex Wavelet Transform and Huffman encoder gives excellent results. By choosing suitable threshold value compression ratio as high as ‘8’ can be achieved.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Threshold_vs_Compression_Ratio.png}
\caption{Threshold Vs compression Ratio}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Threshold_vs_PSNR.png}
\caption{‘Threshold Vs PSNR}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Threshold_vs_BPP.png}
\caption{‘Threshold Vs BPP}
\end{figure}

The curves of ‘Threshold Vs Compression Ratio’, ‘Threshold Vs PSNR’ and ‘Threshold Vs BPP’ and have been calculated and depicted in the fig 6.1 (a), (b) & (c) respectively. Here the Image encoded and compressed using complex wavelet and only Huffman Encoder gives better Compression Ratio, BPP and PSNR values than Image Compressed by combining EZW Encoding with Huffman Encoder.

5. CONCLUSION

An Image compression Technique which uses the 2D Dual Tree Complex Wavelet Transform in combination with Huffman encoder is proposed here. Complex wavelet with Huffman encoder gives effective results in higher compression ratio, good BPP and better PSNR. The algorithm is tested on
different images, and it is observed that the Image Compression Based on 2D Dual Tree Complex Wavelet Transform (2D DT-CWT) performs consistent results compared to the results obtained by Still Image Compression by Combining EZW Encoding with Huffman Encoder. It is also observed that the results are better than those reported in [5].

References


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