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Size Effect and Strength Correction Factors for Normal Weight Concrete Specimens under Uniaxial Compression Stress

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Abstract

The reduction phenomenon of compressive strength of normal weight concrete specimens, particularly with increasing the aspect ratio *h/b* (the height to diameter or side dimension ratio), is considered as a matter of great concern. This paper introduces a shear sliding model to explain the reason for the implementation of the magnitude 1.0 for the strength correction factor when $h/b \geq 2.0$.Theoretical analysis was conducted on the proposed model with idealized shear-failure planes to determine the direction of sliding planes. A general equation is proposed to predict the strength correction factors taking into consideration the influence of aspect ratio and the concrete compressive strength level. A series of concrete prismatic specimens with various depths and constant cross section subjected to axial compressive load have been studied experimentally.

Keywords: Size effect, Aspect ratio, Strength correction factor

1 Introduction

The compressive strength is usually used as a measure to judge the quality of concrete and is calculated simply as the stress at failure based on the transverse cross-sectional area. The 28-days compressive strength is accepted universally as a general index of concrete strength. The size and the shape of tested specimens are two of the most important parameters that influence the result of concrete compressive strength. There are basically three shapes of test specimen; cube, cylinder and prism. In the British approach, 150mm cube is employed as standard specimen while 150x300 mm cylinder is commonly used in the American approach in accordance with ASTM standard C39, whereas in Russia150x150x600mm prismatic specimens are tested. Jordan is one of several countries that use both cylinder and cube as standard test specimens to establish the characteristic compressive strength. One of the significant factors affecting the results of compression test for brittle materials as concrete is the aspect ratio. Experimentally, it was found that the failure load increases as that ratio falls and the attempts to correct to a "standard" value [8] become extremely complicated due to the variation of concrete compressive strength level as well. Some specifications, in determination the axial compressive strength, allow for considerable variation in the aspect ratio even if greater than 2 [14]. A range of 5 > b/h > 1 for the aspect ratio has been suggested in order to avoid buckling and too uneven an end face stress distribution [15].Generally, three types of failure have been observed in hardened concrete specimens under uniaxial compressive stress: pyramidal or conical end caps with radial dissection of the remainder; axial slabbing; and shear on a single oblique plane ([4], [5]). The Pyramidal and conical slip surfaces are well known in both ductile and brittle materials [10]. Angle of failure surfaces has been related to the friction emerged at the end faces. The ASTM C 42-90, AASHTO T 22 and BS1881: Part 120: 1983 standard provision ([1], [3]) specify correction factors for concrete strengths, as shown in Tab.1. Conversion factors are provided in the AASHTO and ASTM codes for evaluating strength test results when h/d < 1.8. These factors convert strength test results to equivalent results for a specimen having h/d = 2.

Table1: Standard correction factors for strength of cylinders with different ratios of height to diameter

Height to diameter ratio (h/d)	Strength correction factor	
	ASTM C 42-90 AASHTO T 22	BS 1881.Part 120
2.00	1.00	1.00
1.75	0,98	0.97
1.50	0.96	0.92
1.25	0.93	0.87
1.00	0.87	0.80

Generally, these factors are used for tests on cores taken from structures, from which it is not always possible to obtain the ratio h/d = 2 for specimens. Murdock and Kesler (1957) found that the correction factor depends also on the strength level of the concrete. In 1966, Neville postulated that strength could be determined as a function of the specimen's height-to-maximum lateral dimension *(h/d)* ratio, maximum lateral dimension d alone, and overall specimen volume. Furthermore, Neville found that there is little difference in strength reduction trends for higher strength concrete and inferred that there is comparatively little difference between the strengths of concrete specimen with variable *h/d* ratios. Gonnerman [11] experimentally showed that the ratio of the compressive failure stress to the compressive strength decreases as the specimen size increases. The general relationship observed between *h/d* and strength is shown in Fig. 1. It is evident from Fig. 1 that the strength ratio is more sensitive to *h/d* < 1.5 than for *h/d* > 1.5.

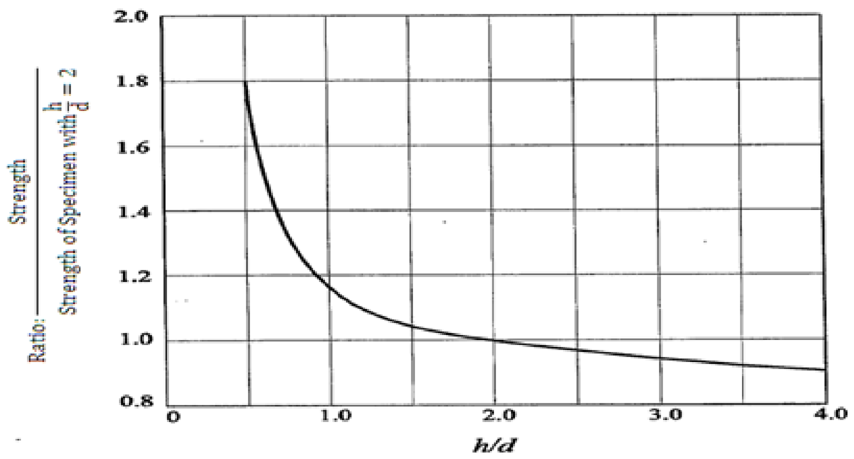


Figure1: General relationship between height/diameter ratio and strength ratio.

In the early 1980's, the behavior of compressive failure has been studied extensively; it became clear that the size effect on the nominal strength of quasibrittle materials failing after large stable crack growth is caused chiefly by energy release (Bazant & Xi 1991). The studies (Cotterell 1972; Jenq & Shah 1991; Bazant & Xiang 1997 [2-13]) on compressive loading based size effect became a focus of interest among researchers. J.k.Kim,andY.Seong[7] investigated the influence of the aspect ratio on the compressive stress assuming that the value of slope angle shown in Fig.2 was approximately selected as 45^o since the confinement effects by frictional force would be negligible if the aspect ratio *h/d* becomes very large. Therefore, a cylinder with an aspect ratio *h/d* =1 will be able to resist higher loads than a cylinder with an aspect ratio of 2.

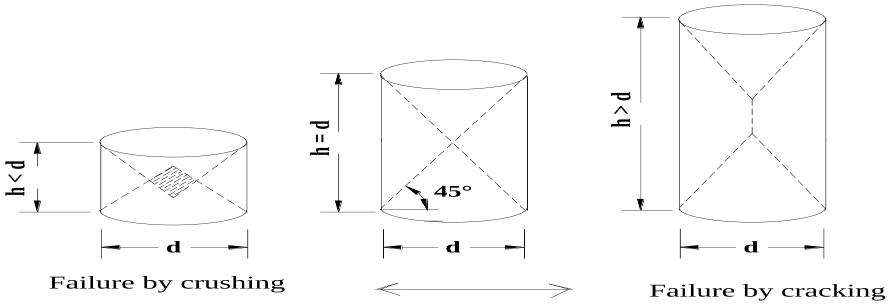


Figure2: Failure modes with specimen geometry. (After Fig.4 from [7])

From the statistical analyses of existing experimental data of Gornnerman (172 specimens), Blanks and McNamara (26 specimens), U.S. Department of the Interior (20 specimens), Kesler (337 specimens), and Murdock and Kesler (123 specimens), the following equation was proposed by J.k.Kim, and Y. Seong:

$$f_0 = \frac{0.4f_c'}{\sqrt{1+(h-d\,)/50}} + 0.8f_c' \qquad (1)$$

Where f_0 is compressive strength of a cylindrical concrete specimen with diameter *d* and height *h*; f_c' is the compressive strength of standard cylinder. f_0 and f_c' are in MPa. and *h* and *d* are in mm. Fig.3 shows the relationship between 1 + (h - d)/50 and f_0/f_c'. Where and diameter of cylinder specimen *d* are in mm. Eq. (1) was compared to the ASTM standard (ASTM C 42) (1999) and it is noted that the prediction values of Eq. (1) are less than those of the ASTM standard, but the difference is minimal. When the value of *h/d* approaches 1.0, it is shown that the scatter of data is increased due to the effects of the confinement and energy release zone.

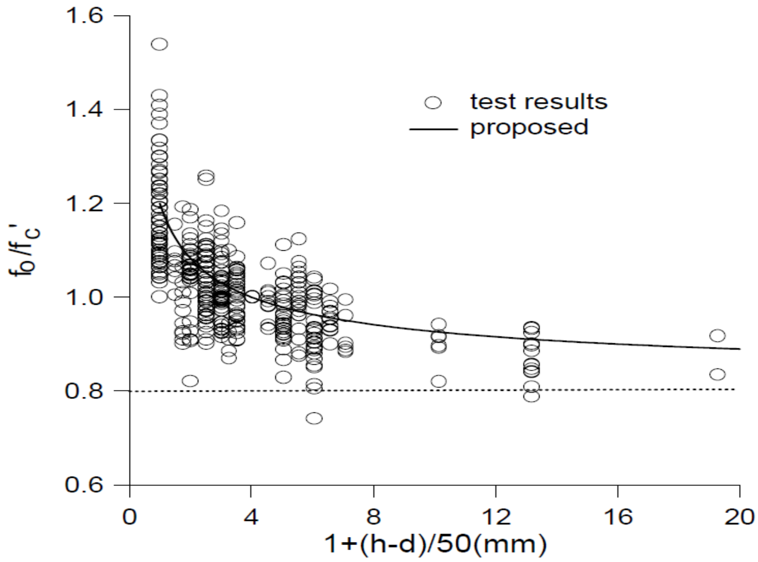


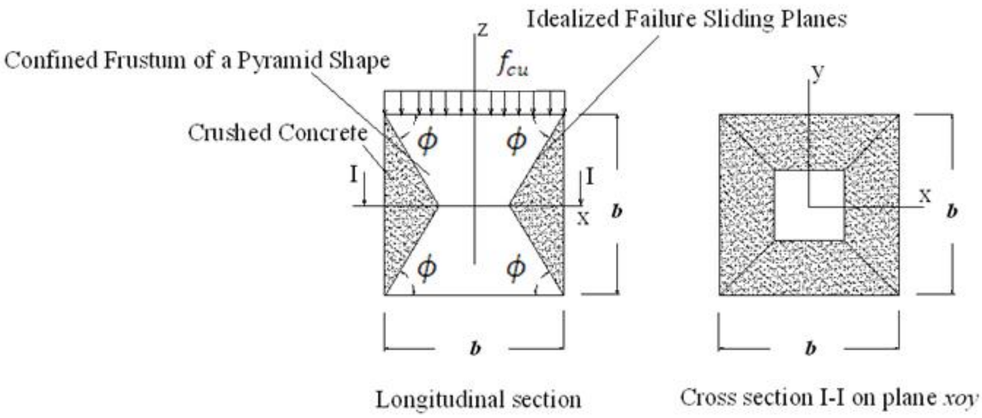
Figure3: Relationship between relative concrete strength f_0/f_c' and 1 + (h - d)/50.

Markeset, Hillerborg [9], Jansen and Shah [6] experimentally showed that the strength reduction is independent of the specimen size when the specimen height/diameter or length/depth is greater than a constant value (i.e. 2.0-2.5 and 3.0 for cylinder and C-shaped specimens, respectively).

Finally, the objective of this study is to experimentally investigate the effect of aspect ratio on the ultimate compressive strength of normal weight concrete and to clarify the reasons for the variety in compressive strength for specimens with various aspect ratio and to put forward an equation to obtain the concrete compressive with different aspect ratios strength from other standard specimen.

2 Proposed model

The shear models consider two main types of failure in concrete members. These are either sliding or crushing. The compression field models assume the concrete crushes at a stress that is some fraction of the concrete compressive strength (Collins 1978, Muttoni et al 1997). The plasticity models (Nielsen, 1984) argue that there is no such thing as a crushing failure, but that all Mohr-Coulomb materials fail along sliding planes and the strength of a concrete element in compression is an apparent compressive strength. A shear sliding model is suggested here to represent the general observation at failure of most test specimens. In general, when a prismatic concrete specimen is subjected to uniaxial compression load, it tends to expand in the lateral direction. However, there exists a frictional force between the machine platens and the specimen. This frictional force creates a lateral compressive force which is responsible for the formation of a pyramid at failure. When the lateral constraint is eliminated, the lateral compressive force disappears and a splitting type rupture is obtained. Though, it seems to be valid to assume that the lateral constraint is produced to some extent since it is very difficult to eliminate the frictional force in practice. This lateral constraint force is responsible for the formation of a general appearance of a quadrangular pyramid or frustum of a pyramid shape at the ends of the specimens at failure of prisms and cubes respectively as shown in Fig.4.
The inclined free surface at failure is assumed to follow a critical inclined crack path associated with shear damage. Assuming that the shear failure plane forms an angel ϕ with the horizontal plane (naturally, sliding may occur by several planes with the same direction simultaneously). If the height h of the prism considerably exceeds the dimension b, then the two quadrangular pyramids will not intersect each other within the limits of prism height, but if this situation is not achieved ,then the generated confinement zone extends through the specimen to lead failure by crushing, not by cracking as shown in Fig.4.case (b).



(b)

Figure 4: Typical prism compressive failure mode: (a) with the appearance of a quadrangular pyramid when $h \geq b \times \tan\phi$; (b) with the appearance of a truncated pyramidal shape when $h < b \times \tan\phi$.

Unless the confinement effect is considered, the prismatic and cube specimens with the same concrete mix show the same size effect because this effect is only a function of the characteristic dimension h on the basis that the cross dimension of the specimens is constant. Therefore, this implies that:

$$f_{cu} \times A^*_{cu} = f_p \times A_b \qquad (2)$$

$$A^*_{cu} = \frac{b^2(2\tan\phi - 1)}{\tan^2\phi} \qquad (3)$$

$$A_b = b^2 \qquad (4)$$

Where f_p is the characteristic prismatic axial compressive strength with $h \geq b \times \tan\phi$; f_{cu} is the characteristic cube compressive strength of the same concrete mix and cross sectional dimensions; A^*_{cu} is the projected surface area of sliding failure planes onto the *xoy* plane for the cube specimen; and A_b is the projected surface area of sliding failure planes onto the *xoy* plane for the prismatic specimen. Assume that the ratio $\frac{f_{cu}}{f_p} = k$, then:

$$k = \frac{\tan^2\phi}{2\tan\phi - 1} \qquad (5)$$

This yields to: $\tan^2\phi - 2k\tan\phi + k = 0$ (6)

Solving for $\tan\phi$ as a function of k

$$\tan\phi = k + \sqrt{k(k-1)} \qquad (7)$$

For example, the variation in the magnitude of sliding fracture plane angle ϕ for normal weight concrete with the ratio k is shown in the Tab.2.

Table 2: The sliding fracture plane angle ϕ versus different values of k

k	$\tan\phi$	ϕ
1.25	1.81	61.08°
1.35	2.04	63.88°
1.45	2.26	66.13°

Consequently, such pattern of failure which is illustrated in Fig. 4 case (a) may occur as soon as the height h is less than $b \times \tan\phi$ .This theoretical result gives an evident explanation to the adopted strength correction factor of 1.0 for the aspect ratio *h/b*=2.0 since the ratio *k* for normal weight concrete ranged from 1.33 to 1.40 as indicated in the Russian design code [12] and the decreasing strength correction factor values for the ratio *h/b*<2.0. Gradually increasing the concrete compressive strength will almost raise the value of ϕ and the magnitude of the ratio *k* as shown in the Tab.3 extracted from [12].

Table 3: The magnitude of the ratio *k* compared with the increment of axial concrete compressive strength

Grade	B10	B20	B25	B30	B35	B40	B45	B50	B55	B60
Nominal f_{cu} (MPa)	10	20	25	30	35	40	45	50	55	60
Nominal f_p (Mpa)	7.5	15.0	18.5	22.0	25.5	29.0	32.0	36.0	39.5	43.0
$k = \frac{f_{cu}}{f_p}$	1.33	1.33	1.35	1.36	1.37	1.37	1.40	1.38	1.39	1.39

When the difference in compressive strength of concrete between the cube strength and the prism strength vanishes as in high strength concrete, then the influence of the aspect ratio for all concrete specimens with *h*≥*b* diminishes. Neville found that the influence of the aspect ratio on the maximum compressive strength of high-strength concrete is not so pronounced as on that of normal strength concrete.

Based on the above assumptions and in case of $h < b \times \tan\phi$, the Eq.2 may be expressed in the next form following the same steps

$$f_m \times A_m^* = f_p \times A_b \qquad (8)$$

$$A_m^* = 2bh\cot\phi - h^2\cot^2\phi \qquad (9)$$

$$f_m = \frac{b^2}{2bh\cot\phi - h^2\cot^2\phi} \times f_p \qquad if\ h \le b \times \tan\phi \qquad (10a)$$

$$f_m = f_p \qquad if\ h > b \times \tan\phi \qquad (10b)$$

Where f_m is the axial compressive strength of a prismatic concrete specimen with side *b* and height *h;* A_m^* is the projected surface area of sliding failure planes onto the *xoy* plane for the prismatic specimen. It should be noted that the application of (10a) is limited for cases $h \le b \times \tan\phi$.

Clearly, the Eq. (10a) involved the effect of the aspect ratio and the concrete strength level which is expressed through the angle of sliding fracture plane ϕ simultaneously.

3 Experimental study

The main purposes of this experimental study are to verify the accuracy of proposed model. For that reason, the sliding angle ϕ is determined by Eq.7 which required the knowledge of axial compressive of f_{cu} (150 x 150 x150 mm specimens) and f_p(150 x150 x300 mm specimens). The experimental result of the ultimate axial compressive strength (150 x 150 x225 mm specimens) is compared with the theoretical result of the suggested model. Therefore, a total of 54 prismatic test specimens were constructed with constant square side dimension (*b* = 150mm) and variable height *h* varied from 150 to 225 to 300 mm. The materials used in the tests involve ordinary Portland cement (type1), crushed limestone with maximum size of 19mm as coarse aggregate, natural sand as fine aggregate and a superplasticizer to improve workability with the samples labeled (H.S.C). A summary of the specimen geometry, labels and the mix designs are listed in Tab.4.The specimens were cured at a controlled temperature of 25°C and 100 % relative humidity until the designated ages of testing, approximately 28 days after casting. The top and bottom surfaces were leveled with cement to distribute the applied load uniformly over the concrete specimens The compression tests were carried out with 3000 kN capacity load testing machine with 10kN/div. Universal Test Machine at a constant rate of stress equal to 0.2 to 0.4 MPa/second. The specimens were first placed concentrically under the loading head; this process involved a manual preloading to secure the specimens in place near the axis of the platen as possible as shown in Fig.5.The load on the specimen was applied until the specimen fails, the maximum load carried by the specimen recorded during the test, and the angle of fracture plane with respect to horizontal plane was approximately measured.

Table 4: Specimen designation, dimensions and variables

Specimen designation	Dimensions* *b* x *b* x *h* (*mm*)	Quantity	Mix Proportion Cement: Aggregates
L.W.C(01-06)	150x150x150	6	1: 7,5
L.W.C(07-12)	150x150x225	6	
L.W.C(13-15)	150x150x300	6	
N.W.C(16-18)	150x150x150	6	1: 6
N.W.C(19-21)	150x150x225	6	
N.W.C(22-24)	150x150x300	6	
H.S.C(25-27)	150x150x150	6	1:4,5
H.S.C(28-30)	150x150x225	6	
H.S.C(31-33)	150x150x300	6	

**b* x *b* is the area of cross section and *h* stands for the variable height.

Figure 5: Test specimens with *h* =150mm, 225mm and 300mm.

It is worth noting that the inclination of the failure surface with respect to the horizontal plane was far from the angle 45° as seen in Fig.6.The experiments demonstrate that the slope of sliding failure plane often ranged from 60° to 70°. Failure of all tested specimens occurred at compressive stress of 12Mpa up to 53Mpa.

Figure 6: Failure pattern of concrete prisms (measurement of fracture angle).

As might be expected, the compressive strength of concrete deceased as *h* increased. The decreasing rate of compressive strength slightly moderated when *h* approaches approximately 300 mm. Tab.5 summarizes the average compressive strength of concrete measured from testing of the six specimens with different aspect ratios and the correction factors to convert concrete strength to equivalent 150 x 150x300 mm standard strength.

Table5**:** Correction factors to convert concrete strength to equivalent 150 x 150x300 mm standard strength

Specimen designation	Dimensions (mm)	Average Compressive strength of Concrete (MPa)	Correction factors for concrete strength
L.W.C(01-06)	150x150x150	17.84	0.80
L.W.C(07-12)	150x150x225	15.51	0.92
L.W.C(13-18)	150x150x300	14.23	1
N.W.C(19-24)	150x150x150	28.89	0.76
N.W.C(25-30)	150x150x225	24.39	0.90
N.W.C(31-36)	150x150x300	21.97	1
H.S.C(37-42)	150x150x150	47.55	0.71
H.S.C(43-48)	150x150x225	40.51	0.84
H.S.C(49-54)	150x150x300	33.76	1

Fig.7 shows the plot of the proposed model of Eq. (10a) for specimens of different aspect ratios.The angle of fracture plane in the model is determined by the test results of specimens with average prism compressive strength 14.23, 21.97 and 33.76 MPa and the average cube strength 17.84, 28.89 and 47.55 MPa as has been shown in the Tab 5. The average compressive test data results for specimens with *h/b*=1.5 (i.e. 150x150x225mm specimens) compared with the proposed model is shown in Fig.7.

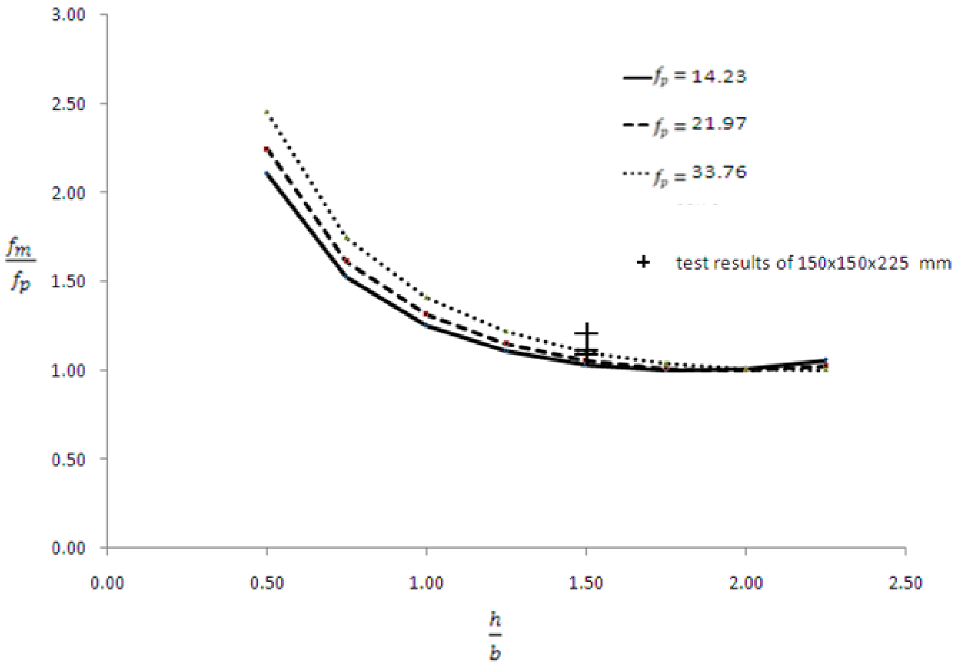


Figure7: Plot of $\frac{f_m}{f_p}$ versus the aspect ratio $\frac{h}{b}$

4 Conclusions

The proposed model of a shearing failure and the analytical analysis afford a new interpretation to elucidate the effect of aspect ratio and strength concrete level on ultimate compressive strength of concrete specimen with different aspect ratios and on the choice of the strength correction factor .The offered model clarify that the size effect vanishes if $h \geq b \times \tan\phi$ but the specimens with $h < b \times \tan\phi$ can attain more compressive strength. Obviously , the model allow for following researches to predict the reasons for the unnoticeable size effect for high strength concrete and can tackle a larger variety of problems. Finally, the proposed Eq. (10a) is commonly applicable to both cylindrical and prismatic specimens. All the research done thus far serves as the basis for future research and additional study may be fruitful.

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