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α-Ideals and Prime Ideals in Almost Distributive Lattices

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Abstract

The sufficient conditions for a prime ideal in R to become an α -ideal are studied. The concept of weakly disjunctive ADL is introduced and characterized in terms of it's α -ideals and prime ideals. A conditional characterization theorem of a $\star$ -ADL is proved.

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1 Introduction

In [7], Swamy. U. M and Rao. G. C introduced the concept of an Almost Distributive Lattice(ADL) while giving an algebraic characterization for sheaves of sets over locally Boolean spaces. Later in the paper [8], Swamy. U. M., Rao. G. C. and Nanaji Rao. G introduced the concept of a $\star$ -ADL and characterized $\star$ -ADL by means of its dense elements in [9]. In the paper [4], the concept of α -ideals in an ADL is introduced, analogous to that in a distributive lattice given in [2] and proved that the set $\mathcal{I}_\alpha(R)$ of all α -ideals in an ADL R with 0 can be made into a complete distributive lattice. In this paper, we observe that a prime ideal in an ADL need not be an α -ideal in general. We give some sufficient conditions for a prime ideal of R to become an α -ideal. We also introduce the concept of a weakly disjunctive ADL and prove the necessary and sufficient conditions for a prime ideal of R to become an α -ideal, as a characterization of a weakly disjunctive ADL. W. H. Cornish[2], observed that every annihilator ideal in a distributive lattice is an α -ideal, whose converse need not be true. Finally, we prove some equivalent conditions for every α -ideal in an ADL R to become an annihilator ideal which lead to a conditional characterization of a $\star$ -ADL.

2 Preliminary Notes

First we give the necessary definitions and results taken mostly from [6] and [7] which will be used in the later text.

Definition 2.1:[7] *An algebra $(R, \vee, \wedge, 0)$ of type (2, 2, 0) is called an Almost Distributive Lattice (ADL) with 0 if it satisfies the following axioms:*

1. $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$
2. $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
3. $(a \vee b) \wedge b = b$
4. $(a \vee b) \wedge a = a$
5. $a \vee (a \wedge b) = a$
6. $0 \wedge a = 0$ for any $a, b, c \in R$

Every non-empty set X can be regarded as an ADL as follows. Let $x_0 \in X$. Define two binary operations $\vee, \wedge$ on X by

$$x \vee y = \begin{cases} x & \text{if } x \neq x_0 \\ y & \text{if } x = x_0 \end{cases} \qquad x \wedge y = \begin{cases} y & \text{if } x \neq x_0 \\ x_0 & \text{if } x = x_0 \end{cases}$$

Then $(X, \vee, \wedge, x_0)$ is an ADL which is called a discrete ADL with zero x_0.

We define a binary relation on R by $a \leq b$ if and only if $a = a \wedge b$ (or equivalently, $a \vee b = b$), then $\leq$ is a partial ordering on R. Throughout this paper R stands for an ADL $(R, \vee, \wedge, 0)$ with zero.

Theorem 2.2:[7] *For any $a, b, c \in R$, we have the following*

1. $a \vee b = a \Leftrightarrow a \wedge b = b$
2. $a \vee b = b \Leftrightarrow a \wedge b = a$
3. $a \wedge b = b \wedge a$ *whenever* $a \leq b$
4. $\wedge$ *is associative in* R.
5. $a \wedge b \wedge c = b \wedge a \wedge c$
6. $(a \vee b) \wedge c = (b \vee a) \wedge c$
7. $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$
8. $a \vee b = b \vee a$ *whenever* $a \wedge b = 0$

9. $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
10. $a \wedge (a \vee b) = a, (a \wedge b) \vee b = b,$ *and* $a \vee (b \wedge a) = a$
11. $a \leq a \vee b$ *and* $a \wedge b \leq b$
12. $a \wedge a = a$ *and* $a \vee a = a$
13. $0 \vee a = a$ *and* $a \wedge 0 = 0$
14. *If* $a \leq c$ *and* $b \leq c$ *then* $a \wedge b = b \wedge a$ *and* $a \vee b = b \vee a$
15. $a \vee b = a \vee b \vee a.$

A non-empty subset I of R is called an ideal(filter)of R if $a \vee b \in I (a \wedge b \in I)$ and $a \wedge x \in I (x \vee a \in I)$ whenever $a, b \in I$ and $x \in R$. If I is an ideal of R and $a, b \in R$, then $a \wedge b \in I \Leftrightarrow b \wedge a \in I$. The set $\mathcal{I}(R)$ of all ideals of R is a complete distributive lattice with least element $\{0\}$ and the greatest element R under set inclusion in which, for any $I, J \in \mathcal{I}(R), I \cap J$ is the infimum of I, J and the supremum is given by $I \vee J = \{ i \vee j \mid i \in I, j \in J \}$. $\mathcal{I}(R)$ is called semi-complemented if for every $R \neq I \in \mathcal{I}(R)$, there exists an ideal $J \neq (0]$ such that $I \cap J = (0]$. For any $a \in R$, $(a] = \{ a \wedge x \mid x \in R \}$ is the principal ideal generated by a. Similarly, $[a) = \{ x \vee a \mid x \in R \}$ is the principal filter generated by a. The set $\mathcal{PI}(R)$ of all principal ideals of R is a sublattice of $\mathcal{I}(R)$. A proper ideal P of R is said to be prime if for any $x, y \in R$, $x \wedge y \in P \Rightarrow$ either $x \in P$ or $y \in P$. It is clear that a subset P of R is a prime ideal if and only if $R - P$ is a prime filter. A prime ideal P of R is called minimal if there exists no prime ideal Q of R such that $Q \subset P$.

Theorem 2.4:[7] *If* I *is an ideal and* F *a filter in an ADL* R *such that* $I \cap F = \emptyset$*, then there exists a prime ideal* P *of* R *such that* $I \subseteq P$ *and* $P \cap F = \emptyset$.

Theorem 2.5:[5] *Let* P *be a prime ideal of* R *with 0. Then* P *is minimal if and only if for each* $x \in P$ *there exists* $y \notin P$ *such that* $x \wedge y = 0$.

For any $A \subseteq R$, $A^* = \{x \in R / a \wedge x = 0 \text{ for all } a \in A\}$ is an ideal of R. We write $(a]^*$ for $\{a\}^*$ and this is called an annulet in R. Clearly $(0]^* = R$ and $R^* = (0]$. Now we recall some more definitions and results from [3], [4] and [8].

Lemma 2.6:[3] *For any non-empty subsets* I*,* J *of* R*, we have*

1). $I \subseteq J \Rightarrow J^* \subseteq I^*$.
2). $I \subseteq I^{**}$
3). $I^{***} = I^*$
4). $I \cap J = (0] \Leftrightarrow I \subseteq J^*$
5). $(I \vee J)^* = I^* \cap J^*$.

An ideal I of R is called an annihilator ideal if $I = I^{**}$ or equivalently, $I = S^*$ for some non-empty subset S of R. The set of $\mathcal{A}(R)$ of all annihilator ideals of R forms a complete Boolean algebra with bounds $\{0\}, R$ and the complement of any $I \in \mathcal{A}(R)$ is I^* with respect to the operations $\wedge$ and $\underline{\vee}$ given by $I \wedge J = I \cap J$ and $I \,\underline{\vee}\, J = (I^* \cap J^*)^*$.

An ADL R with 0 is called disjunctive iff for all $a, b \in R$, $(a]^* = (b]^*$ implies that $a = b$. An element $a \in R$ is called dense if $(a]^* = (0]$ and the set all dense elements of R is denoted by D. Then D is a filter, whenever $D \neq \emptyset$. An ADL R with 0 is called a $\star$-ADL [8], if to each $x \in R$, $(x]^{**} = (x']^*$ for some $x' \in R$. An ADL R with 0 is a $\star$-ADL iff to each $x \in R$, there exists $y \in R$ such that $x \wedge y = 0$ and $x \vee y \in D$. Every $\star$-ADL possesses a dense element.

An ideal I of an ADL R is called an α-ideal [4] if $(x]^{**} \subseteq I$, for any $x \in I$. For any ideal I of R, the set $I^e = \{\ x \in R \mid (a]^* \subseteq (x]^* \text{ for some } a \in I\ \}$ is the smallest α-ideal of R such that $I \subseteq I^e$.

Lemma 2.7:[4] *Let R be an ADL with 0. Then for any ideals I, J of R, we have the following :*

(a). $I \subseteq J \Rightarrow I^e \subseteq J^e$
(b). $(I \cap J)^e = I^e \cap J^e$
(c). $(I^e)^e = I^e$.

The set $I_\alpha(R)$ of all α-ideals of R forms a complete distributive lattice with respect to the operations $\wedge$ and $\tilde{\vee}$ given by $I \wedge J = I \cap J$ and $I \,\tilde{\vee}\, J = (I \vee J)^e$.

Theorem 2.8:[4] *Let R be an ADL with 0. Then for any ideal I of R, I is an α-ideal if and only if for $x, y \in R, (x]^* = (y]^*$ and $x \in I$ imply that $y \in I$.*

3 α -ideals and Prime ideals

In this section we study the properties of prime ideals and α-ideals in an ADL R. We derive some conditions for prime ideals to become α-ideals. Finally, we characterize weakly disjunctive ADL and $\star$-ADL in terms of α-ideals.

We start this section with the following theorem.

Theorem 3.1: *If I is an α-ideal and F a filter of an ADL R with 0 such that $I \cap F = \emptyset$, then there exists an α-ideal P such that $I \subseteq P$ and $P \cap F = \emptyset$. Moreover P is prime.*

Proof: Let I be an α-ideal and F a filter of R such that $I \cap F = \emptyset$.
Take $\Sigma = \{\ J \mid J \text{ is an } \alpha\text{-ideal}, I \subseteq J \text{ and } J \cap F = \emptyset\ \}$. Then Σ satisfies the hypothsis of zorn's lemma. Let P be a maximal element of Σ. Choose $a, b \in R$ such that $a \notin P$ and $b \notin P$. Then $P \subset P \vee (a] \subseteq \{P \vee (a]\}^e$ and $P \subset P \vee (b] \subseteq \{P \vee (b]\}^e$. Hence $\{P \vee (a]\}^e$ and $\{P \vee (b]\}^e$ are not in Σ. Thus

$\{P \vee (a]\}^e \cap F \neq \emptyset$ and $\{P \vee (b]\}^e \cap F \neq \emptyset$. So choose $x \in \{P \vee (a]\}^e \cap F$ and $y \in \{P \vee (b]\}^e \cap F$. Now $x \wedge y \in \{P \vee (a]\}^e \cap \{P \vee (b]\}^e = \{P \vee (a \wedge b]\}^e$.
If $a \wedge b \in P$, then $x \wedge y \in P^e = P$, because of P is an α-ideal. Which is a controdiction, because of $x \wedge y \in F$. Thus P is prime. $\square$

Definition 3.2: *Let R be an ADL with 0. An ideal I of R is called dense if $I^* = (0]$. Otherwise, I is called a non-dense ideal.*
Let us denote the set of all dense ideals of R by $\mathcal{DI}(R)$.

We now prove some sufficient conditions for a prime ideal of an ADL R to become an α-ideal. First we note that a prime ideal in general, need not be an α-ideal. For, consider the following example.

Example 3.3: *Let $R = \{0, a, b, c\}$ and define $\vee$ and $\wedge$ on R as follows:*

$\vee$	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	b	b	b
c	c	a	b	c

$\wedge$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
c	0	c	c	c

*Then clearly $(R, \vee, \wedge, 0)$ is an ADL with 0. Consider the ideal $P = \{0, c\}$.
Now, we make the following observations.
First, P is a prime ideal of R and $P^* = (0]$. So P is a dense prime ideal.
Second, P is not a minimal prime ideal, because $\{0\}$ is also a prime ideal, and third, R is a $\star$-ADL and c is a dense element in P.
But P is not an α-ideal in R, because $(c]^{**} = (0]^* = R$.*

However, the following theorem explains the reasons for why the prime ideal P in the above example is not an α-ideal.

Theorem 3.4: *Let R be an ADL with 0 and D be the set of all dense elements of R. Then we have*

(a). Every non-dense prime ideal of R is an α-ideal
(b). Every minimal prime ideal of R is an α-ideal
(c). If R is a $\star$-ADL, then every prime ideal P of R with $P \cap D = \emptyset$ is an α-ideal.

Proof: (a). Let P be a non-dense prime ideal of R. So, there exists an element $0 \neq x \in R$ such that $x \in P^*$. Hence $P \subseteq P^{**} \subseteq (x]^*$. Now, let $a \in (x]^*$. Then $a \wedge x = 0 \in P$. Since $x \in P^*$ and $x \neq 0$, we get that $a \in P$. Hence $(x]^* \subseteq P$. Thus $P = (x]^*$. Now, If $a \in P = (x]^*$, then clearly $(a]^{**} \subseteq (x]^{***} = (x]^* = P$. Therefore P is an α-ideal of R.

(b). Let P be a minimal prime ideal of R. Let $x, y \in R$ such that $(x]^* = (y]^*$ and $x \in P$. Suppose $y \notin P$. Then $(y]^* \subseteq P$. Hence $(x]^* \subseteq P$ which implies that $x \notin P$. Which is a controdiction. Hence $y \in P$. Therefore P is an α-ideal.
(c). Let R be a $\star$-ADL and P a prime ideal of R such that $P \cap D = \emptyset$. Now, let $x \in P$. Since R is a $\star$-ADL, there exists $y \in R$ such that $x \wedge y = 0$ and $x \vee y \in D$. Since $P \cap D = \emptyset$, we get $x \vee y \notin P$. So $x \notin P$ and $y \notin P$. That is, to each $x \in P$, there exists $y \notin P$ such that $x \wedge y = 0$. Hence P is a minimal prime ideal. Therefore by (b), P is an α-ideal of R. □

We now introduce the concept of weakly disjunctive ADL in the following.

Definition 3.5: *An ADL R with 0 is called a weakly disjunctive ADL if and only if for all $x, y \in R$, $(x]^* = (y]^*$ implies that $(x] = (y]$.*

Example 3.6: *Let $A = \{0, a\}$ and $B = \{0, b_1, b_2\}$ be two discrete ADLs. Write $R = A \times B = \{(0,0), (0,b_1), (0,b_2), (a,0), (a,b_1), (a,b_2)\}$. Then $(R, \vee, \wedge, 0')$ is an ADL where $0' = (0,0)$, under point-wise operations.*
We have $((0,b_1)]^* = ((0,b_2)]^* = \{(0,0), (a,0)\}$ and
$((0,b_1)] = ((0,b_2)] = \{(0,0), (0,b_1), (0,b_2)\}$.
Also $((a,b_1)]^* = ((a,b_2)]^* = \{(0,0)\}$ and $((a,b_1)] = ((a,b_2)] = R$.
Therefore R is a weakly disjunctive ADL.

It can be easily observed that every disjunctive ADL is weakly disjunctive. But the converse is not true. For, the ADL R in the above example is weakly disjunctive but not a disjunctive ADL.

We now characterize a weakly disjunctive ADL in terms of it's α-ideals and prime ideals.

Theorem 3.7: *Let R be an ADL with 0. Then the following conditions are equivalent:*

(a). R is weakly disjunctive.
(b). Each ideal is an α-ideal.
(c). Each prime ideal is an α-ideal.

Proof: $(a) \Rightarrow (b)$: Assume that R is weakly disjunctive. Let I be an ideal of R. Suppose $x, y \in R$ such that $(x]^* = (y]^*$. Hence we get that $(x] = (y]$. Suppose $x \in I$. Then $y \in (y] = (x] \subseteq I$. Therefore I is an α-ideal.
$(b) \Rightarrow (c)$: It is trivial.
$(c) \Rightarrow (a)$: Assume that every prime ideal of R is an α-ideal.
Let $x, y \in R$ such that $(x] \neq (y]$. Without loss of generality assume that $(x] \nsubseteq (y]$. Let $\Sigma = \{\ I \in I(R) \mid x \wedge y \in I \text{ and } x \notin I\ \}$. Clearly $(x \wedge y] \in \Sigma$. Let P be a maximal element of Σ. Let $a, b \in R$ with $a \notin P$ and $b \notin P$. Therefore

by the maximality of P, $P \vee (a]$ and $P \vee (b]$ are not in Σ. Hence $x \in P \vee (a]$ and $x \in P \vee (b]$. So $x \in \{P \vee (a]\} \cap \{P \vee (b]\} = P \vee (a \wedge b]$. If $a \wedge b \in P$, then $x \in P \vee (a \wedge b] = P$. Which is a controdiction. So $a \wedge b \notin P$. Hence P is a prime ideal. Thus by the hypothesis, P is an α-ideal. Now $x \wedge y \in P$ implies that $y \in P$ because of $x \notin P$. Now suppose $(x]^* = (y]^*$. Since $y \in P$ and P is an α-ideal, we get that $x \in P$. Which is a controdiction. Hence $(x]^* \neq (y]^*$. Thus R is weakly disjunctive. □

We now define relative annihilators in an ADL R, which lead to another important characterization of α-ideals.

Definition 3.8: *Let A be a non-empty subset and J an ideal of R. Then define the annihilator of A relative to J as follows*

$$\langle A, J\rangle = \{x \in R / x \wedge a \in J, \ \textit{for all } a \in A\}.$$

If $J = \{0\}$, then we get that $\langle A, \{0\}\rangle = A^*$.

Theorem 3.9: *Let R be an ADL with 0. If J is an α-ideal of R, so is $\langle A, J\rangle$ for any subset A of R.*

Proof: Let J be an α-ideal of R. We have clearly $\langle A, J\rangle$ is an ideal of R. Now, let $x, y \in R$ such that $(x]^* = (y]^*$ and $x \in \langle A, J\rangle$.

$$\begin{aligned}
\text{Now } x \in \langle A, J\rangle \ &\Rightarrow\ x \wedge a \in J \quad \text{for all } a \in A \\
&\Rightarrow\ (x]^{**} \cap (a]^{**} = (x \wedge a]^{**} \subseteq J \quad \text{for all } a \in A \\
&\Rightarrow\ (y]^{**} \cap (a]^{**} \subseteq J \quad \text{for all } a \in A \\
&\Rightarrow\ y \wedge a \in (y \wedge a]^{**} \subseteq J \quad \text{for all } a \in A \\
&\Rightarrow\ y \in \langle A, J\rangle \quad \text{for all } a \in A
\end{aligned}$$

Hence $\langle A, J\rangle$ is an α-ideal of R. □

Corollary 3.10: *Every annihilator ideal is an α-ideal.*

The converse of the above corollary is not true. That is every α-ideal of R need not be an annihilator ideal. For example, a proper dense α-ideal is not an annihilator ideal. However, we now prove in the following theorem, some equivalent conditions for each α-ideal to be an annihilator ideal. Recall that D is the set of all dense elements of R and $\mathcal{DI}(R)$ is the set of all dense ideals of R.

Theorem 3.11: *Let R be an ADL with 0 in which $D \neq \emptyset$. Then the following conditions are equivalent:*

(1). *R is $\star$-ADL and $I^* \neq (0]$ for each proper α-ideal I of R*
(2). *$I \cap D \neq \emptyset$ for each $I \in \mathcal{DI}(R)$*
(3). *Every α-ideal is an annihilator ideal*

(4). $\mathcal{I}_\alpha(R)$ *is semicomplemented*
(5). $\mathcal{I}_\alpha(R)$ *has a unique dense element.*

Proof: (1) $\Rightarrow$ (2): Assume the condition (1). Let $I \in \mathcal{DI}(R)$. Suppose $I \cap D = \emptyset$. Then by theorem 2.4, there exists a prime ideal P such that $I \subseteq P$ and $P \cap D = \emptyset$. Suppose $x, y \in R$ such that $(x]^* = (y]^*$. Assume $x \in P$. Since R is $\star$-ADL, there exists $x' \in R$ such that $x \wedge x' = 0$ and $x \vee x' \in D$. Hence $x' \in (x]^* = (y]^*$ and $x \vee x' \notin P$. Since $x \in P$, we get $x' \notin P$. Hence $x' \wedge y = 0 \in P$ and $x' \notin P$. So y must be in P. Thus P is a proper α-ideal. Therefore by (1), $P^* \neq (0]$. But $I \subseteq P$ implies that $P^* \subseteq I^* = (0]$. Which is a controdiction. Therefore $I \cap D \neq \emptyset$, for each $I \in \mathcal{DI}(R)$.
(2) $\Rightarrow$ (3): Assume the condition (2). Let I be an α-ideal of R. Always we have $I \subseteq I^{**}$. Let $x \in I^{**}$. Since $I \vee I^*$ is a dense ideal, by (2) it has a dense element, say d. Hence $d = r \vee s$ where $r \in I$ and $s \in I^*$. Thus $(r]^* \cap (s]^* = (r \vee s]^* = (d]^* = (0]$. Therefore $(s]^* \subseteq (r]^{**}$.
Now $x \in I^{**}$ and $s \in I^*$ imply that $x \wedge s = 0$. Hence $x \in (s]^* \subseteq (r]^{**} \subseteq I$, because of $r \in I$ and I is an α-ideal. Hence $I^{**} \subseteq I$. Therefore $I = I^{**}$.
(3) $\Rightarrow$ (4): Assume the condition (3). Let $I \in \mathcal{I}_\alpha(R)$. Then by (3), I is an annihilator ideal of R. Since the set of all annihilator ideals forms a complete Boolean algebra, it follows that $\mathcal{I}_\alpha(R)$ is semicomplemented.
(4) $\Rightarrow$ (5): Assume that $\mathcal{I}_\alpha(R)$ is semicomplemented. Clearly $R \in \mathcal{I}_\alpha(R)$ and R is dense in $\mathcal{I}_\alpha(R)$. Suppose $I(\neq R)$ is a dense element in $\mathcal{I}_\alpha(R)$. By (4), there exists an α-ideal $J \neq (0]$ such that $I \cap J = (0]$. Hence $J \subseteq I^* = (0]$ implies that $J = (0]$. Which is a controdiction. Thus $I_\alpha(R)$ has a unique dense element, precisely R.
(5) $\Rightarrow$ (1): Assume that $\mathcal{I}_\alpha(R)$ has a unique dense element, precisely R.
It is clear that every proper α-ideal is nondense. So it is enough to prove that R is a $\star$-ADL. Let $x \in R$. Clearly $\{(x]^* \vee (x]^{**}\}^e \in I_\alpha(R)$.

$$
\begin{aligned}
\text{Now } (x]^*, (x]^{**} \subseteq \{(x]^* \vee (x]^{**}\}^e &\Rightarrow \{\{(x]^* \vee (x]^{**}\}^e\}^* \subseteq (x]^{**}, (x]^* \\
&\Rightarrow \{\{(x]^* \vee (x]^{**}\}^e\}^* \subseteq (x]^* \cap (x]^{**} = (0]
\end{aligned}
$$

That is $\{(x]^* \vee (x]^{**}\}^e$ is a dense element in $I_\alpha(R)$. Hence by (5), we get that $\{(x]^* \vee (x]^{**}\}^e = R$. Therefore $\{(x]^* \vee (x]^{**}\}^e$ has a dense element, say d.

$$
\begin{aligned}
&\Rightarrow (r \vee s]^* \subseteq (d]^* = (0] \qquad \text{for some } r \in (x]^* \text{ and } s \in (x]^{**} \\
&\Rightarrow (r]^* \cap (s]^* = (r \vee s]^* \subseteq (0] \\
&\Rightarrow (r]^* \subseteq (s]^{**} \subseteq (x]^{**}, \text{ since } s \in (x]^{**}
\end{aligned}
$$

Again, since $r \in (x]^*$, we get that $(x]^{**} \subseteq (r]^*$.
Hence $(x]^{**} = (r]^*$. Therefore R is a $\star$-ADL. □

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