Abstract. Let $M$ be a semiprime $\Gamma$-ring satisfying a certain assumption. Then we prove that every Jordan left $K$-centralizer on $M$ is a left $K$-centralizer on $M$. We also prove that every Jordan $K$-centralizer of a 2-torsion free semiprime $\Gamma$-ring $M$ satisfying a certain assumption is a $K$-centralizer.

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Keywords: semiprime $\Gamma$-ring, left centralizer, centralizer, $K$-centralizer, Jordan $K$-centralizer

1. Introduction

In 1969, Nobusawa[10] gave the notion of a $\Gamma$-ring. This concept is more general than the concept of a ring. Barnes [1] generalized this concept and obtained many important basic properties of $\Gamma$-ring. Moreover, during the past few decades several researchers have studied the extensions and generalizations of various important results of the theory of rings to the theory of $\Gamma$-rings (see [4, 7, 8, 11] and references there in).

Sapanci and Nakajima [14] studied the notions of derivations and Jordan derivation, previously studied for rings, near rings and C*-algebras, for $\Gamma$-rings and discussed some related properties. Hoque and Paul [5] studied Jordan left centralizer of semiprime $\Gamma$-rings, previously studied in [2, 3, 6, 9, 15, 16, 17, 18, 19] for prime and semiprime rings.
In this paper the concept of a $K$-centralizer on a semiprime Γ-ring is introduced and some related identities are investigated. This concept is more general than the concept of a centralizer.

The motivation behind this paper is to initiate a study of the properties of $K$-centralizer on semiprime Γ-rings and prove certain results.

2. Preliminaries

In this section we describe some notions and known results which will be used in the sequel.

**Definition 2.1.** Let $M$ and $Γ$ be additive abelian groups. If there exists a mapping $(x, α, y) → xαy$ of $M × Γ × M → M$, satisfying the following conditions

(i) $xαy ∈ M$,

(ii) $(x + y)αz = xαz + yαz, x(α + β)z = xαz + xβz, xα(y + z) = xαy + xαz$

(iii) $(xαy)βz = xα(yβz)$ for all $x, y, z ∈ M$ and $α, β ∈ Γ$,

then $M$ is called a $Γ$-ring.

Every ring $M$ is a $Γ$-ring with $M = Γ$. However a $Γ$-ring need not be a ring.

**Definition 2.2.** Let $M$ be a $Γ$-ring. An additive subgroup $U$ of $M$ is called a left (right) ideal of $M$ if $MΓU ⊂ U (UΓM ⊂ U)$. If $U$ is both a left and a right ideal, then $U$ is called an ideal of $M$.

**Definition 2.3.** A $Γ$-ring $M$ is said to be 2-torsion free if $2x = 0$ implies $x = 0$ for all $x ∈ M$.

**Definition 2.4.** An ideal $P$ of a $Γ$-ring $M$ is said to be prime if for any ideals $A$ and $B$ of $M$, $AΓB ⊆ P$ implies $A ⊆ P$ or $B ⊆ P$.

**Definition 2.5.** An ideal $P$ of a $Γ$-ring $M$ is said to be semiprime if for any ideals $A$ of $M$, $AΓA ⊆ P$ implies $A ⊆ P$.

**Definition 2.6.** A $Γ$-ring $M$ is said to be prime if $aΓMΓb = \{0\}$, $a, b ∈ M$, implies $a = 0$ or $b = 0$.

**Definition 2.7.** A $Γ$-ring $M$ is said to be semiprime if $aΓMΓa = \{0\}$, $a ∈ M$, implies $a = 0$.

**Definition 2.8.** A $Γ$-ring $M$ is said to be commutative if $xαy = yαx$ for all $x, y ∈ M$ and $α ∈ Γ$.
Definition 2.9. Let $M$ be a $\Gamma$-ring. Then the set $Z(M) = \{x \in M : x\alpha y = y\alpha x \text{ for all } y \in M \text{ and } \alpha \in \Gamma\}$ is called the centre of the $\Gamma$-ring $M$.

Definition 2.10. Let $M$ be a $\Gamma$-ring. Then $[x, y]_\alpha = x\alpha y - y\alpha x$ is called the commutator of $x$ and $y$ with respect to $\alpha$, $x, y \in M$ and $\alpha \in \Gamma$.

The following commutator identities follow easily from above definition

(i) $[x\alpha y, z]_\beta = [x, z]_\beta \alpha y + x[\alpha, \beta]_\beta y + x\alpha [y, z]_\beta$
(ii) $[x, y\alpha z]_\beta = [x, y]_\beta \alpha z + y[\alpha, \beta]_\beta z + y\alpha [x, z]_\beta$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.11. An additive mapping $T : M \to M$ is called a left (right) centralizer on the $\Gamma$-ring $M$ if $T(x\alpha y) = T(x)\alpha y$ ($T(x\alpha y) = x\alpha T(y)$) holds for all $x, y \in M$ and $\alpha \in \Gamma$. $T$ is called a centralizer if it is both a left and a right centralizer.

For any fixed $a \in M$ and $\alpha \in \Gamma$, the mapping $T(x) = a\alpha x$ is a left centralizer and $T(x) = x\alpha a$ is a right centralizer.

Definition 2.12. An additive mapping $T : M \to M$ is called a Jordan left (right) centralizer if $T(x\alpha x) = T(x)\alpha x$ ($T(x\alpha x) = x\alpha T(x)$) holds for all $x, y \in M$, and $\alpha \in \Gamma$.

Obviously every centralizer is a Jordan centralizer but Jordan centralizer is not, in general, a centralizer.

Definition 2.14. An additive mapping $D : M \to M$ is called a derivation on a $\Gamma$-ring $M$ if $D(x\alpha y) = D(x)\alpha y + x\alpha D(y)$ holds for all $x, y \in M$, and $\alpha \in \Gamma$.

Definition 2.15. An additive mapping $D : M \to M$ is called a Jordan derivation on a $\Gamma$-ring $M$ if $D(x\alpha x) = D(x)\alpha x + x\alpha D(x)$ holds for all $x \in M$, and $\alpha \in \Gamma$.

Lemma 2.16. [5] Let $M$ be a semiprime $\Gamma$-ring. If $a, b \in M$ and $\alpha, \beta \in \Gamma$ are such that $a\alpha x\beta b = 0$ for all $x \in M$, then $a\alpha b = b\alpha a = 0$.

Lemma 2.17. [5] Let $M$ be a semiprime $\Gamma$-ring and $A : M \times M \to M$ a biadditive mapping. If $A(x, y)\alpha w\beta B(x, y) = 0$ for all $x, y, w \in M$ and $\alpha, \beta \in \Gamma$, then $A(x, y)\alpha w\beta B(u, v) = 0$ for all $x, y, u, v \in M$ and $\alpha, \beta \in \Gamma$. 
Lemma 2.18. [5] Let \( M \) be a semiprime \( \Gamma \)-ring satisfying the assumption \( x\alpha y\beta z = x\beta y\alpha z \), for all \( x, y, z \in M \) and \( \alpha, \beta \in \Gamma \). If \( a \in M \) is a fixed element such that \( a\alpha[x, y]_{\beta} = 0 \) for all \( x, y \in M \) and \( \alpha, \beta \in \Gamma \), then there exists an ideal \( U \) of \( M \) such that \( a \in U \subset Z(M) \).

Lemma 2.19. [5] Let \( M \) be a semiprime \( \Gamma \)-ring satisfying the assumption \( x\alpha y\beta z = x\beta y\alpha z \), for all \( x, y, z \in M \) and \( \alpha, \beta \in \Gamma \). Let \( D \) be a derivation of \( M \) and \( \alpha \in M \), a fixed element.

(i) If \( D(x)\alpha D(y) = 0 \) for all \( x, y \in M \), \( \alpha \in \Gamma \), then \( D = 0 \).

(ii) If \( a\alpha x - x\alpha a \in Z(M) \) for all \( x \in M \), \( \alpha \in \Gamma \), then \( \alpha \in Z(M) \).

Lemma 2.20. [5] Let \( M \) be a semiprime \( \Gamma \)-ring satisfying the assumption \( x\alpha y\beta z = x\beta y\alpha z \), for all \( x, y, z \in M \) and \( \alpha, \beta \in \Gamma \). Let \( a, b \in M \) be two fixed elements such that \( a\alpha x = x\alpha b \) for all \( x \in M \) and \( \alpha \in \Gamma \), then \( a = b \in Z(M) \).

3. Left \( K \)-Centralizers of Semiprime Gamma Rings

In this section we prove some properties of left \( K \)-Centralizers of semiprime gamma rings.

**Definition 3.1.** Let \( M \) be a \( \Gamma \)-ring and \( K : M \to M \) an automorphism such that \( K(x\alpha y) = K(x)\alpha K(y) \) for all \( x, y \in M \) and \( \alpha \in \Gamma \). An additive mapping \( T : M \to M \) is a left (right) \( K \)-centralizer if \( T(x\alpha y) = T(x)\alpha K(y) \) (\( T(x\alpha y) = K(x)\alpha T(y) \)) holds for all \( x, y \in M \) and \( \alpha \in \Gamma \). \( T \) is called a \( K \)-centralizer if it is both a left and a right \( K \)-centralizer.

For any fixed \( a \in M \) and \( \alpha \in \Gamma \), the mapping \( T(x) = a\alpha K(x) \) is a left \( K \)-centralizer and \( T(x) = K(x)a\alpha \) is a right \( K \)-centralizer.

**Definition 3.2.** An additive mapping \( T : M \to M \), \( M \) is a \( \Gamma \)-ring, is a Jordan left (right) \( K \)-centralizer if \( T(x\alpha y + y\alpha x) = T(x)\alpha K(y) + K(y)\alpha T(x) \) holds for all \( x, y \in M \) and \( \alpha \in \Gamma \).

**Definition 3.3.** An additive mapping \( T : M \to M \), \( M \) is a \( \Gamma \)-ring, is called a Jordan \( K \)-centralizer if \( T(x\alpha y + y\alpha x) = T(x)\alpha K(y) + K(y)\alpha T(x) \), for all \( x, y \in M \) and \( \alpha \in \Gamma \).

**Lemma 3.4.** Let \( M \) be a semiprime \( \Gamma \)-ring satisfying the assumption \( x\alpha y\beta z = x\beta y\alpha z \), for all \( x, y, z \in M \), and \( \alpha, \beta \in \Gamma \). Let \( T : M \to M \) be a Jordan left \( K \)-centralizer, then

**a)** \( T(x\alpha y + y\alpha x) = T(x)\alpha K(y) + T(y)\alpha K(x) \),

**b)** \( T(x\alpha y\beta x + x\beta y\alpha x) = T(x)\alpha K(y)\beta K(x) + T(x)\beta K(y)\alpha K(x) \),
(c) If $M$ is a $2$-torsion free $\Gamma$-ring satisfying the above assumption, then
(i) $T(x\alpha y\beta x) = T(x)\alpha K(y)\beta K(x)$
(ii) $T(x\alpha y\beta z + z\beta y\alpha x) = T(x)\alpha K(y)\beta K(z) + T(z)\beta K(y)\alpha K(x)$.

**Proof.** Since $T$ is a Jordan left $K$-centralizer, therefore

\[(1)\quad T(x\alpha x) = T(x)\alpha K(x).\]

(a) Replacing $x$ by $x + y$ in (1) , we get

\[(2)\quad T(x\alpha y + y\alpha x) = T(x)\alpha K(y) + T(y)\alpha K(x)\]

for all $x, y \in M$ and $\alpha \in \Gamma$.

(b) Replacing $y$ by $x\alpha y + y\alpha x$ and $\alpha$ by $\beta$ in (2),we get

\[T(x\beta x\alpha y\alpha x + x\alpha y\beta x + y\alpha x\beta x) = T(x)\beta K(x)\alpha K(y) + T(x)\beta K(y)\alpha K(x) + T(y)\alpha K(y)\beta K(x) + T(y)\alpha K(x)\beta K(y),\]

which gives

\[(3)\quad T(x\beta y\alpha x + x\alpha y\beta x) = T(x)\beta K(y)\alpha K(x) + T(x)\alpha K(y)\beta K(x) + T(x)\alpha K(y)\beta K(x) + T(y)\alpha K(x)\beta K(y).\]

The last relation along with (2) implies

\[(4)\quad T(x\beta y\alpha x + x\alpha y\beta x) = T(x)\beta K(y)\alpha K(x) + T(x)\alpha K(y)\beta K(x).\]

Replacing $x$ by $x + z$ in (4), we get

\[T((x + z)\beta y\alpha (x + z)) = T(x + z)\beta K(y)\alpha K(x + z),\]

which implies

\[T(x\beta y\alpha z + z\beta y\alpha x) = T(x)\beta K(y)\alpha K(z) + T(z)\beta K(y)\alpha K(x).\]

The last relation along with the assumption $x\alpha y\beta z = x\beta y\alpha z$ gives

\[(5)\quad T(x\alpha y\beta z + z\beta y\alpha x) = T(x)\alpha K(y)\beta K(z) + T(z)\beta K(y)\alpha K(x).\]

**Theorem 3.5.** Let $M$ be a semiprime $\Gamma$-ring satisfying the assumption $x\alpha y\beta z = x\beta y\alpha z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$ and let $T : M \rightarrow M$ be a Jordan left $K$-centralizer. Then $T$ is a left $K$-centralizer.

**Proof.** Using Lemma 3.4-c(i) , we have

\[(6)\quad T(x\alpha y\beta z\gamma y\alpha x + y\alpha x\beta z\gamma x\alpha y) = T(x)\alpha K(y)\beta K(z)\gamma K(y)\alpha K(x) + T(y)\alpha K(x)\beta K(z)\gamma K(x)\alpha K(y).\]
Moreover, Lemma 3.4-c(ii) gives
\[
(7) \quad T(x_\alpha y \beta z \gamma y \alpha x + y_\alpha x \beta z \gamma x_\alpha y) = T(x_\alpha y) \beta K(z) \gamma K(y) \alpha K(x) + T(y_\alpha x) \beta K(z) \gamma K(x) \alpha K(y).
\]

Subtracting (6) from (7), we get \((T(x_\alpha y) - T(x)\alpha K(y))\beta K(z)\gamma K(y)\alpha K(x) + (T(y_\alpha x) - T(y)\alpha K(x))\beta K(z)\gamma K(x)\alpha K(y) = 0\), which implies
\[
(8) \quad H(x, y)\beta K(z)\gamma K(y)\alpha K(x) + H(y, x)\beta K(z)\gamma K(x)\alpha K(y) = 0.
\]

Where \(H(x, y) = T(x_\alpha y) - T(x)\alpha K(y)\), which alongwith (2) implies \(H(x, y) = -H(y, x)\). Using the last relation, from (8) we get \(H(x, y)\beta K(z)\gamma [K(x), K(y)]_\alpha = 0\).

Replacing \(x\) by \(K^{-1}(x)\), \(y\) by \(K^{-1}(y)\) and \(z\) by \(K^{-1}(z)\) in the last relation, we get \(H(K^{-1}(x), K^{-1}(y))\beta z \gamma [u, v]_\alpha = 0\). Replacing \(x\) by \(K(x)\) and \(y\) by \(K(y)\) in the last relation, we get
\[
(9) \quad H(x, y)\beta z \gamma [u, v]_\alpha = 0.
\]

Using Lemma 2.16 in (9), we get
\[
(10) \quad H(x, y)\beta [u, v]_\alpha = 0.
\]

We now fix some \(x, y \in M\) and denote \(H(x, y)\) by \(H\). Using Lemma 2.18, we get the existence of an ideal \(U\) such that \(H \in U \subseteq Z(M)\). In particular, \(\alpha H, H \alpha b \in Z(M)\) for all \(b \in M\), then \(x_\alpha (H \beta H \gamma y) = (H \beta H \gamma y)\alpha x = (y_\gamma H \beta H \alpha x) = (H \beta H \alpha x)\gamma y\), which implies \(4T(x_\alpha (H \beta H \gamma y)) = 4T(y_\gamma (H \beta H \alpha x))\), which gives
\[
2T(x_\alpha H \beta H \gamma y + H \beta H \gamma y_\alpha x) = 2T(y_\gamma H \beta H \alpha x + H \beta H \alpha x_\gamma y).
\]

Using (2) in the last relation, we get
\[
2T(x_\alpha K(H)\beta K(H)\gamma K(y) + 2T(H \beta H \gamma y)\alpha K(x) = 2T(y_\gamma K(H)\beta K(H)\alpha K(x) + 2T(H \beta H \alpha x)\gamma K(y),
\]

which implies
\[
2T(x_\alpha K(H)\beta K(H)\gamma K(y) + T(H)\beta K(H)\gamma K(y)\alpha K(x) + T(y)\gamma K(H)\beta K(H)\alpha K(x) = 2T(y_\gamma K(H)\beta K(H)\alpha K(x) + T(H)\beta K(H)\beta K(H)\gamma K(y),
\]

which implies
\[
T(x_\alpha K(H)\beta K(H)\gamma K(y) + T(H)\beta K(H)\gamma K(y)\alpha K(x) = T(y_\gamma K(H)\beta K(H)\alpha K(x) + T(H)\beta K(H)\alpha K(x)\gamma K(y).
\]

Replacing \(H\) by \(K^{-1}(H)\) in the last relation, we get \(T(x_\alpha H \beta H \gamma K(y) + \)

$T(K^{-1}(H))\beta H \gamma K(y)\alpha K(x) = T(y)\gamma H \beta H \alpha K(x) + T(K^{-1}(H))\beta H \alpha K(x)\gamma K(y)$.

Since $H \in U \subseteq Z(M)$ and $x\alpha y\beta z = x\beta y\alpha z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, therefore $H\gamma K(y)\alpha K(x) = (H\gamma K(y))\alpha K(x) = K(x)\alpha (H\gamma K(y)) = (K(x)\alpha H)\gamma K(y) = H\alpha K(x)\gamma K(y)$, using this in the last relation we get

$$(11) \quad T(x)\alpha K(y)\gamma H \beta H = T(y)\gamma H \beta H \alpha K(x).$$

Now since $H \in U \subseteq Z(M)$, one has $x\alpha y\gamma H \beta H = x\alpha (y\gamma H)\beta H = (x\alpha H)\gamma (y\beta H) = (H\alpha x)\gamma (H\beta y)$, therefore $4T(x\alpha y\gamma H \beta H) = 4T((H\alpha x)\gamma (H\beta y))$, which implies

$2T(x\alpha y\gamma H \beta H + H \beta H \gamma x\alpha y) = 2T(H\alpha x\gamma H \beta y + H \beta y\gamma H \alpha x).$ The last relation along with (2) gives

$2T(x\alpha y)\gamma K(H)\beta K(H) + 2T(H)\beta K(H)\gamma K(x)\alpha K(y) = 2T(H\alpha x)\gamma K(H)\beta K(y) + 2T(H)\beta K(H)\gamma K(x)\alpha K(y) = T(x\alpha H + H\alpha x)\gamma K(H)\beta K(y) + T(y\beta H + H \beta y)\gamma K(H)\alpha K(x),$ which further gives

$2T(x\alpha y)\gamma K(H)\beta K(H) + 2T(H)\beta K(H)\gamma K(x)\alpha K(y) = T(x)\alpha K(H)\gamma K(H)\beta K(y) + T(H)\alpha K(x)\gamma K(H)\beta K(y) + T(y)\beta K(H)\gamma K(x)\alpha K(x) + T(H)\beta K(y)\gamma K(H)\alpha K(x).$

Replacing $H$ by $K^{-1}(H)$ in the last relation, we get

$2T(x\alpha y)\gamma H \beta H + 2T(K^{-1}(H))\beta H \gamma K(x)\alpha K(y) = T(x)\alpha H \gamma H \beta H,$

$+ T(K^{-1}(H))\alpha K(x)\gamma H \beta K(y) + T(y)\beta H \gamma H \alpha K(x) + T(K^{-1}(H))\beta K(y)\gamma H \alpha K(x).$

Since $H \in U \subseteq Z(M)$ and $x\alpha y\beta z = x\beta y\alpha z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, therefore $2T(x\alpha y)\gamma H \beta H = T(x)\alpha K(y)\gamma H \beta H + T(y)\gamma H \beta H \alpha K(x).$

The last relation along with (11) gives $T(x\alpha y)\gamma H \beta H = T(x)\alpha K(y)\gamma H \beta H,$ that is $H \gamma H \beta H = 0$. Using Lemma 2.16 in the last relation, we get $H \beta H = 0$. Now $H \beta M \alpha H = (H \beta H)\alpha M = \{0\}$. Thus $H = 0$, that is $T(x\alpha y) = T(x)\alpha K(y)$.

4. **The K-Centralizers of Semiprime Gamma Rings.**

**Lemma 4.1.** Let $M$ be a semiprime $\Gamma$-ring satisfying the assumption $x\alpha y\beta z = x\beta y\alpha z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$ and for some fixed element $m \in M$ if $T(x) = m\alpha K(x) + K(x)\alpha m$ is a Jordan $K$-centralizer, then $m \in Z(M)$.

**Proof.** By hypothesis

$$(12) \quad T(x) = m\alpha K(x) + K(x)\alpha m.$$

Since $T$ is a Jordan $K$-centralizer, therefore

$$(13) \quad T(x\beta y + y\beta x) = T(x)\beta K(y) + K(y)\beta T(x).$$
Using (12) in (13), we get
\[ mαK(xβy+yβx)+K(xβy+yβx)αm = (mαK(x)+K(x)αm)βK(y)+K(y)β(mαK(x)+K(x)αm), \]
which implies \( mαK(x)βK(y)+mαK(y)βK(x)+K(x)βK(y)αm+K(y)βK(x)αm = (mαK(x)+K(x)αm)βK(y)+K(y)β(mαK(x)+K(x)αm), \)
which further gives \( mαK(y)βK(x)+K(x)βK(y)αm = K(x)αmβK(y)+K(y)βmαK(x). \)
Using the assumption \( xαyβz = xβyαz \) in the last relation, we get
\( (mαK(y)−K(y)αm)βK(x)−K(x)β(mαK(y)−K(y)αm) = 0, \)
which implies \( mαK(y)−K(y)αm ∈ Z(M). \) The last relation along with Lemma 2.19 implies \( m ∈ Z(M). \)

**Lemma 4.2.** Let \( M \) be a semiprime \( Γ \)-ring satisfying the assumption \( xαyβz = xβyαz, \) for all \( x, y, z ∈ M \) and \( α, β ∈ Γ. \) Then every Jordan \( K \)-centralizer of \( M \) maps \( Z(M) \) into \( Z(M). \)

**Proof.** Let \( m ∈ Z(M). \) Then
\[ 2T(max) = T(max + xam) = T(m)αK(x) + K(x)αT(m). \]
Let \( S(x) = 2T(max). \) Then \( S(xβy+yβx) = 2T(mα(xβy+yβx)) = 2T(maxβy+macβx). \) Since \( m ∈ Z(M) \) and \( xαβz = xβyαz, \) one has
\[ S(xβy+yβx) = 2T((xαm)βy+yβ(xαm)) = 2T(xαm)βk(y)+2k(y)βT(xαm) = S(x)βk(y)+k(y)βS(x). \]
Hence \( S \) is a Jordan \( K \)-centralizer. So (14) along with Lemma 4.1 gives \( T(m) ∈ Z(M). \)

**Theorem 4.3.** Every Jordan \( K \)-centralizer of a 2-torsion free semiprime \( Γ \)-ring \( M \) satisfying the assumption \( xαyβz = xβyαz, \) for all \( x, y, z ∈ M \) and \( α, β ∈ Γ \) is a \( K \)-centralizer.

**Proof.** Suppose that \( T \) is a Jordan \( K \)-centralizer, then
\[ T(xαy + yαx) = T(x)αK(y) + K(y)αT(x) = K(x)αT(y) + T(y)αK(x). \]
Replacing \( y \) by \( xβy + yβx \) in the last relation, we get
\[ T(x)αK(x)βK(y)+T(x)αK(y)βK(x)+K(x)βK(y)αT(x)+K(y)βK(x)αT(x) \]
\[ = (T(x)βK(y)+K(y)βT(x))αK(x) + K(x)α(T(x)βK(y)+K(y)βT(x)). \]
Using the assumption \( xαyβz = xβyαz, \) from the last relation we get
\[ T(x)αK(x)βK(y)+K(y)βK(x)αT(x) = K(y)βT(x)αK(x)+K(x)αT(x)βK(y). \]
That is, \([T(x), K(x)]αβK(y) = K(y)β[T(x), K(x)]α, \) which implies \([T(x), K(x)]α ∈ Z(M). \)
Now we prove that $[T(x), K(x)]_\alpha = 0$. Let $m \in Z(M)$. Lemma 4.2 implies that $T(m) \in Z(M)$. Thus
$$2T(max) = T(max + xam) = T(m)\alpha K(x) + K(x)\alpha T(m) = 2T(x)\alpha k(m),$$
which implies
$$T(max) = T(x)\alpha K(m) = T(m)\alpha K(x).$$

Now $[T(x), K(x)]_\alpha \beta K(m) = T(x)\alpha K(x)\beta K(m) - K(x)\alpha T(x)\beta K(m)$. The last relation along with (15) implies $[T(x), K(x)]_\alpha \beta K(m) = 0$.

Since $[T(x), K(x)]_\alpha$ itself is a central element, one has $[T(x), K(x)]_\alpha = 0$. Now
$$2T(x\alpha x) = T(x\alpha x + x\alpha x) = T(x)\alpha K(x) + K(x)\alpha T(x) = 2T(x)\alpha K(x) = 2K(x)\alpha T(x).$$
That is, $T(x\alpha x) = K(x)\alpha T(x)$. Hence $T$ is a Jordan left $K$-centralizer. By Theorem 3.5, $T$ is a left $K$-centralizer. Similarly, we can prove that $T$ is a right $K$-centralizer. Therefore $T$ is a $K$-centralizer.

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