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Finite Groups in which (S-)Semipermutability is a Transitive Relation

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Abstract

Let G be a finite group. A subgroup H of G is called *semipermutable* (or *S-semipermutable*) in G if H permutes with every subgroup K (or every Sylow p-subgroup) of G with $(|K|, |H|) = 1$ (or $(p, |H|) = 1$, where $p \in \pi(G)$). A group G is called an $BT-$group (or SBT-group) if semipermutability (S-semipermutability) is a transitive relation in G. In this paper, we determine the structure of $BT-$group (or SBT-group) and classify the minimal non$-BT-$groups (or minimal non-SBT-group).

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1 Introduction

All groups considered in this paper are finite. Our notations and notion are standard and taken mainly from [4]. Throughout this article, G stands for a finite group and $\pi(G)$ denotes the primes dividing $|G|$ and G_p a Sylow p-subgroup of G for some prime $p \in \pi(G)$.

Two subgroups H and K of G are said to be permutable if $\langle H, K\rangle = HK = KH$. A subgroup H of G is called *permutable* if H permutes with every subgroup of G. Permutable subgroups were initially studied by Ore[8], who called them *quasinormal*, in 1939. A subgroup H of G is called $S-$*permutable* if H permutes with every *Sylow* subgroup of G. This concept was introduced by Kegel in [7], who called them S-*quasinormal*. As the generalizations of permutability and S-permutability, Zhongmu Chen in [2] introduced the following: A subgroup H of G is called *semipermutable* in G if H permutes with every subgroup K of G with $(|H|, |K|) = 1$, and S-*semipermutable* if H permutes with every *Sylow* $p-$ subgroup of G with $(p, |H|) = 1$, who called them *seminormal,* S-*seminormal* subgroup of G, respectively. It is easy to see that a permutable (resp. S-permutable) subgroup of G is a semipermutable (resp. S-semipermutable) subgroup of G. The converse is not true in general. Usually (S-)seminormality cannot imply subnormality while S-permutablity can by a result of Kegel [7]. For example, a Sylow $2-$subgroup of the symmetric group S_3 of degree 3 is semipermutable in S_3 but not S-permutable in S_3. Many authors studied the influence of ($S-$)semipermutability of some subgroups on the structure of groups. For example, Chen, in [2], proved that G is supersolvable if every maximal subgroup of any Sylow subgroup of G is semipermutable in G. Zhang and Wang [14] proved that G is supersolvable if one of following holds: (1) every maximal subgroup of any Sylow subgroup of G is S-semipermutable in G; (2) every minimal subgroup and cyclic subgroup of order 4 of G is S-semipermutable in G. Zhang and Wang in [14] classified the finite nonabelian simple groups which contain a nontrivial semipermutable (s-semipermutable) subgroup.

Recall that G is said to be a $T-$*group* (resp. $PT-$*group*, $PST-$*group*), if normality (resp. permutability, $S-$permutability) is a transitive relation in G. From 1950s, especially in recent years, due to the efforts of many leading mathematicians, such as Gaschätz, Robinson, Cossey, Ballester-Bolinches, ect, many characterizations of T-group, PT-group, PST-group were discovered . The study of these classes of groups has undoubtedly constituted a fruitful topic in group theory. The structure of solvable T-groups was determined by Gaschätz [5] in 1957, he showed that these are exactly the groups with an abelian normal Hall subgroup L of odd order such that G/L is a Dedekind group and the elements of G induce power automorphisms in L. The corresponding theorem for solvable PT-group (resp. PST-group) is due to Zacher

[11](resp. Agrawal [1]): here one has to replace "Dedekind" in Gaschätz's theorem by "nilpotent modular" (resp. "nilpotent"). Robinson also has given the structure of minimal non-T-group, minimal non-PT-group and minimal non-PST-group [9, 15].

Stimulated by results in above, we naturally consider the parallelism questions about semipermutability and S-semipermutability. Semipermutability and S-semipermutablity, like normality, permutability and $S-$permutability, is not a transitive relation in an arbitrary group. For example, S_4, the symmetric group of degree 4, is a counterexample. In this paper, we study the structure of groups in which semipermutability or S-semipermutablity is transitive, that is, groups G such that H semipermutable (resp. S-semipermutable) in K and K semipermutable (resp. S-semipermutable) in G imply that H is semipermutable (resp. S-semipermutable) in G. Such groups are called *$BT-$groups* (resp. *SBT-groups*). A byproduct of our results is that solvable BT-group and solvable SBT-group coincide. We also give the structure of the minimal non$-BT-$groups (resp. minimal non$-SBT-$groups), i.e., the group which is not a BT-group (resp. SBT-group), but every proper subgroup of which is a BT-group (resp. SBT-group).

Remark: It is easy to see that every nilpotent group is a BT-group (SBT-group).

2 Preliminaries

In this section, we give some results which will be useful in the sequel.

Deducing from the definitions directly, we get:

Lemma 2.1 *Suppose that H is (S-)semipermutable in a G, $K \leq G$. Then:*
(1) If $H \leq K$, then H is (S-)semipermutable in K;
(2) If N is a normal $p-$subgroup of G, then HN/N is (S-)semipermutable in G/N.

Lemma 2.2 *Suppose that N is a normal $p-$subgroup of G, where $p \in \pi(G)$. If all $p'-$elements of G induce power automorphisms in N by conjugate, then $G/C_G(N)$ is nilpotent.*

Proof. If N is non-abelian, all $p'-$power automorphisms are trivial by [6]. Hence, $G/C_G(N)$ is a $p-$group. If N is abelian, the power automorphisms are in the center of $\mathrm{Aut}(N)$. Hence $G/C_G(N)$ is nilpotent.

Lemma 2.3 *Let G be a group. Then the following statements are equivalent:*
(i) Every subgroup of G is semipermutable in G;

(ii) For any $p, q \in \pi(G)$, and $p \neq q$, let $\langle x\rangle$ be a $p-$subgroup and $\langle y\rangle$ a $q-$subgroup. Then $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$.

Proof. $(i) \Longrightarrow (ii)$. It is clear.

$(ii) \Longrightarrow (i)$. Take any subgroup A of G. Let B be a subgroup of G such that $(|A|, |B|) = 1$. Clearly, any group can be generated by some elements of prime power order. Hence, we can assume that $A = \langle a_1, a_2, \cdots, a_m\rangle$ and $B = \langle b_1, b_2, \cdots, b_n\rangle$, where a_i, b_j are elements of G of prime power order. Since $(|A|, |B|) = 1$, it follows that $(o(a_i), o(b_j)) = 1$. By hypotheses, we have $\langle a_i\rangle\langle b_j\rangle = \langle b_j\rangle\langle a_i\rangle$. So

$$AB = \langle a_1, a_2, \cdots, a_m\rangle\langle b_1, b_2, \cdots, b_n\rangle = \langle b_1, b_2, \cdots, b_n\rangle\langle a_1, a_2, \cdots, a_m\rangle = BA.$$

Therefore, A is semipermutable in G. Thus, Lemma 2.3 is proved.

Lemma 2.4 *Let N be a solvable minimal normal subgroup of a $BT-$group. Then N is a cyclic subgroup of prime order.*

Proof. By hypotheses, we can suppose that N is an elementary abelian $p-$group for some $p \in \pi(G)$. Since N is a $p-$subgroup, every subgroup of N is semipermutable in N. It is clear that N is semipermutable in G. Since G is a $BT-$group, it follows that every subgroup of N is semipermtable in G.

Let G_p be a Sylow $p-$subgroup of G. Then $N \leq G_p$ and $N \cap Z(G_p) \neq 1$. Take a subgroup A of $N \cap Z(G_p)$ of order p. Then A is semipermutable in G. Let Q be any $p'-$subgroup of G. Then $AQ = QA$. Since $[Q, A] \leq N \cap AQ = A$, we have that $Q \leq N_G(A)$. Hence $O^p(G) \leq N_G(A)$. Since $A \leq Z(G_p)$, it follows that $A \trianglelefteq G$. By the minimality of N, we have that $N = A$ is a cyclic subgroup of order p.

Lemma 2.5 *If G is a solvable $BT-$group, then G is supersolvable.*

Proof. Let N be a minimal normal subgroup of G. By Lemma 2.1, we have that G/N is a $BT-$group. Hence G/N is supersolvable by the induction on the order of G. From Lemma 2.4, we know that N is a cyclic subgroup of prime order. Hence G is supersolvable.

Lemma 2.6 *Suppose that G is a group. Then G is supersolvable if every cyclic subgroup of G of prime order or order 4 is S-semipermutable in G.*

Proof. See [14, Lemma 5].

Lemma 2.7 *Suppose that G is a group and P is a normal p-subgroup of G for some $p \in \pi(G)$. If $G/C_G(P)$ is a p-group, then $P \leq Z_\infty(G)$.*

Proof: See [10, Theorem VI. 3]].

3 The structure of $BT-$group or BST-group

Theorem 3.1 *Let G be a group. The following statements are equivalent:*

(1) G is a solvable BT-group;

(2) G is a solvable SBT-group;

(3) every subgroup of G of prime power order is semipermutable in G;

(4) every subgroup of G is semipermutable in G;

(5) every subgroup of G is S- semipermutable in G;

(6) every subgroup of G of prime power order is S-semipermutable in G;

(7) there exists an abelian normal Hall subgroup L of G of odd order such that G/L is nilpotent and the elements of G induce power automorphisms in L. Moreover, for any two distinct primes $p, q \notin \pi(L)$, $[G_p, G_q] = 1$.

Proof: We get the theorem by proving (1)⇒ (3) ⇒ (4) ⇒ (5) ⇒ (6) ⇒ (7) ⇒ (4) ⇒ (2) ⇒ (3) and (4) ⇒ (1).

(1)⇒ (3) Suppose that $\pi(G) = \{p_1, p_2, \cdots, p_n\}$, where $p_1 < p_2 < \cdots < p_n$. Applying Lemma 2.5, we have that the Sylow p_n-subgroup G_{p_n} of G is normal in G. Therefore, G_{p_n} is semipermutable in G. Clearly, every subgroup of G_{p_n} is semipermutable in G_{p_n}. Hence, every subgroup of G_{p_n} is semipermutable in G as G is a BT-group.

Again by Lemma 2.5, $G_{p_n}G_{p_{n-1}} \trianglelefteq G$, where $G_{p_{n-1}}$ is a Sylow $p_{n-1}-$subgroup of G. Since every subgroup of P_n is semipermutable in G, we have that every subgroup of P_{n-1} is semipermutable in $P_{n-1}P_n$. Hence every subgroup of P_{n-1} is semipermutable in G as G is a BT-group.

Continuing these procedure, we can obtain that every subgroup of G_{p_i} is semipermutable in G, where $G_{p_i} \in \mathrm{Syl}_{p_i}(G)$, $i = 1, 2, \cdots, n$. Therefore every subgroup of G of prime power order is semipermutable in G.

(3)⇒(4). Let $H \leq G$. Since G is solvable, we can choose a Sylow base $\{H_{p_1}, H_{p_2}, \cdots, H_{p_s}\}$ of H. Then $H = H_{p_1}H_{p_2}\cdots H_{p_s}$. By the hypotheses, every H_{p_i} is semipermutable in G. Therefore, H is semipermutable in G.

(4) ⇒ (5) and (5) ⇒ (6) are obvious.

(6) ⇒ (7) Suppose that every subgroup of G of prime order is semipermutable in G. We prove (7) in following steps:

(i) G is supersolvable.

By hypotheses, we know that every cyclic subgroup of G of prime order or order 4 is S-semipermutable. By Lemma 2.6, we have that G is supersolvable.

(ii) If N is a normal $p-$subgroup of G, then $p'-$elements of G induce power automorphisms in N.

Take any $a \in N$. Let x be a $p'-$element of G. Then $x \in G_{p'}$ for some

p'-Hall subgroup of G. Then $\langle a\rangle G_{p'}$ is a group. Hence

$$a^{\langle x\rangle} = a^{\langle x\rangle} \cap \langle a\rangle G_{p'} = \langle a\rangle(a^{\langle x\rangle} \cap G_{p'}) = \langle a\rangle(N \cap G_{p'}) = \langle a\rangle.$$

Therefore x induce a power automorphism in $\langle a\rangle$.

(iii) Let $L = G^{\mathcal{N}}$ be the nilpotent residual of G. Then L is a abelian Hall subgroup of G.

Let p be the largest prime dividing $|G|$, $P \in \mathrm{Syl}_p(G)$. Since G is supersolvable by step (i), we know that $P \trianglelefteq G$. Now, we consider the quotient group G/P. By Lemma 2.1, all subgroups of G/P is semipermutable in G/P. By induction, $(G/P)^{\mathcal{N}} = G^{\mathcal{N}}P/P = LP/P$ is an abelian Hall subgroup of G/P.

(a) If $G/C_G(P)$ is a $p-$group, then $G = PC_G(P)$. By Lemma 2.7, $P \leq Z_\infty(G)$. Let $L_p \in \mathrm{Syl}_p(L)$. Suppose that $L_p \neq 1$, by $L_p \leq P$ and $G = PC_G(P)$, we have $L_p = [L_p, G] = [L_p, PC_G(P)] = [L_p, P] < L_p$, a contradiction. Hence $L_p = 1$ and L is a $p'-$group. Therefore, $L \cong LP/P$ is a normal Hall subgroup of G.

(b) If $G/C_G(P)$ is not a $p-$subgroup, then there exists a $p'-$element x such that $P = [P, x]$ since $G/C_G(P) \lesssim \mathrm{Aut}(P)$. Hence $P = [P, G] \leq L$. By (ii) and Lemma 2.2, we know that $G/C_G(P)$ is nilpotent. So $L \leq C_G(P)$ and $P \leq Z(L)$. Since L/P is a normal abelian Hall subgroup of G/P, it follows that $L = P \times L_{p'}$ is a normal Hall subgroup of G, where $L_{p'}$ is the $p'-$Hall subgroup of L.

(iv) L has odd order.

Let L_2 be a Sylow $2-$subgroup of L. If $L_2 \neq 1$, then all $2'-$elements of G induce power automorphism in L_2 by (ii). Since 2 is the smallest prime dividing $|G|$, then $[L_2, G_{2'}] = 1$, where $G_{2'}$ is a Hall $2'-$Hall subgroup of G. Hence, $L_2 = [L_2, G] = [L_2, G_2] < L_2$, a contradiction, where $G_2 \in \mathrm{Syl}_2(G)$. So, $L_2 = 1$ and L has odd order.

(v) The elements of G induce power automorphisms in L.

This follows from (ii) and (iii).

(vi) For any two distinct primes $p, q \notin \pi(L)$, $[G_p, G_q] = 1$.

By the hypotheses, G_pG_q is a group. Since $G_pG_q \cong G_pG_qL/L \leq G/L$, G_pG_q is nilpotent. Hence $[G_p, G_q] = 1$.

(vii) $\Rightarrow$ (iv) we prove that every subgroup of G is semipermutable in G. By Lemma 2.3, we only need to prove $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$, for any $p-$element x and $q-$element y in G.

If $p, q \in \pi(L)$, since L is a normal abelian Hall subgroup of G, we have that $x, y \in L$ and $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$.

If $p, q \notin \pi(L)$, by hypotheses, $[x, y] = 1$. Hence $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$.

Suppose that $p \in \pi(L)$ or $q \in \pi(L)$. Without lose generality, let $p \in \pi(L)$. Then $x \in L$ and $\langle x\rangle \trianglelefteq G$. Hence $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$.

(4) ⇒ (2) and (4) ⇒ (1) Obviously.

(2) ⇒ (6) Repeating the argument in (1) ⇒ (3).

Hence we get the equivalence of (1)-(7).

Corollary 3.2 Every solvable BT-group is a subgroup closed and quotient closed BT-group.

From Theorem 3.1 and the structure of PST-group [1], we can obtain the following result:

Corollary 3.3 *If* G *is a finite solvable* $BT-$*group, then* G *is a* $PST-$*group.*

The following example indicates that BT-group is a proper subclass of PST-group.

Example. Let $G = \langle a\rangle \rtimes (\langle x\rangle \times \langle y\rangle)$, where $a^7 = x^2 = y^3 = 1, a^x = a^{-1}, a^y = a^4$. Since the nilpotent residual L of G is $\langle a\rangle$. By Agrawal's theorem [1], we know that G is a $PST-$group. However, since $\langle x^a\rangle\langle y\rangle \neq \langle y\rangle\langle x^a\rangle$, by Theorem 3.1, we know that G is not a $BT-$group.

The following result tells us under certain conditions $BT-$groups and $PST-$groups may coincide.

Corollary 3.4 *Let* L *be the nilpotent residual of a solvable group* G*. If* $\pi(G)/\pi(L) = \{p\}$*, then* G *is a* $BT-$*group if and only if* G *is a* $PST-$*group.*

4 Minimal non-$BT-$group

In this section, we use the Theorem 3.1 to determine the structure of minimal non-$BT-$group. First, we give the following result:

Theorem 4.1 *Let* G *be a minimal non-BT$-$group. Then*
(i) G *is solvable;*
(ii) G *has a normal Sylow subgroup;*
(iii) $|\pi(G)| = 2$ *or* 3.

Proof. Assume that G is insolvable. Then G contains a minimal insolvable subgroup G_1. Since G is a minimal non-$BT-$group, it follows that each proper subgroup of G_1 is a solvable $BT-$group. By Lemma 2.5, each proper subgroup of G_1 is supersolvable. Hence G_1 is a solvable, a contradiction. Therefore, G is solvable. (i) is proved.

If G is supersolvable, then G has a normal Sylow subgroup. If G is nonsupersolvable, then G is a minimal nonsupersolvable group by Theorem 3.1. Applying a result of Doerk [3], G has a normal Sylow subgroup. Hence (ii) is proved.

Denote the normal $p-$Sylow subgroup of G by P.

Since G is a minimal non-$BT-$group, from Theorem 3.1 and Lemma 2.3 there exist elements x, y of G of prime power order such that $(o(x), o(y)) = 1$ and $\langle x\rangle\langle y\rangle \neq \langle y\rangle\langle x\rangle$. By the minimality of G and without losing generality, we can suppose that $G = \langle x, y\rangle$.

If one of x, y is a $p-$element, x say, then $x \in P$. This implies that $G = \langle x, y\rangle = P\langle y\rangle$. Hence $|\pi(G)| = 2$.

Therefore assume that both x and y are not $p-$elements. Let H be a $p'-$Hall subgroup of G. Then $G/P \cong H$ is an $BT-$group. By Theorem 3.1, each subgroup of G/P is semipermutable. This implies that $(\langle x\rangle P/P) \cdot (\langle y\rangle P/P) = (\langle y\rangle P/P) \cdot (\langle x\rangle P/P)$. Since $G = \langle x, y\rangle$, it follows that $G = \langle x\rangle\langle y\rangle P$. Hence $|\pi(G)| = 3$. (iii) is proved.

Next, we study the structure of the minimal non-$BT-$group G.

Theorem 4.2 *Suppose G is a minimal non-$BT-$group.*

(1) If $|\pi(G)| = 2$, then G is a minimal non-$PST-$group.

(2) If $|\pi(G)| = 3$, then $G = P \rtimes (Q \times R)$, where P is a normal abelian $p-$Sylow subgroup (p is odd), $Q = \langle a\rangle, R = \langle b\rangle$ are cyclic $q-$Sylow subgroup and $r-$Sylow subgroup, respectively. The elements of G induce power automorphisms in P, and for any $g \in G$, $[a^q, b^g] = 1$, $[a^g, b^r] = 1$.

Proof. (1) Suppose that $|\pi(G)| = 2$. By Corollary 3.3, we have that each proper subgroup of G is a $PST-$group. Clearly, the nilpotent residual $G^{\mathcal{N}}$ of G is nontrivial. So, if G is a $PST-$group, then by Corollary 3.4, G is an $BT-$group, which is a contradiction. Hence, G is a minimal non-$PST-$group.

By the proof of Theorem 4.1, we have that $G = \langle x, y\rangle = P\langle x\rangle\langle y\rangle$, where P is the normal $p-$Sylow subgroup, $\langle x\rangle, \langle y\rangle$ are cyclic $q-$Sylow subgroup and $r-$Sylow subgroup, respectively, for some prime $p, q, r \in \pi(G)$ and $\langle x\rangle$ and $\langle y\rangle$ don't permute.

Since G is a minimal non-$BT-$group, it follows from Corollary 3.3 that each proper subgroup of G is $PST-$group. By [9, Theorem 1], we have that G is a $PST-$group. Let $L = G^{\mathcal{N}}$ be the nilpotent residual of G. By [1], L is an abelian normal Hall subgroup of odd order and the elements of G induce power automorphisms in L.

Suppose that $q||L|$. Since L is a Hall subgroup of G, we have that $\langle x\rangle \leq L$. So $\langle x\rangle \trianglelefteq G$ and $G = \langle x, y\rangle = \langle x\rangle\langle y\rangle$, a contradiction. Hence $q \nmid |L|$. Similarly, $r \nmid |L|$.

If $L = 1$, then G is nilpotent and G is an $BT-$group, a contradiction.

Hence $L \neq 1$. Since $q \nmid |L|$, $r \nmid |L|$ and L is a Hall subgroup of G, it follows that $L = P$ is abelian and the elements of G induce power automorphisms in P.

Let H be a $p'-$Hall subgroup of G such that $\langle x\rangle \leq H$. Then there exists $g_0 \in G$ such that $\langle y\rangle^{g_0} \leq H$. Clearly, $H = \langle x\rangle\langle y\rangle^{g_0}$. Since $G/P \cong H$ is nilpotent, $H = \langle x\rangle \times \langle y\rangle^{g_0}$. Let $x = a$, $y^{g_0} = b$, $Q = \langle a\rangle$, $R = \langle b\rangle$, then $G = P \rtimes (Q \times R)$.

Let $M_1 = P\langle a^q\rangle\langle b\rangle$ and $M_2 = P\langle a\rangle\langle b^r\rangle$, then M_1, M_2 are proper normal subgroups of G. So M_1, M_2 are $BT-$groups. By $L = P$, we have that the nilpotent residual $M_i^{\mathcal{N}} \leq P$, $i = 1, 2$. Since M_i is an $BT-$group, $M_i^{\mathcal{N}}$ is Hall subgroup of M_i. Hence $M_i^{\mathcal{N}} = 1$ or P. If $M_1^{\mathcal{N}} = 1$, then M_1 is nilpotent and $\langle b\rangle$ is the normal $q-$subgroup of M_1 and $\langle b\rangle \trianglelefteq G$, then this will imply that $\langle x\rangle$ and $\langle y\rangle$ permute, a contradiction. Hence $M_1^{\mathcal{N}} \neq 1$. Similarly, $M_2^{\mathcal{N}} \neq 1$. So $M_1^{\mathcal{N}} = M_2^{\mathcal{N}} = P$. By $M_1 \trianglelefteq G$, $M_2 \trianglelefteq G$, we have that $\langle b^g\rangle \leq M_1$, $\langle a^g\rangle \leq M_2$ for any $g \in G$. Applying Theorem 3.1, $[a^q, b^g] = 1$ and $[a^g, b^r] = 1$. The proof is complete.

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