International Journal of Algebra, Vol. 4, 2010, no. 3, 99 - 108

On Jordan $(\alpha, \beta)^*-$Derivations in Semiprime $^*-$Rings

Shakir Ali

Department of Mathematics
Aligarh Muslim University, Aligarh-202002, India
shakir50@rediffmail.com, shakir.ali.mm@amu.ac.in

Ajda Fošner

Institute of Mathematics, Physics and Mechanics
Jadranska 19, SI-1000 Ljubljana, Slovenia
ajda.fosner@uni-mb.si

Abstract

In this paper we introduce the notion of $(\alpha, \beta)^*-$derivation (resp. Jordan $(\alpha, \beta)^*-$derivation) and prove the following result: Let R be a $6-$torsion free semiprime $^*-$ring and let α be an endomorphism of R and β an automorphism of R. Then an additive mapping $d : R \to R$ is a Jordan $(\alpha, \beta)^*-$derivation on R if and only if $d(xyx) = d(x)\alpha(y^*x^*) + \beta(x)d(y)\alpha(x^*) + \beta(xy)d(x)$ for all $x, y \in R$. As an application of this result we establish that on a $6-$torsion free semiprime $^*-$ring every Jordan triple left (resp. right) α^*-centralizer is a Jordan left (resp. right) α^*-centralizer.

Mathematics Subject Classification: 16N60, 16W10, 16W25

Keywords: Semiprime $^*-$ring, $^*-$derivation, $(\alpha, \beta)^*-$derivation, Jordan $(\alpha, \beta)^*-$derivation, Jordan triple $(\alpha, \beta)^*-$derivation, Jordan left (right) α^*-centralizer, Jordan triple left (right) α^*-centralizer

1 Introduction

Throughout, R will represent an associative ring. Let $n \geq 2$ be an integer. A ring R is said to be $n-$torsion free if for $x \in R$, $nx = 0$ implies $x = 0$. Recall that R is prime if $aRb = \{0\}$ implies $a = 0$ or $b = 0$. A ring R is called semiprime if $aRa = \{0\}$ implies $a = 0$. An additive mapping $x \mapsto x^*$ satisfying

$(xy)^* = y^*x^*$ and $(x^*)^* = x$ for all $x, y \in R$ is called an involution. A ring equipped with an involution is called a $^*-$ring or a ring with involution.

An additive mapping $d : R \to R$ is called a derivation (resp. Jordan derivation) if $d(xy) = d(x)y + xd(y)$ (resp. $d(x^2) = d(x)x + xd(x)$) holds for all $x, y \in R$. Let α and β be endomorphisms of R. An additive mapping $d : R \to R$ is said to be a $(\alpha, \beta)-$derivation (resp. Jordan $(\alpha, \beta)-$derivation) if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ (resp. $d(x^2) = d(x)\alpha(x) + \beta(x)d(x)$) holds for all $x, y \in R$. Note that for I_R, the identity map on R, the $(I_R, I_R)-$derivations (resp. Jordan $(I_R, I_R)-$derivations) are derivations (resp. Jordan derivations) on R. One can easily prove that every derivation is a Jordan derivation, but converse is in general not true. A famous result due to Herstein [8, Theorem 3.3] asserts that a Jordan derivation on a $2-$torsion free prime ring is a derivation. A brief proof of Herstein's result can also be found in [4]. This result was extended to a $2-$torsion free semiprime rings by Cusack [6] (see [1] for an alternative proof). An additive mapping $d : R \to R$ is called a Jordan triple derivation if $d(xyx) = d(x)yx + xd(y)x + xyd(x)$ holds for all $x, y \in R$. One can easily prove that every Jordan derivation of a $2-$torsion free ring is a Jordan triple derivation. In [2] Brešar proved that every Jordan triple derivation on a $2-$torsion free semiprime ring is a derivation.

Let R be a $^*-$ring. An additive mapping $d : R \to R$ is said to be a $^*-$derivation (resp. Jordan $^*-$derivation) if $d(xy) = d(x)y^* + xd(y)$ (resp. $d(x^2) = d(x)x^* + xd(x)$) holds for all $x, y \in R$. By our knowledge the concept of Jordan $^*-$derivations appears for the first time in the work of Brešar and Vukman [5]. The notion of Jordan $^*-$derivations arise naturally in the theory of representability of quadratic functionals with sesquilinear functionals (see Šemrl's work [10] and [11]). For results concerning this theory we refer to [12], [13], and [16]. More results on Jordan $^*-$derivations can be found in [3], [14], and [16]. The work of Brešar and Zalar [3] and Šemrl [11] inspired us to introduce and study more general concept of Jordan $^*-$derivations. Let α, β be endomorphisms of R. An additive mapping $d : R \to R$ is said to be a $(\alpha, \beta)^*-$derivation (resp. Jordan $(\alpha, \beta)^*-$derivation) if $d(xy) = d(x)\alpha(y^*) + \beta(x)d(y)$ (resp. $d(x^2) = d(x)\alpha(x^*) + \beta(x)d(x)$) holds for all $x, y \in R$. Note that if α and β are automorphisms of R, the mapping $x \mapsto a\alpha(x^*) - \beta(x)a$, where a is a fixed element, is called a Jordan $(\alpha, \beta)^*-$inner derivation. Of course, for I_R the identity map on R, $(I_R, I_R)^*-$derivation (resp. Jordan $(I_R, I_R)^*-$derivation) is a $^*-$derivation (resp. Jordan $^*-$derivation). Obviously, every $(\alpha, \beta)^*-$derivation is a Jordan $(\alpha, \beta)^*-$derivation. The converse is in general not true. For example, the mapping $x \mapsto a\alpha(x^*) - \beta(x)a$ is a Jordan $(\alpha, \beta)^*-$derivation but not a $(\alpha, \beta)^*-$derivation. An additive mapping d on a $^*-$ring R is called a Jordan triple $^*-$derivation if $d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$ holds for all $x, y \in R$. We call d a Jordan triple $(\alpha, \beta)^*-$derivation if $d(xyx) = d(x)\alpha(y^*x^*) + \beta(x)d(y)\alpha(x^*) + \beta(xy)d(x)$ is fulfilled for all $x, y \in R$. Note that for I_R the identity map on R, a Jordan triple $(I_R, I_R)^*-$derivation is a Jordan triple $^*-$derivation. Clearly, every $^*-$derivation on a 2$-$torsion free ring is a Jordan triple $^*-$derivation but not conversely. In [14] Vukman proved the following result. Let R be a 6$-$torsion free semiprime $^*-$ring and let $d : R \to R$ be an additive mapping satisfying the relation $d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$ for all $x, y \in R$. Then d is a Jordan $^*-$derivation.

The purpose of this paper is to extend the above mentioned result in the setting of Jordan triple $(\alpha, \beta)^*-$derivation.

2 Jordan $(\alpha, \beta)^*-$derivations

Let us state the main result of the paper.

Theorem 2.1 . *Let R be a $6-$torsion free semiprime $^*-$ring and let α be an endomorphism of R and β an automorphism of R. An additive mapping $d : R \to R$ is a Jordan $(\alpha, \beta)^*-$derivation on R if and only if* $d(xyx) = d(x)\alpha(y^*x^*) + \beta(x)d(y)\alpha(x^*) + \beta(xy)d(x)$ *for all $x, y \in R$.*

In order to prove the above theorem we need the following lemma which is a generalization of [14, Lemma 1].

Lemma 2.2 . *Let R be a $2-$torsion free semiprime $^*-$ring and let α be an endomorphism of R and β an automorphism of R. If there exist elements $a, b \in R$ such that $a\alpha(x^*b^*)+\beta(bx)a = 0$ for all $x \in R$, then $a\beta(b) = \beta(b)a = 0$. If R is prime, then either $a = 0$ or $\beta(b) = 0$.*

Proof. By the hypothesis, we have

$$a\alpha(x^*b^*) + \beta(bx)a = 0 \text{ for all } x \in R. \tag{1}$$

Replacing x by ybx in (1), we obtain $a\alpha(x^*b^*y^*)\alpha(b^*) + \beta(b)\beta(ybx)a = 0$ for all $x, y \in R$. In view of (1), the last expression yields that

$$\beta(b)\{\beta(xby) + \beta(ybx)\}a = 0 \text{ for all } x, y \in R. \tag{2}$$

Taking $y = x$ in (2) and using the fact that R is 2-torsion free, we find that $\beta(bx)\beta(bx)a = 0$ for all $x \in R$. This implies that

$$\beta(bx)a\alpha(x^*b^*) = 0 \text{ for all } x \in R. \tag{3}$$

Now, taking $y = x\beta^{-1}(a)y$ in (2) and using (1), we have

$$-\beta(bx)a\alpha(x^*b^*)\beta(y)a + \beta(bx)a\beta(y)\beta(bx)a = 0 \text{ for all } x, y \in R.$$

In view of (3), the last expression yields that $\beta(bx)a\beta(y)\beta(bx)a = 0$ for all $x, y \in R$. Since β is an automorphism of R, we find $\beta(bx)aR\beta(bx)a = \{0\}$ for all $x \in R$. Thus, the semiprimeness of R forces that $\beta(bx)a = 0$ for all $x \in R$. Hence, the last expression implies that $\beta(b)a = 0$. Similarly, we can prove that $a\beta(b) = 0$.

Proof of Theorem 2.1. Suppose that

$$d(xyx) = d(x)\alpha(y^*x^*) + \beta(x)d(y)\alpha(x^*) + \beta(xy)d(x) \tag{4}$$

for all $x, y \in R$. Replacing y by xyx in (4) and using (4), we obtain

$$\begin{aligned} d(x^2yx^2) \ &= \ d(x)\alpha(x^*y^*)\alpha(x^{*2}) + \beta(x)d(x)\alpha(y^*x^{*2}) + \\ &+\beta(x^2)d(y)\alpha(x^{*2}) + \beta(x^2)\beta(y)d(x)\alpha(x^*) + \beta(x^2)\beta(yx)d(x) \end{aligned} \tag{5}$$

for all $x, y \in R$. Again, replace x by x^2 in (4) to get

$$\begin{aligned} d(x^2yx^2) = d(x^2)\alpha(y^*)\alpha(x^{*2}) + \beta(x^2)d(y)\alpha(x^{*2}) + \\ +\beta(x^2)\beta(y)d(x^2) \end{aligned} \tag{6}$$

for all $x, y \in R$. On combining (5) and (6), we find that

$$\begin{aligned} (d(x^2) - d(x)\alpha(x^*) - \beta(x)d(x))\alpha(y^*)\alpha(x^{*2}) + \\ +\beta(x^2y)(d(x^2) - d(x)\alpha(x^*) - \beta(x)d(x)) = 0 \end{aligned}$$

for all $x, y \in R$. This can be rewritten as

$$A(x)\alpha(y^*x^{*2}) + \beta(x^2y)A(x) = 0 \tag{7}$$

for all $x, y \in R$, where

$$A(x) = d(x^2) - d(x)\alpha(x^*) - \beta(x)d(x) \tag{8}$$

for all $x \in R$. Using Lemma 2.2, we get

$$A(x)\beta(x^2) = 0 \tag{9}$$

and

$$\beta(x^2)A(x) = 0 \tag{10}$$

for all $x \in R$.

Put $x = y + x$ in (9) to get

$$A(x+y)\beta(x^2) + A(x+y)\beta(y^2) + A(x+y)\beta(xy+yx) = 0 \tag{11}$$

for all $x, y \in R$.

Now, we find

$$A(x+y) = d(x+y)^2 - d(x+y)\alpha(x+y)^* - \beta(x+y)d(x+y) \tag{12}$$

$$= (d(x^2) - d(x)\alpha(x^*) - \beta(x)d(x)) + (d(y^2) - d(y)\alpha(y^*) - \beta(y)d(y))$$

$$+(d(xy+yx) - d(x)\alpha(y^*) - d(y)\alpha(x^*) - \beta(x)d(y) - \beta(y)d(x))$$

$$= A(x) + A(y) + B(x, y)$$

for all $x, y \in R$, where

$$B(x, y) = d(xy+yx) - d(x)\alpha(y^*) - d(y)\alpha(x^*) - \beta(x)d(y) - \beta(y)d(x)$$

for all $x, y \in R$.

On combining (9), (11), and (12), we find that

$$A(x)\beta(y^2) + A(y)\beta(x^2) + B(x, y)\beta(x^2) + B(x, y)\beta(y^2)+ \tag{13}$$

$$+A(x)\beta(xy+yx) + A(y)\beta(xy+yx) + B(x, y)\beta(xy+yx) = 0$$

for all $x, y \in R$.

Replacing x by $-x$ in (13) and using the fact that $A(-x) = A(x)$ and $B(-x, y) = -B(x, y)$, we get

$$A(x)\beta(y^2) + A(y)\beta(x^2) - B(x, y)\beta(x^2) - B(x, y)\beta(y^2)- \tag{14}$$

$$-A(x)\beta(xy+yx) - A(y)\beta(xy+yx) + B(x, y)\beta(xy+yx) = 0$$

for all $x, y \in R$.

From (13) and (14) we obtain

$$2\{B(x, y)\beta(x^2) + B(x, y)\beta(y^2) + A(x)\beta(xy+yx) + A(y)\beta(xy+yx)\} = 0$$

for all $x, y \in R$. Since R is 6-torsion free, the above relation yields that

$$B(x, y)\beta(x^2) + B(x, y)\beta(y^2) + A(x)\beta(xy+yx)+ \tag{15}$$

$$+A(y)\beta(xy+yx) = 0$$

for all $x, y \in R$.

Further, replace x by $2x$ in (15) and use the facts that $B(2x, y) = 2B(x, y)$ and $A(2x) = 4A(x)$ to get

$$4B(x, y)\beta(x^2) + B(x, y)\beta(y^2)+ \tag{16}$$

$$+4A(x)\beta(xy+yx)+A(y)\beta(xy+yx)=0$$

for all $x, y \in R$.

On subtracting (15) from (16) and using the fact that R is $6-$torsion free we obtain

$$B(x,y)\beta(x^2)+A(x)\beta(xy+yx)=0 \tag{17}$$

for all $x, y \in R$.

Right multiplication of equation (17) by $A(x)$ gives

$$B(x,y)\beta(x^2)A(x)+A(x)\beta(xy+yx)A(x)=0 \tag{18}$$

for all $x, y \in R$.

Using (10) in (18) we have

$$A(x)\beta(xy)A(x)+A(x)\beta(yx)A(x)=0 \tag{19}$$

for all $x, y \in R$.

Put $y = yx$ in (19) to get

$$A(x)\beta(x)\beta(y)\beta(x)A(x)+A(x)\beta(y)\beta(x^2)A(x)=0 \tag{20}$$

for all $x, y \in R$.

In view of equation (10) the last relation reduces to

$$A(x)\beta(x)\beta(y)\beta(x)A(x)=0$$

which yields

$$\beta(x)A(x)\beta(x)R\beta(x)A(x)\beta(x)=\{0\}$$

for all $x \in R$.

Semiprimeness of R implies that

$$\beta(x)A(x)\beta(x)=0 \tag{21}$$

for all $x \in R$.

Again, multiplying (19) by $\beta(x)$ from right and using (21) we find

$$A(x)\beta(x)\beta(y)A(x)\beta(x)=0$$

for all $x, y \in R$.

Since β is an automorphism and R is semiprime, the last expression yields that

$$A(x)\beta(x)=0 \tag{22}$$

for all $x \in R$.

Using (22) in relation (17) we find that

$$B(x, y)\beta(x^2) + A(x)\beta(y)\beta(x) = 0$$

for all $x, y \in R$.

Right multiplication by $A(x)$ to above relation gives that

$$A(x)\beta(y)\beta(x)A(x) = 0 \tag{23}$$

for all $x, y \in R$.

This implies that

$$\beta(x)A(x)\beta(y)\beta(x)A(x) = 0$$

for all $x, y \in R$.

Since R is semiprime and β is an automorphism of R, we conclude that

$$\beta(x)A(x) = 0 \tag{24}$$

for all $x \in R$.

Replacing x by $x + y$ in (22) and using (22) and (24) we obtain

$$A(y)\beta(x) + B(x, y)\beta(y) + A(x)\beta(y) + B(x, y)\beta(x) = 0$$

for all $x, y \in R$.

Further, replace x by $-x$ in the above equation to get

$$-A(y)\beta(x) - B(x, y)\beta(y) + A(x)\beta(y) + B(x, y)\beta(x) = 0$$

for all $x, y \in R$.

Adding last two equations and using the fact that R is 6−torsion free we find that

$$A(x)\beta(y) + B(x, y)\beta(x) = 0$$

for all $x, y \in R$.

Multiplying the last equation by $A(x)$ from right and applying (24) we arrive at

$$A(x)\beta(y)A(x) = 0$$

for all $x, y \in R$. Since β is an automorphism and R is semiprime the last relation implies that $A(x) = 0$ for all $x \in R$. So, $d(x^2) = d(x)\alpha(x^*) + \beta(x)d(x)$ for all $x \in R$. Hence, d is a Jordan $(\alpha, \beta)^*$−derivation on R.

Conversely, suppose that $d(x^2) = d(x)\alpha(x^*) + \beta(x)d(x)$ for all $x \in R$. The linearization of the last expression yields

$$d(xy + yx) = d(x)\alpha(y^*) + \beta(x)d(y) + d(y)\alpha(x^*) + \beta(y)d(x)$$

for all $x, y \in R$. Replacing y by $xy + yx$ and using the above equation we get the required result.

The next corollary proved Vukman in [14] and it is a direct consequence of Theorem 2.1.

Corollary 2.3. Let R be a 6−torsion free semiprime *−ring and let $d : R \to R$ be an additive mapping satisfying the relation

$$d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$$

for all $x, y \in R$. Then d is a Jordan *−derivation on R.

The following example demonstrates that Theorem 2.1 does not hold true for arbitrary rings.

Example 2.4. Let S be a commutative ring. Next, let

$$R = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} \mid a, b, c \in S \right\}.$$

Define maps $d : R \longrightarrow R$ and $\alpha,\ \beta,\ * : R \longrightarrow R$ as follows:

$$d\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \alpha\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -a & b \\ 0 & 0 & -c \\ 0 & 0 & 0 \end{pmatrix},$$

$$\beta\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -a & -b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & c & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}.$$

Then it is straightforward to check that d satisfies the requirements of Theorem 2.1. However, d is not a Jordan $(\alpha, \beta)^*$−derivation on R.

3 Jordan left (right) α^*−centralizers

Let α be an automorphism of R. Following [17], an additive mapping $T : R \to R$ is called a left (resp. right) centralizer if $T(xy) = T(x)y$ (resp. $T(xy) = xT(y)$) holds for all $x, y \in R$. An additive mapping $T : R \to R$ is called a Jordan left (resp. right) centralizer if $T(x^2) = T(x)x$ (resp. $T(x^2) = xT(x)$) holds for all $x \in R$. Let $T : R \to R$ be an additive mapping. We call T a Jordan left (resp. right) α−centralizer if $T(x^2) = T(x)\alpha(x)$ (resp. $T(x^2) = \alpha(x)T(x)$)

holds for all $x \in R$. Now, let R be a $^*-$ring and $T : R \rightarrow R$ an additive mapping. We call T a left (resp. right) $^*-$centralizer if $T(xy) = T(x)y^*$ (resp. $T(xy) = x^*T(y)$) holds for all $x, y \in R$. An additive mapping $T : R \rightarrow R$ is called a Jordan left (resp. right) $^*-$centralizer if $T(x^2) = T(x)x^*$ (resp. $T(x^2) = x^*T(x)$) holds for all $x \in R$. An additive mapping $T : R \rightarrow R$ is called a Jordan left (resp. right) α^*-centralizer if $T(x^2) = T(x)\alpha(x^*)$ (resp. $T(x^2) = \alpha(x^*)T(x)$) holds for all $x \in R$. If T is both Jordan left as well as Jordan right α^*-centralizer, then T is a Jordan α^*-centralizer. Furthermore, an additive mapping $T : R \rightarrow R$ is called a Jordan triple left (resp. right) centralizer if $T(xyx) = T(x)yx$ (resp. $T(xyx) = xyT(x)$) holds for all $x, y \in R$ and if R is a $*-$ring an additive mapping $T : R \rightarrow R$ is called a Jordan triple left (resp. right) α^*-centralizer if $T(xyx) = T(x)\alpha(y^*x^*)$ (resp. $T(xyx) = \alpha(x^*y^*)T(x)$) holds for all $x, y \in R$.

The following result is in the spirit of Theorem 2 in [9] which states that on a $2-$torsion free semiprime ring every Jordan triple left (resp. right) centralizer is a left (resp. right) centralizer (see also [15]).

Theorem 3.1. Let R be a $6-$torsion free semiprime $^*-$ring and let α be an automorphism of R. An additive mapping $T : R \rightarrow R$ is a Jordan left α^*-centralizer on R if and only if $T(xyx) = T(x)\alpha(y^*x^*)$ for all $x, y \in R$.

Proof. In the proof of Theorem 2.1 we take $\beta = 0$.

Remark 3.2. Note that the above theorem holds true also for Jordan right α^*-centralizers: On a $6-$torsion free semiprime $^*-$ring every Jordan triple right α^*-centralizer is a Jordan right α^*-centralizer.

Corollary 3.3. Let R be a $6-$torsion free semiprime $^*-$ring. An additive mapping $T : R \rightarrow R$ is a Jordan left (resp. right) $^*-$centralizer if and only if $T(xyx) = T(x)y^*x^*$ (resp. $T(xyx) = x^*y^*\mathrm{T(x)}$) for all $x, y \in R$.

ACKNOWLEDGEMENTS. The authors are greatly indebted to the referee for his careful reading the paper.

References

[1] M. Brešar, Jordan derivations on semiprime rings, *Proc. Amer. Math. Soc.* **104** (1998), 1003–1006.

[2] M. Brešar, Jordan mappings of semiprime rings, *J. Algebra* **127** (1989), 128–228.

[3] M. Brešar, B. Zalar, On the structure of Jordan $^*-$derivation, *Colloq. Math.* **63** (1992), 163–171.

[4] M. Brešar, J. Vukman, Jordan derivations on prime rings, *Bull. Austral. Math. Soc.* **37** (1988), 321–322.

[5] M. Brešar, J. Vukman, On some additive mappings in rings with involution, *Aequ. Math.* **38** (1989), 178–185.

[6] J. Cusack, Jordan derivations on semiprime rings, *Proc. Amer. Math. Soc.* **53** (1975), 321–324.

[7] I. N. Herstein, *Rings with involution*, The University of Chicago Press, Chicago (1976).

[8] I. N. Herstein, Jordan derivations of prime rings, *Proc. Amer. Math. Soc.* **8** (1957), 1104–1110.

[9] C. K. Liu, W. K. Shiue, Generalized Jordan triple $(\theta, \phi)-$derivations of semiprime rings, *Taiwanese J. Math.* **11** (2007), 1397–1406.

[10] P. Šemrl, On Jordan $^*-$derivations and application, *Colloq. Math.* **59** (1990), 241–251.

[11] P. Šemrl, Quadratic functionals and Jordan $^*-$derivations, *Studia Math.* **97** (1991), 157–165.

[12] J. Vukman, A result concerning additive functions in hermitian Banach $^*-$algebras and an application, *Proc. Amer. Math. Soc.* **91** (1984), 367–372.

[13] J. Vukman, Some functional equations in Banach algebras and an application, *Proc. Amer. Math. Soc.* **100** (1987), 133–136.

[14] J. Vukman, A note on Jordan $^*-$derivations in semiprime rings with involution, *Int. Math. Forum.* **13** (2006), 617–622.

[15] J. Vukman, I. Kosi-Ulbl, A remark on a paper of L. Molnár, *Publ. Math. (Debr.)* **67** (2005), 419–421.

[16] B. Zalar, Jordan *-derivation pairs and quadratic functionals on modules over $*-$rings, *Aequ. Math.* **54** (1997), 31–43.

[17] B. Zalar, On centralizers of semiprime rings, *Comment. Math. Univ. Carol.* **32** (1991), 609–614.

Received: July, 2009