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The Sequential Core of an Economy with Environmental Externalities

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Abstract

In this paper we apply a coalitional stability analysis to a Cobb-Douglas economy with multilateral environmental externalities. We find that the sequential core of the economy can be unique, empty or nonempty depending on the intensity of agents' preferences on private and public goods.

Keywords: Core, Environmental Externalities, Cobb-Douglas Economy

1. Introduction

In this paper we apply the concept of sequential core (introduced by Currarini and Marini, 2003) to an economy with negative environmental externalities. Such approach assumes that, when the members of a coalition deviate from the grand coalition they possess a first-mover advantage in selecting their joint strategy profile. Moreover, it is assumed that outside players respond to any deviating coalition by optimally reacting as singletons. We test this coalitional approach on a specific Cobb-Douglas economy with negative multilateral externalities (e.g., pollutants emissions) originated as side-effect of the production activity. Similar types of economies were studied, among others, by Maler (1992) and Chander-Tulkens (1997). In Maler (1992), it is shown that the concept of α-core (Aumann, 1959) coincides with the set of Pareto-optima of the economy. Chander-Tulkens (1997) prove that the γ-core is nonempty. In what follows we show that the sequential core of the economy (here denoted λ-core) can be, respectively, unique, nonempty or empty depending on the intensity of agents' preferences on private and public goods.

2. An Economy with Environmental Externalities

We consider an economy with finite set of agents $N = \{1,..i,..,n\}$ possessing preferences described by a Cobb-Douglas utility function

$$u_i(q, y_i, m_i) = q^a y_i^b + m_i,$$

where $q \geq 0$ denotes the overall environmental quality enjoyed by each agent, $y_i \geq 0$ the consumption of private good and m_i money. Let also $p \geq 0$ denote a polluting emission originated as side-effect of the production of *y*. The existing technology is described by the simple production function

$$y = p^\gamma,$$

and emissions are assumed to accumulate according to the following additive law

$$q = \mu - \sum_{i \in N} p_i$$

where μ is a constant expressing the quality of a pollution-free environment. We will assume that γ, a and b are all positive constant and that $\gamma \leq 1$, $a \leq 1$ and $b \leq 1$. We can now associate to this economy a strategic form game G with players set *N*, strategy space $P_i = [0, p^0{}_i]$ for each $i \in N$ and $\sum_{i \in N} p^0{}_i < \mu$, and players' payoffs $U_i(p_1,..., p_n) = q^a p_i^d + m_i$, where $d = b\gamma$. Now, we can easily obtain the following expression for the characteristic function $v_\lambda(S)$ of any nonempty coalition $S \subset N$ deviating as leader when the remaining agents in $N \setminus S$ optimally react as individual followers:[1]

$$v_\lambda(S) = s^{1-d} \mu^{a+d} a^{2a} (a+d)^{-a-d} (a + d(n-s))^{-a} d^d, \qquad (1)$$

where *s* denotes the size of coalition *S*.

3. Main Results

In this Section we prove that the λ-core of the game G can be characterized for three possible ranges of the economy parameters *a*, *b* and γ: (i) the case $a = b\gamma$, for

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[1]See also Currarini and Marini (2004) for derivation details.

which the core is nonempty and consists of the equal split allocation assigning $v_\lambda(N)/n$ to each player; (ii) the case *a* > *bγ*, for which the core strictly include the equal split allocation; (iii) the case *a* < *bγ*, for which the core is empty. Due to the non-polynomial form of $v_\lambda(S)$, we are able to obtain closed form results only for the case *a* =*bγ*. For all other cases, we use numerical simulations.

Proposition 1. *If a = bγ the* λ-core *consists of the symmetric allocation giving to each player the payoff* $v_\lambda(N)/n$.

Proof. We show that no allocation but the equal split (giving to each player the payoff $v_\lambda(N)/n$) belongs to the λ-core. By (1) we obtain

$$\frac{v_\lambda(N)}{n} - v(\{i\}) = \frac{a^a \mu^{\alpha+\delta}(a+d)^{-a-d} d^d \left[(a+d(n-1))^a - a^a n^d\right]}{n^d (a+d(n-1))^a}$$

from which, using the fact that *d* = *bγ* we get

$$\frac{v_\lambda(N)}{n} - v_\lambda(\{i\}) = 0 \Leftrightarrow \left[(a+d(n-1))^a - a^a n^d\right] = 0.$$

This directly implies that no allocation other than the equal split belongs to the λ-core. To show that the equal split belongs to the λ-core, it suffices to show that $v_\lambda(S) \le s\frac{v_\lambda(N)}{n}$ for all coalitions *S* with *s* > 1. Using (1) we obtain

$$\frac{v_\lambda(N)}{n} - \frac{v_\lambda(S)}{s} \ge 0 \Leftrightarrow \left[s^d (a+d(n-s))^a - a^a n^d\right] \ge 0$$

which, using again the fact that *d* = *bγ* reduces to

$$\frac{v_\lambda(N)}{n} - \frac{v_\lambda(S)}{s} \ge 0 \Leftrightarrow [s(a+a(n-s))]^a \ge (an)^a.$$

Proposition 2. *If a > bγ the λ-*core *strictly includes the symmetric allocation giving to each player the payoff* $v_\lambda(N)/n$.

Proof. We proceed with a numerical simulation. We need to show that whenever *a* > *bγ* the difference $\Delta = (v_\lambda(N)/n - v_\lambda(S)/s)$ is strictly positive for every s. We first consider the case *s* = 1. For *d* = 0.5, $n \in (1{,}10000)$ and $a \in (0{,}1)$ the simulation shows that $\Delta_i(n,a) = (\frac{v_\lambda(N)}{n} - v_\lambda(\{i\})) > 0$ whenever $a > 0.5$. Similar results are obtained for other values of *d* in (0,1) and for coalitions of size $s > 1$. We can conclude that, for $s \geq 1$, the λ-core is non-empty whenever *a* > *d*.

Proposition 3. *If a < bγ the λ-*core *of G is empty.*

Proof We again proceed by numerical simulation and evaluate the function $\Delta_i(n,\alpha)$ for any player $i \in N$ and fixed values of *d*. Numerical evaluations around the point *a* = 0.5 show that for any value of $n, v_\lambda(N)/n < v_\lambda(\{i\})$ whenever *a* < *d*. We thus conclude that, for such values of *a*, the λ-core is empty.

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