

# A Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space by Using Sub-Compatible Maps

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**Abstract:** In this paper, we introduce the new concepts of subcompatibility and subsequencial continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four maps. Our results extend and intuitionistic fuzzify the results of [4].

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**Keywords:** Intuitionistic fuzzy metric space, Subcompatibility and Subsequencial continuity, common fixed point theorem

## 1. Introduction

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [14]. In 2004, Park [9] defined the notion of intuitionistic fuzzy metric space with the help of continuous  $t$ -norms and continuous  $t$ -conorms. Recently, in 2006, Alaca et al.[1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous  $t$ -norm and continuous  $t$ -conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [6]. Further, Alaca et al.[1] proved Intuitionistic fuzzy Banach and Intuitionistic

fuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [9].

In this paper, we introduce the new concepts of subcompatibility and subsequential continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four maps. Our results extend and intuitionistic fuzzify the results of [4].

## 2. Preliminaries

**Definition 2.1 [10].** A binary operation  $*$  :  $[0,1] \times [0,1] \rightarrow [0,1]$  is a continuous  $t$ -norm if  $*$  is satisfying the following conditions:

- (a)  $*$  is commutative and associative;
- (b)  $*$  is continuous;
- (c)  $a * 1 = a$  for all  $a \in [0, 1]$ ;
- (d)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0,1]$ .

Examples of  $t$ -norm are  $a * b = \min\{a, b\}$  and  $a * b = ab$ .

**Definition 2.2 [10].** A binary operation  $\diamond$  :  $[0,1] \times [0,1] \rightarrow [0,1]$  is a continuous  $t$ -conorm if  $\diamond$  is satisfying the following conditions:

- (a)  $\diamond$  is commutative and associative;
- (b)  $\diamond$  is continuous;
- (c)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ;
- (d)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0,1]$ .

Examples of  $t$ -conorm are  $a \diamond b = \max\{a, b\}$  and  $a \diamond b = \min\{1, a + b\}$ .

Alaca et al. [1] defined the notion of intuitionistic fuzzy metric space as follows:

**Definition 2.3[1].** A 5-tuple  $(X, M, N, *, \diamond)$  is said to be an IFM-space if  $X$  is an arbitrary set,  $*$  is a continuous  $t$ -norm,  $\diamond$  is a continuous  $t$ -conorm and  $M, N$  are fuzzy sets on  $X^2 \times [0, \infty)$  satisfying the following conditions: for all  $x, y, z \in X$  and  $t, s > 0$ ,

- (i)  $M(x, y, t) + N(x, y, t) \leq 1$ ;
- (ii)  $M(x, y, 0) = 0$ ;
- (iii)  $M(x, y, t) = 1$  if and only if  $x = y$ ;
- (iv)  $M(x, y, t) = M(y, x, t)$ ;
- (v)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ ;
- (vi)  $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is left continuous;
- (vii)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ ;
- (viii)  $N(x, y, 0) = 1$ ;
- (ix)  $N(x, y, t) = 0$  if and only if  $x = y$ ;

- (x)  $N(x, y, t) = N(y, x, t)$ ;
- (xi)  $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ ;
- (xii)  $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is right continuous;
- (xiii)  $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ .

Then  $(M, N)$  is called an intuitionistic fuzzy metric on  $X$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and the degree of non-nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

**Remark 2.1.** Every fuzzy metric space  $(X, M, *)$  is an IFM-space of the form  $(X, M, 1 - M, *, \diamond)$  such that  $t$ -norm  $*$  and  $t$ -conorm  $\diamond$  are associated, i.e.  $x \diamond y = 1 - ((1 - x) * (1 - y))$  for any  $x, y \in X$ .

**Example 2.1.** Let  $(X, d)$  be a metric space. Define  $t$ -norm  $a * b = \min\{a, b\}$  and  $t$ -conorm  $a \diamond b = \max\{a, b\}$  and for all  $x, y \in X$  and  $t > 0$ ,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}.$$

Then  $(X, M, N, *, \diamond)$  is an IFM-space and the intuitionistic fuzzy metric  $(M, N)$  induced by the metric  $d$  is often referred to as the standard intuitionistic fuzzy metric.

**Remark 2.2.** In IFM-space  $(X, M, N, *, \diamond)$ ,  $M(x, y, \cdot)$  is non-decreasing and  $N(x, y, \cdot)$  is non-increasing for all  $x, y \in X$ .

**Definition 2.4 [1].** Let  $(X, M, N, *, \diamond)$  be an IFM-space. Then

- (i) a sequence  $\{x_n\}$  in  $X$  is said to be Cauchy sequence if for all  $t > 0$  and  $p > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1, \quad \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0.$$

- (ii) a sequence  $\{x_n\}$  in  $X$  is said to be convergent to a point  $x \in X$  if for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1, \quad \lim_{n \rightarrow \infty} N(x_n, x, t) = 0.$$

Since  $*$  and  $\diamond$  are continuous, the limit is uniquely determined from (v) and (xi) respectively.

**Definition 2.5 [1].** An IFM-space  $(X, M, N, *, \diamond)$  is said to be complete if and only if every Cauchy sequence in  $X$  is convergent.

**Example 2.2.** Let  $X = \{ \frac{1}{n} : n = 1, 2, 3, \dots \} \cup \{0\}$  and let  $*$  be the continuous  $t$ -norm and  $\diamond$  be the continuous  $t$ -conorm defined by  $a * b = ab$  and  $a \diamond b = \min\{1, a + b\}$  respectively, for all  $a, b \in [0, 1]$ . For each  $t \in (0, \infty)$  and  $x, y \in X$ , define  $(M, N)$  by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}.$$

Clearly,  $(X, M, N, *, \diamond)$  is complete intuitionistic fuzzy metric space.

The following definition of weakly commuting mappings in intuitionistic fuzzy metric space is given on the lines of Sessa [11].

**Definition 2.6[11].** Let  $A$  and  $S$  be maps from an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  into itself. The maps  $A$  and  $S$  are said to be weakly commuting if  $M(ASz, SAz, t) \geq M(Az, Sz, t)$  and  $N(ASz, SAz, t) \leq N(Az, Sz, t)$  for all  $z \in X$  and  $t > 0$ .

**Definition 2.7[13].** Let  $A$  and  $S$  be maps from an IFM-space  $(X, M, N, *, \diamond)$  into itself. The maps  $A$  and  $S$  are said to be compatible if for all  $t > 0$ ,  $\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1$  and  $\lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0$ , whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$  for some  $z \in X$ .

**Definition 2.8[7]:** Two mappings  $A$  and  $S$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  will be called reciprocally continuous if  $ASu_n \rightarrow Az$  and  $SAu_n \rightarrow Sz$ , whenever  $\{u_n\}$  is a sequence such that  $Au_n, Su_n \rightarrow z$  for some  $z$  in  $X$ .

If  $A$  and  $S$  are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings  $A$  and  $S$  implies their reciprocal continuity but not conversely.

**Definition 2.9:** Let  $(X, M, N, *, \diamond)$  be a intuitionistic fuzzy metric space.  $A$  and  $S$  be self maps on  $X$ . A point  $x$  in  $X$  is called a coincidence point of  $A$  and  $S$  iff  $Ax = Sx$ . In this case,  $w = Ax = Sx$  is called a point of coincidence of  $A$  and  $S$ .

**Definition 2.10:** A pair of self mappings  $(A, S)$  of a intuitionistic fuzzy metric space

$(X, M, N, *, \diamond)$  is said to be weakly compatible if they commute at the coincidence points

i.e., if  $Au = Su$  for some  $u$  in  $X$ , then  $ASu = SAu$ .

It is easy to see that two compatible maps are weakly compatible but converse is not true.

**Definition 2.11[2]:** Two self mappings  $A$  and  $S$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  are said to be occasionally weakly compatible (owc) iff there is a point  $x$  in  $X$  which is coincidence point of  $A$  and  $S$  at which  $A$  and  $S$  commute.

In this paper, we weaken the above notion by introducing a new concept called subcompatibility just as defined by H. Bouhadjera[4] in metric space, as follows:

**Definition 2.12:** Let  $(X, M, N, *, \diamond)$  be a intuitionistic fuzzy metric space. Self maps  $A$  and  $S$  on  $X$  are said to be subcompatible iff there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z, z \in X$  and satisfy  $\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1,$   
 $\lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0.$

Obviously, two owc maps are subcompatible, however the converse is not true in general. The example below shows that there exist subcompatible maps which are not owc.

**Example 2.3:** Let  $X = [0, \infty)$ . For each  $t \in (0, \infty)$  and  $x, y \in X$ , define  $(M, N)$  by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0, \\ 0 & t=0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0, \\ 1 & t=0 \end{cases}$$

Define  $A$  and  $S$  as follows:

$$A(x) = x^2, S(x) = \begin{cases} x+2 & \text{if } x \in [0, 4] \cup (9, \infty) \\ x+12 & \text{if } x \in (4, 9] \end{cases}.$$

Let  $\{x_n\}$  be a sequence in  $X$  defined by  $x_n = 2 + \frac{1}{n}$  for  $n = 1, 2, 3, \dots$

Then,  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 4, 4 \in X$  and

$$ASx_n \rightarrow 16, SAx_n \rightarrow 16$$

when  $n \rightarrow \infty$ . Thus,  $\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1, \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0.$

i.e.  $A$  and  $S$  are subcompatible. On the other hand, we have

$Ax = Sx$  iff  $x = 2$  and  $AS(2) \neq SA(2)$ , hence  $A$  and  $S$  are not owc.

Now, our second objective is to introduce subsequential continuity in intuitionistic fuzzy metric space which weaken the concept of reciprocal continuity which was introduced by Pant[8] just as introduced by H. Bouhadjera[4] in metric space, as follows:

**Definition 2.13:** Let  $(X, M, N, *, \diamond)$  be a intuitionistic fuzzy metric space. Self maps  $A$  and  $S$  on  $X$  are said to be subsequentially continuous iff there exist a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t, t \in X$  and satisfy  $\lim_{n \rightarrow \infty} ASx_n = At, \lim_{n \rightarrow \infty} SAx_n = St$ .

Clearly, if  $A$  and  $S$  are continuous or reciprocally continuous then they are obviously subsequentially continuous. The next example shows that there exist subsequential continuous pairs of maps which are neither continuous nor reciprocally continuous.

**Example 2.4:** Let  $X = [0, \infty)$ . For each  $t \in (0, \infty)$  and  $x, y \in X$ , define  $(M, N)$  by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0, \\ 0 & t=0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0, \\ 1 & t=0 \end{cases}$$

Define A and S as follows:

$$A(x) = \begin{cases} 1+x & \text{if } x \in [0,1] \\ 2x-1 & \text{if } x \in (1,\infty) \end{cases}, S(x) = \begin{cases} 1-x & \text{if } x \in [0,1] \\ 3x-2 & \text{if } x \in [1,\infty). \end{cases}$$

Clearly A and S are discontinuous at  $x = 1$ .

Let  $\{x_n\}$  be a sequence in X defined by  $x_n = \frac{1}{n}$  for  $n = 1, 2, 3, \dots$

Then,  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 1, 1 \in X$  and

$ASx_n \rightarrow 2 = A(1), SAx_n \rightarrow 1 = S(1)$  when  $n \rightarrow \infty$ , therefore, A and S are subsequential continuous.

Now, let  $\{x_n\}$  be a sequence in X defined by  $x_n = 1 + \frac{1}{n}$  for  $n = 1, 2, 3, \dots$

Then,  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 1, 1 \in X$  and

$ASx_n \rightarrow 1 \neq 2 = A(1)$  when  $n \rightarrow \infty$ , so A and S are not reciprocally continuous.

**Lemma 2.1** Let  $\{u_n\}$  be a sequence in an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$ . If there exists a constant  $k \in (0,1)$  such that  $M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t)$  and  $N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$ , for all  $t > 0$  and  $n=1,2,3,\dots$

Then  $\{u_n\}$  is a Cauchy sequence in X.

**Lemma 2.2** Let  $(X, M, N, *, \diamond)$  be an IFM-space and for all  $x, y \in X$ ,  $t > 0$  and if for a number  $k \in (0,1)$ ,

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t)$$

then  $x = y$ .

### 3. Main Result

Now, we prove our main theorem using definition of subcompatible and subsequential continuous maps as follows:

**Theorem 3.1:** Let A, B, S and T be four self maps of a Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  with continuous t-norm  $*$  and continuous t-conorm  $\diamond$  defined by  $t * t \geq t$  and  $(1-t)\diamond(1-t) \leq (1-t)$  for all  $t \in [0,1]$ . If the pairs (A, S) and (B, T) are subcompatible and subsequentially continuous, then

- (a) A and S have a coincidence point;
- (b) B and T have a coincidence point.

Further, let for all  $x, y$  in  $X, k \in (0,1), t > 0$

$$(c) \begin{aligned} M(Ax, By, kt) &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t), \\ N(Ax, By, kt) &\leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, 2t) \diamond N(Ax, Ty, t). \end{aligned}$$

Then,  $A, B, S$  and  $T$  have a unique common fixed point.

**Proof:** Since, the pairs  $(A, S)$  and  $(B, T)$  are subcompatible and subsequentially continuous, then, there exists two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z, z \in X \text{ and satisfy}$$

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1, \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = N(Az, Sz, t) = 0;$$

$$\lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z', z' \in X \text{ and which satisfy}$$

$$\lim_{n \rightarrow \infty} M(BTy_n, TBy_n, t) = M(Bz', Tz', t) = 1; \lim_{n \rightarrow \infty} N(BTy_n, TBy_n, t) = N(Bz', Tz', t) = 0.$$

Therefore,  $Az = Sz$  and  $Bz' = Tz'$ ; that is,  $z$  is a coincidence point of  $A$  and  $S$  and  $z'$  is a coincidence point of  $B$  and  $T$ .

Now, we prove that  $z = z'$ . By inequality (c), we have take  $x = x_n$  and  $y = y_n$ ,

$$\begin{aligned} M(Ax_n, By_n, kt) &\geq M(Sx_n, Ty_n, t) * M(Ax_n, Sx_n, t) * M(By_n, Ty_n, t) * M(By_n, Sx_n, 2t) * M(Ax_n, Ty_n, t), \\ N(Ax_n, By_n, kt) &\leq N(Sx_n, Ty_n, t) \diamond N(Ax_n, Sx_n, t) \diamond N(By_n, Ty_n, t) \diamond N(By_n, Sx_n, 2t) \diamond N(Ax_n, Ty_n, t). \end{aligned}$$

taking the limit as  $n \rightarrow \infty$  yields

$$M(z, z', kt) \geq M(z, z', t) * M(z, z, t) * M(z', z', t) * M(z', z, 2t) * M(z, z', t),$$

$$N(z, z', kt) \leq N(z, z', t) \diamond N(z, z, t) \diamond N(z', z', t) \diamond N(z', z, 2t) \diamond N(z, z', t)$$

$$\Rightarrow M(z, z', kt) \geq M(z, z', t), N(z, z', kt) \leq N(z, z', t).$$

By lemma 2.2, we have  $z = z'$ .

Also, we claim that  $Az = z$ . By inequality (c), take  $x = z$  and  $y = y_n$ , we get

$$M(Az, By_n, kt) \geq M(Sz, Ty_n, t) * M(Az, Sz, t) * M(By_n, Ty_n, t) * M(By_n, Sz, 2t) * M(Az, Ty_n, t),$$

$$N(Az, By_n, kt) \leq N(Sz, Ty_n, t) \diamond N(Az, Sz, t) \diamond N(By_n, Ty_n, t) \diamond N(By_n, Sz, 2t) \diamond N(Az, Ty_n, t).$$

taking the limit as  $n \rightarrow \infty$  yields

$$M(Az, z', kt) \geq M(Az, z', t) * M(Az, Az, t) * M(z', z', t) * M(z', Az, 2t) * M(Az, z', t),$$

$$N(Az, z', kt) \leq N(Az, z', t) \diamond N(Az, Az, t) \diamond N(z', z', t) \diamond N(z', Sz, 2t) \diamond N(Az, z', t).$$

$$\Rightarrow M(Az, z', kt) \geq M(Az, z', t), N(Az, z', kt) \leq N(Az, z', t)$$

By lemma 2.2,  $Az = z' = z$ .

Again, we claim that  $Bz = z$ . Using (c), take  $x = z$  and  $y = z$ , we get

$$M(Az, Bz, kt) \geq M(Sz, Tz, t) * M(Az, Sz, t) * M(Bz, Tz, t) * M(Bz, Sz, 2t) * M(Az, Tz, t),$$

$$N(Az, Bz, kt) \leq N(Sz, Tz, t) \diamond N(Az, Sz, t) \diamond N(Bz, Tz, t) \diamond N(Bz, Sz, 2t) \diamond N(Az, Tz, t).$$

$$M(z, Bz, kt) \geq M(z, Bz, t) * M(Az, Az, t) * M(Bz, Bz, t) * M(Bz, z, 2t) * M(z, Bz, t),$$

$$N(z, Bz, kt) \leq N(z, Bz, t) \diamond N(Az, Az, t) \diamond N(Bz, Bz, t) \diamond N(Bz, z, 2t) \diamond N(z, Bz, t).$$

$$\Rightarrow M(z, Bz, kt) \geq M(z, Bz, t), N(z, Bz, kt) \leq N(z, Bz, t).$$

By lemma 2.2,  $z = Bz = Tz$ .

Therefore,  $z = Az = Bz = Sz = Tz$ ; that is  $z$  is common fixed point of  $A, B, S$  and  $T$ .

**For Uniqueness:** Suppose that there exist another fixed point  $w$  of  $A, B, S$  and  $T$ .

By condition (c), take  $x = z, y = w$ , we have

$$M(Az, Bw, kt) \geq M(Sz, Tw, t) * M(Az, Sz, t) * M(Bw, Tw, t) * M(Bw, Sz, 2t) * M(Az, Tw, t),$$

$$N(Az, Bw, kt) \leq N(Sz, Tw, t) \diamond N(Az, Sz, t) \diamond N(Bw, Tw, t) \diamond N(Bw, Sz, 2t) \diamond N(Az, Tw, t).$$

$$\Rightarrow M(z, w, kt) \geq M(z, w, t), N(z, w, kt) \leq N(z, w, t)$$

Hence, by lemma 2.2,  $z = w$ . Therefore, uniqueness follows.

If we put  $S = T$ , in Theorem 3.1, we get the following result:

**Corollary 3.2:** Let  $A, B$  and  $S$  be three self maps of a Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  with continuous  $t$ -norm  $*$  and continuous  $t$ -conorm  $\diamond$  defined by  $t * t \geq t$  and  $(1-t) \diamond (1-t) \leq (1-t)$  for all  $t \in [0, 1]$ . If the pairs  $(A, S)$  and  $(B, S)$  are subcompatible and subsequentially continuous, then

(d)  $A$  and  $S$  have a coincidence point;

(e)  $B$  and  $S$  have a coincidence point.

Further, let for all  $x, y$  in  $X, k \in (0, 1), t > 0$

$$(f) \quad M(Ax, By, kt) \geq M(Sx, Sy, t) * M(Ax, Sx, t) * M(By, Sy, t) * M(By, Sx, 2t) * M(Ax, Sy, t),$$

$$N(Ax, By, kt) \leq N(Sx, Sy, t) \diamond N(Ax, Sx, t) \diamond N(By, Sy, t) \diamond N(By, Sx, 2t) \diamond N(Ax, Sy, t).$$

Then,  $A, B$  and  $S$  have a unique common fixed point.

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