

A Common Fixed Point Theorem for Three Pairs of Maps in \mathcal{M} -Fuzzy Metric Spaces

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Abstract

In this paper, we mainly give a common fixed point theorem for three pairs of weakly compatible mappings in \mathcal{M} -fuzzy metric spaces by introducing common property (E) for two pairs of mappings.

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1 Introduction

In 1992, Dhage [1] introduced the notion of generalized metric or D-metric spaces and proved several fixed point theorems in it. Since D-metric space do not possess some topological properties (see [4,5,6]), recently Sedghi and Shobe [8] introduced D^* -metric space as a probable modification of D-metric space and studied some topological properties which are not valid in D-metric spaces. Based on D^* -metric concepts, they [8] define \mathcal{M} -fuzzy metric space and proved a common fixed point theorem in it.

In this paper, we prove a common fixed point theorem for six weakly compatible mappings of which two pairs of mappings satisfy common property (E). We also give an example to illustrate our main theorem. First we state

some known definitions and results in \mathcal{M} -fuzzy metric space given by Sedghi and Shobe [8].

Definition 1.1 ([7]) A binary operation $*$: $[0, 1] \times [0, 1] \longrightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions

- (1) $*$ is associative and commutative,
- (2) $*$ is continuous,
- (3) $a * 1 = a$ for all $a \in [0, 1]$,
- (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 1.2 ([8]) A 3-tuple $(X, \mathcal{M}, *)$ is called a \mathcal{M} -fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and \mathcal{M} is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$,

- (1) $\mathcal{M}(x, y, z, t) > 0$,
- (2) $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z$,
- (3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, (symmetry) where p is a permutation function,
- (4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$,
- (5) $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Remark 1.3 ([8]) Let $(X, \mathcal{M}, *)$ be a \mathcal{M} -fuzzy metric space. Then for every $t > 0$ and for every $x, y \in X$ we have $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$.

Definition 1.4 ([8]) Let $(X, \mathcal{M}, *)$ be a \mathcal{M} -fuzzy metric space . For $t > 0$, the open ball $B_{\mathcal{M}}(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B_{\mathcal{M}}(x, r, t) = \{y \in X : \mathcal{M}(x, y, y, t) > 1 - r\}.$$

A subset A of X is called open set if for each $x \in A$ there exist $t > 0$ and $0 < r < 1$ such that $B_{\mathcal{M}}(x, r, t) \subseteq A$.

Definition 1.5 ([8]) A sequence $\{x_n\}$ in X converges to x if and only if $\mathcal{M}(x, x, x_n, t) \longrightarrow 1$ as $n \longrightarrow \infty$, for each $t > 0$. It is called a Cauchy sequence if for each $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbf{N}$ such that $\mathcal{M}(x_n, x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$. The \mathcal{M} -fuzzy metric space $(X, \mathcal{M}, *)$ is said to be complete if every Cauchy sequence is convergent.

Lemma 1.6 ([8]). Let $(X, \mathcal{M}, *)$ be a \mathcal{M} -fuzzy metric space. Then $\mathcal{M}(x, y, z, t)$ is nondecreasing with respect to t , for all x, y, z in X .

Lemma 1.7 ([8]). Let $(X, \mathcal{M}, *)$ be a \mathcal{M} -fuzzy metric space. Then \mathcal{M} is continuous function on $X^3 \times (0, \infty)$.

Definition 1.8 ([8]) Let f and g be two self maps of $(X, \mathcal{M}, *)$. Then f and g are said to satisfy property (E), if there exists a sequence $\{x_n\}$ in X such that $\mathcal{M}(fx_n, u, u, t) \rightarrow 1$ and $\mathcal{M}(gx_n, u, u, t) \rightarrow 1$ as $n \rightarrow \infty$ for some u in X and for every $t > 0$.

Liu et.al [3] defined common property(E) for two pairs of maps in a metric space. In 1998, Jungck and Rhoades [2] introduced the concept of weakly compatibility of pair of self mappings in a metric space.

Definition 1.9 ([8]) Let f and g be two self maps of $(X, \mathcal{M}, *)$. Then f and g are said to be weakly compatible if there exists u in X with $fu = gu$ implies $fgu = gfu$.

2 Main Results

Now we give the following definition.

Definition 2.1 Let P, Q, f and g be self mappings on \mathcal{M} -fuzzy metric space $(X, \mathcal{M}, *)$. We say that the pairs (P, f) and (Q, g) satisfy common property(E) if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\mathcal{M}(Px_n, u, u, t) \rightarrow 1$, $\mathcal{M}(fx_n, u, u, t) \rightarrow 1$, $\mathcal{M}(Qy_n, u, u, t) \rightarrow 1$ and $\mathcal{M}(gy_n, u, u, t) \rightarrow 1$ as $n \rightarrow \infty$ for some u in X and for every $t > 0$.

Example 2.2 Let $X = \mathbf{R}$ and

$$\mathcal{M}(x, y, z, t) = \frac{t}{t + |x - y| + |y - z| + |x - z|}$$

for all $t > 0$ and $x, y, z \in X$. Let $P, Q, f, g : X \rightarrow X$ be defined by $Px = 2x + 1$, $fx = x + 2$, $Qx = 2x + 5$ and $gx = 2 - x$.

Consider the sequences $\{x_n\} = \{1 + \frac{1}{n}\}$ and $\{y_n\} = \{-1 + \frac{1}{n}\}$. Then $\mathcal{M}(Px_n, 3, 3, t) \rightarrow 1$, $\mathcal{M}(fx_n, 3, 3, t) \rightarrow 1$, $\mathcal{M}(Qy_n, 3, 3, t) \rightarrow 1$ and $\mathcal{M}(gy_n, 3, 3, t) \rightarrow 1$ as $n \rightarrow \infty$ for every $t > 0$.

Thus the pairs (P, f) and (Q, g) satisfy common property(E).

Similarly we can define common property(E) for three pairs of maps.

Theorem 2.3 Let P, Q, R, f, g and h be self mappings of a \mathcal{M} -fuzzy metric space $(X, \mathcal{M}, *)$ satisfying

- (i) $P(X) \subseteq g(X), Q(X) \subseteq h(X), R(X) \subseteq f(X)$
and $f(X)$ or $g(X)$ or $h(X)$ is a closed subspace of X ,
- (ii) the pairs $(P, f), (Q, g)$ and (R, h) are weakly compatible,

(iii) any two pairs from $(P, f), (Q, g)$ and (R, h) satisfy common property(E) and
(iv)

$$\mathcal{M}(Px, Qy, Rz, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(fx, gy, hz, t), \mathcal{M}(fx, Px, Qy, t), \\ \mathcal{M}(gy, Qy, Rz, t), \mathcal{M}(hz, Rz, Px, t), \\ \mathcal{M}(fx, gy, Rz, t), \mathcal{M}(Px, gy, hz, t), \\ \mathcal{M}(fx, Qy, hz, t), \mathcal{M}(fx, Qy, Rz, t), \\ \mathcal{M}(Px, gy, Rz, t), \mathcal{M}(Px, Qy, hz, t) \end{array} \right\}$$

$\forall x, y, z \in X, \forall t > 0$, where $\phi : (0, \infty) \rightarrow (0, \infty)$ is such that $\phi(t) < t$ for every $t > 0$.

Then P, Q, R, f, g and h have a unique common fixed point in X .

Proof.

Suppose the pairs (P, f) and (Q, g) satisfy common property(E). Then there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} Qy_n = \lim_{n \rightarrow \infty} gy_n = \alpha$$

for some $\alpha \in X$.

Since $Q(X) \subseteq h(X)$, there exists a sequence $\{z_n\}$ in X such that $Qy_n = hz_n, \forall n$.

Hence

$$\lim_{n \rightarrow \infty} hz_n = \alpha.$$

Let $\lim_{n \rightarrow \infty} Rz_n = \gamma$. Now from (iv), we have

$$\mathcal{M}(Px_n, Qy_n, Rz_n, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(fx_n, gy_n, hz_n, t), \mathcal{M}(fx_n, Px_n, Qy_n, t), \\ \mathcal{M}(gy_n, Qy_n, Rz_n, t), \mathcal{M}(hz_n, Rz_n, Px_n, t), \\ \mathcal{M}(fx_n, gy_n, Rz_n, t), \mathcal{M}(Px_n, gy_n, hz_n, t), \\ \mathcal{M}(fx_n, Qy_n, hz_n, t), \mathcal{M}(fx_n, Qy_n, Rz_n, t), \\ \mathcal{M}(Px_n, gy_n, Rz_n, t), \mathcal{M}(Px_n, Qy_n, hz_n, t) \end{array} \right\}$$

Letting $n \rightarrow \infty$, we get

$$\mathcal{M}(\alpha, \alpha, \gamma, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \gamma, t), \mathcal{M}(\alpha, \gamma, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \gamma, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \gamma, t), \\ \mathcal{M}(\alpha, \alpha, \gamma, t), \mathcal{M}(\alpha, \alpha, \alpha, t) \end{array} \right\}$$

$$\mathcal{M}(\alpha, \alpha, \gamma, \phi(t)) \geq \mathcal{M}(\alpha, \alpha, \gamma, t)$$

so that $\gamma = \alpha$. Thus $\lim_{n \rightarrow \infty} Rz_n = \alpha$.

Suppose $f(X)$ is a closed subspace of X . Then $\alpha = fu$ for some $u \in X$. Now

$$\mathcal{M}(Pu, Qy_n, Rz_n, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(fu, gy_n, hz_n, t), \mathcal{M}(fu, Pu, Qy_n, t), \\ \mathcal{M}(gy_n, Qy_n, Rz_n, t), \mathcal{M}(hz_n, Rz_n, Pu, t), \\ \mathcal{M}(fu, gy_n, Rz_n, t), \mathcal{M}(Pu, gy_n, hz_n, t), \\ \mathcal{M}(fu, Qy_n, hz_n, t), \mathcal{M}(fu, Qy_n, Rz_n, t), \\ \mathcal{M}(Pu, gy_n, Rz_n, t), \mathcal{M}(Pu, Qy_n, hz_n, t) \end{array} \right\}$$

Letting $n \rightarrow \infty$, we get

$$\mathcal{M}(Pu, \alpha, \alpha, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, Pu, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, Pu, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(Pu, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(Pu, \alpha, \alpha, t), \\ \mathcal{M}(Pu, \alpha, \alpha, t) \end{array} \right\}$$

$$\mathcal{M}(Pu, \alpha, \alpha, \phi(t)) \geq \mathcal{M}(Pu, \alpha, \alpha, t)$$

so that $Pu = \alpha$.

Since the pair (P, f) is weakly compatible and $Pu = fu = \alpha$, we have $P\alpha = f\alpha$. Since $P(X) \subseteq g(X)$, there exists $v \in X$ such that $\alpha = Pu = gv$. Now we have

$$\mathcal{M}(Pu, Qv, Rz_n, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(fu, gv, hz_n, t), \mathcal{M}(fu, Pu, Qv, t), \\ \mathcal{M}(gv, Qv, Rz_n, t), \mathcal{M}(hz_n, Rz_n, Pu, t), \\ \mathcal{M}(fu, gv, Rz_n, t), \mathcal{M}(Pu, gv, hz_n, t), \\ \mathcal{M}(fu, Qv, hz_n, t), \mathcal{M}(fu, Qv, Rz_n, t), \\ \mathcal{M}(Pu, gv, Rz_n, t), \mathcal{M}(Pu, Qv, hz_n, t) \end{array} \right\}$$

Letting $n \rightarrow \infty$, we get

$$\mathcal{M}(\alpha, Qv, \alpha, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, Qv, t), \mathcal{M}(\alpha, Qv, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, Qv, \alpha, t), \mathcal{M}(\alpha, Qv, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, Qv, \alpha, t) \end{array} \right\}$$

$$\mathcal{M}(\alpha, Qv, \alpha, \phi(t)) \geq \mathcal{M}(\alpha, Qv, \alpha, t)$$

so that $Qv = \alpha$.

Since the pair (Q, g) is weakly compatible and $Qv = gv = \alpha$, we have $Q\alpha = g\alpha$. Since $Q(X) \subseteq h(X)$, there exists $w \in X$ such that $\alpha = Qv = hw$. Now we have

$$\mathcal{M}(Pu, Qv, Rw, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(fu, gv, hw, t), \mathcal{M}(fu, Pu, Qv, t), \\ \mathcal{M}(gv, Qv, Rw, t), \mathcal{M}(hw, Rw, Pu, t), \\ \mathcal{M}(fu, gv, Rw, t), \mathcal{M}(Pu, gv, hw, t), \\ \mathcal{M}(fu, Qv, hw, t), \mathcal{M}(fu, Qv, Rw, t), \\ \mathcal{M}(Pu, gv, Rw, t), \mathcal{M}(Pu, Qv, hw, t) \end{array} \right\}$$

$$\mathcal{M}(\alpha, \alpha, Rw, \phi(t)) \geq \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, Rw, t), \\ \mathcal{M}(\alpha, Rw, \alpha, t), \mathcal{M}(\alpha, \alpha, Rw, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, Rw, t), \\ \mathcal{M}(\alpha, \alpha, \alpha, t) \end{array} \right\}$$

$\mathcal{M}(\alpha, \alpha, Rw, \phi(t)) \geq \mathcal{M}(\alpha, \alpha, Rw, t)$ so that $Rw = \alpha$.

Since the pair (R, h) is weakly compatible, we have $R\alpha = h\alpha$.

From (iv), we have

$$\begin{aligned}
\mathcal{M}(P\alpha, \alpha, \alpha, \phi(t)) &\geq \mathcal{M}(P\alpha, Qv, Rv, \phi(t)) \\
&\geq \min \left\{ \begin{array}{l} \mathcal{M}(f\alpha, gv, hw, t), \mathcal{M}(f\alpha, P\alpha, Qv, t), \mathcal{M}(gv, Qv, Rv, t), \\ \mathcal{M}(hw, Rv, P\alpha, t), \mathcal{M}(f\alpha, gv, Rv, t), \mathcal{M}(P\alpha, gv, hw, t), \\ \mathcal{M}(f\alpha, Qv, hw, t), \mathcal{M}(f\alpha, Qv, Rv, t), \mathcal{M}(P\alpha, gv, Rv, t), \\ \mathcal{M}(P\alpha, Qv, hw, t) \end{array} \right\} \\
&= \min \left\{ \begin{array}{l} \mathcal{M}(P\alpha, \alpha, \alpha, t), \mathcal{M}(P\alpha, P\alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, P\alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(P\alpha, \alpha, \alpha, t), \\ \mathcal{M}(P\alpha, \alpha, \alpha, t), \mathcal{M}(P\alpha, \alpha, \alpha, t), \mathcal{M}(P\alpha, \alpha, \alpha, t), \\ \mathcal{M}(P\alpha, \alpha, \alpha, t) \end{array} \right\} \\
&= \mathcal{M}(P\alpha, \alpha, \alpha, t) \quad \text{from Remark (1.3)}
\end{aligned}$$

so that $P\alpha = \alpha$ and hence $P\alpha = \alpha = f\alpha$. Now, we have

$$\begin{aligned}
\mathcal{M}(\alpha, Q\alpha, \alpha, \phi(t)) &\geq \mathcal{M}(P\alpha, Q\alpha, Rv, \phi(t)) \\
&\geq \min \left\{ \begin{array}{l} \mathcal{M}(f\alpha, g\alpha, hw, t), \mathcal{M}(f\alpha, P\alpha, Q\alpha, t), \mathcal{M}(g\alpha, Q\alpha, Rv, t), \\ \mathcal{M}(hw, Rv, P\alpha, t), \mathcal{M}(f\alpha, g\alpha, Rv, t), \mathcal{M}(P\alpha, g\alpha, hw, t), \\ \mathcal{M}(f\alpha, Q\alpha, hw, t), \mathcal{M}(f\alpha, Q\alpha, Rv, t), \mathcal{M}(P\alpha, g\alpha, Rv, t), \\ \mathcal{M}(P\alpha, Q\alpha, hw, t) \end{array} \right\} \\
&= \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, Q\alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, Q\alpha, t), \mathcal{M}(Q\alpha, Q\alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, Q\alpha, \alpha, t), \mathcal{M}(\alpha, Q\alpha, \alpha, t), \\ \mathcal{M}(\alpha, Q\alpha, \alpha, t), \mathcal{M}(\alpha, Q\alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, Q\alpha, \alpha, t) \end{array} \right\} \\
&= \mathcal{M}(\alpha, Q\alpha, \alpha, t) \quad \text{from Remark (1.3)}
\end{aligned}$$

so that $Q\alpha = \alpha$ and hence $Q\alpha = \alpha = g\alpha$. Now we have

$$\begin{aligned}
\mathcal{M}(\alpha, \alpha, R\alpha, \phi(t)) &\geq \mathcal{M}(P\alpha, Q\alpha, R\alpha, \phi(t)) \\
&\geq \min \left\{ \begin{array}{l} \mathcal{M}(f\alpha, g\alpha, h\alpha, t), \mathcal{M}(f\alpha, P\alpha, Q\alpha, t), \mathcal{M}(g\alpha, Q\alpha, R\alpha, t), \\ \mathcal{M}(h\alpha, R\alpha, P\alpha, t), \mathcal{M}(f\alpha, g\alpha, R\alpha, t), \mathcal{M}(P\alpha, g\alpha, h\alpha, t), \\ \mathcal{M}(f\alpha, Q\alpha, h\alpha, t), \mathcal{M}(f\alpha, Q\alpha, R\alpha, t), \mathcal{M}(P\alpha, g\alpha, R\alpha, t), \\ \mathcal{M}(P\alpha, Q\alpha, h\alpha, t) \end{array} \right\} \\
&= \min \left\{ \begin{array}{l} \mathcal{M}(\alpha, \alpha, R\alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, R\alpha, t), \\ \mathcal{M}(R\alpha, R\alpha, \alpha, t), \mathcal{M}(\alpha, \alpha, R\alpha, t), \mathcal{M}(\alpha, \alpha, R\alpha, t), \\ \mathcal{M}(\alpha, \alpha, R\alpha, t), \mathcal{M}(\alpha, \alpha, R\alpha, t), \mathcal{M}(\alpha, \alpha, R\alpha, t), \\ \mathcal{M}(\alpha, \alpha, R\alpha, t) \end{array} \right\} \\
&= \mathcal{M}(\alpha, \alpha, R\alpha, t) \quad \text{from Remark (1.3)}
\end{aligned}$$

so that $R\alpha = \alpha$ and hence $R\alpha = \alpha = h\alpha$.

Thus $P\alpha = Q\alpha = R\alpha = f\alpha = g\alpha = h\alpha = \alpha$.

Suppose $\beta \neq \alpha$ is a common fixed point of P, Q, R, f, g and h . Then

$$\begin{aligned} & \mathcal{M}(\beta, \alpha, \alpha, \phi(t)) \geq \mathcal{M}(P\beta, Q\alpha, R\alpha, \phi(t)) \\ & \geq \min \left\{ \begin{array}{l} \mathcal{M}(f\beta, g\alpha, h\alpha, t), \mathcal{M}(f\beta, P\beta, Q\alpha, t), \mathcal{M}(g\alpha, Q\alpha, R\alpha, t), \\ \mathcal{M}(h\alpha, R\alpha, P\beta, t), \mathcal{M}(f\beta, g\alpha, R\alpha, t), \mathcal{M}(P\beta, g\alpha, h\alpha, t), \\ \mathcal{M}(f\beta, Q\alpha, h\alpha, t), \mathcal{M}(f\beta, Q\alpha, R\alpha, t), \mathcal{M}(P\beta, g\alpha, R\alpha, t), \\ \mathcal{M}(P\beta, Q\alpha, h\alpha, t) \end{array} \right\} \\ & = \min \left\{ \begin{array}{l} \mathcal{M}(\beta, \alpha, \alpha, t), \mathcal{M}(\beta, \beta, \alpha, t), \mathcal{M}(\alpha, \alpha, \alpha, t), \\ \mathcal{M}(\alpha, \alpha, \beta, t), \mathcal{M}(\beta, \alpha, \alpha, t), \mathcal{M}(\beta, \alpha, \alpha, t), \\ \mathcal{M}(\beta, \alpha, \alpha, t), \mathcal{M}(\beta, \alpha, \alpha, t), \mathcal{M}(\beta, \alpha, \alpha, t), \\ \mathcal{M}(\beta, \alpha, \alpha, t) \end{array} \right\} \\ & = \mathcal{M}(\beta, \alpha, \alpha, t) \quad \text{from Remark (1.3)} \end{aligned}$$

so that $\beta = \alpha$. Thus α is the unique common fixed point of P, Q, R, f, g and h .

Now we give an example to illustrate our Theorem 2.3.

Example 2.4 Let $X = \mathbf{R}$ and

$$\mathcal{M}(x, y, z, t) = \frac{t}{t + |x - y| + |y - z| + |x - z|}$$

for all $t > 0$ and $x, y, z \in X$. Let $P, Q, R, f, g, h : X \rightarrow X$ be defined by $Px = Qx = Rx = 1$ and

$$fx = \begin{cases} 1, & \text{if } x \in [1, \infty) \\ 0, & \text{otherwise,} \end{cases}$$

$$gx = \begin{cases} 1, & \text{if } x \in [1, \infty) \\ \frac{1}{2}, & \text{otherwise,} \end{cases}$$

$$hx = \begin{cases} 1, & \text{if } x \in [1, \infty) \\ \frac{1}{3}, & \text{otherwise.} \end{cases}$$

Define $\phi : (0, \infty) \rightarrow (0, \infty)$ as $\phi(t) = kt, 0 < k < 1$. Clearly (i) and (ii) are satisfied. Consider the sequences $\{x_n\} = \{1 + \frac{1}{n}\}$ and $\{y_n\} = \{1 + \frac{2}{n}\}$. Then the pairs (P, f) and (Q, g) satisfy common property(E). The inequality(iv) is satisfied since the L.H.S. of inequality(iv) is 1. Clearly 1 is the unique common fixed point of P, Q, R, f, g and h .

Remark 2.5 Theorem 2.3 is also true if (i) and (iii) are replaced by (i)¹ $f(X), g(X)$ and $h(X)$ are closed subspaces of X , (iii)¹ the three pairs $(P, f), (Q, g)$ and (R, h) satisfy common property(E).

References

- [1] B.C.Dhage , Generalised metric spaces and mappings with fixed point, *Bull. Calcutta Math. Soc.***84** (1992),no.4,329-336.
- [2] G.Jungck and B.E.Rhoades , Fixed points for set valued functions without continuity,*Indian J. Pure Appl. Math.* **29**(1998), no. 3,227-238.
- [3] Yicheng Liu, Jun Wu and Zhixiangli, Common fixed points of single valued and multi valued maps, *Internat.J.Math.Math.Sci* **2005:19**(2005),3045-3055.
- [4] S.V.R.Naidu , K.P.R.Rao and N.Srinivasa Rao , On the topology of D-metric spaces and the generation of D-metric spaces from metric spaces, *Internat.J.Math. Math.Sci.***2004**(2004), No.51,2719-2740.
- [5] S.V.R.Naidu ,K.P.R.Rao and N.Srinivasa Rao , On the concepts of balls in a D- metric space, *Internat.J.Math.Math.Sci.*,**2005, No.1**(2005)133-141.
- [6] S.V.R.Naidu ,K.P.R. Rao and N.Srinivasa Rao , On convergent sequences and fixed point theorems in D-Metric spaces, *Internat.J.Math.Math.Sci.*, **2005:12**(2005),1969-1988.
- [7] B. Schweizer and A. Sklar, Statistical metric spaces, *Pacific J. Math.* **10** (1960), 313-334.
- [8] S.Sedghi and N.Shobe , Fixed point theorem in \mathcal{M} -fuzzy metric spaces with property(E), *Advances in Fuzzy Mathematics*, **Vol.1**,No.1(2006),55-65.

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