On Left, Right Weakly Prime Hyperideals on Semihypergroups

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Abstract

In this paper, we define the concepts of left(right) weakly prime and left(right) semiregular on semihypergroup. Also we investigate the properties of them.

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1 Introduction

Hyperstructures represent a natural extension of classical algebraic structures and they were introduced by the French mathematician F. Marty [3]. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. In this paper, we define the concepts of left(right) weakly prime and left(right) semiregular. Also we investigate the properties of them.

2 Preliminary Notes

Definition 2.1 A map \( o : H \times H \to \mathcal{P}^*(H) \) is called hyperoperation on the set \( H \), where \( H \) is a nonempty set and \( \mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\} \) denotes the set of all nonempty subset of \( H \).
Definition 2.2 A hyperstructure is called the pair \((H, \circ)\), where \(\circ\) is a hyperoperation on the set \(H\).

Definition 2.3 A hyperstructure \((H, \circ)\) is called a semihypergroup if for all \(x, y, z \in H\)
\[(x \circ y) \circ z = x \circ (y \circ z),\]
which means that
\[\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.\]

If \(x \in H\) and \(A, B\) are nonempty subsets of \(H\), then
\[A \circ B = \bigcup_{a \in A, b \in B} a \circ b,\quad A \circ x = A \circ \{x\},\quad x \circ B = \{x\} \circ B.\]

Definition 2.4 A nonempty subset \(A\) of a semihypergroup \(H\) is called a subsemihypergroup of \(H\) if \(A \circ A \subseteq A\).

Definition 2.5 Let \(H\) be a semihypergroup. \(\emptyset \neq L \subseteq H\), \(L\) is called a left hyperideal of \(H\) if \(H \circ L \subseteq L\).

Definition 2.6 Let \(H\) be a semihypergroup. \(\emptyset \neq R \subseteq H\), \(R\) is called a right hyperideal of \(H\) if \(R \circ H \subseteq R\).

If \(A\) is a right and a left hyperideal of \(H\), then it is called an hyperideal of \(H\).

Definition 2.7 An element \(a\) of a semihypergroup \(H\) is a right (resp. left) semiregular element if \(a \subseteq a \circ H \circ a \circ H\) (resp. \(a \in H \circ a \circ H \circ a\)).

A subsemihypergroup \(T\) of \(H\) is right (resp. left) semiregular if all elements of \(T\) are right (resp. left) semiregular.

Definition 2.8 Let \(H\) be a semihypergroup and \(T \subseteq H\). \(T\) is called left (resp. right) weakly prime if for all left (resp. right) hyperideals \(A, B\) of \(H\) such that
\[A \circ B \subseteq T \Rightarrow A \subseteq T\] or \(B \subseteq T\).

\(T\) is called a left (resp. right) weakly prime hyperideal of \(H\) if \(T\) is a left (resp. right) hyperideal which is weakly prime.

Let \(H\) be a semihypergroup and \(a \in H\). The right (resp. left) hyperideal of \(H\) generated by \(a\), denoted by \(R(a)\) (resp. \(L(a)\)) is of the form:
\[R(a) = a \cup a \circ H\] and \(L(a) = a \cup H \circ a\).
Lemma 2.9 Let \( T \) be a subsemihypergroup of a semihypergroup \( H \). Then we have the following:

1. \( T \) is left (resp. right) semiregular if and only if for any \( a \in T \),
   \[ a \in T \circ a \circ T \circ a (\text{resp. } a \in a \circ T \circ a \circ T) . \]

2. \( T \) is semiregular if and only if for any \( a \in T \),
   \[ a \in T \circ a \circ T \circ a \text{ or } a \in a \circ T \circ a \circ T . \]

3 Main Results

Now we give some characterizations of the right (left) weakly prime.

Theorem 3.1 Let \( H \) be a semihypergroup and \( T \) a left hyperideal of \( H \). Then the following are equivalent:

1. \( T \) is left weakly prime.

2. If \( a \circ H \circ b \subseteq T \) for some \( a \) and \( b \) in \( H \), then \( a \in T \) or \( b \in T \).

3. If \( L(a) \circ L(a) \subseteq T \) for some \( a \) and \( b \) in \( H \), then \( a \in T \) or \( b \in T \).

4. If \( A \) is any subset of \( H \) and \( B \) is a left ideal of \( H \) such that \( A \circ B \subseteq T \), then \( A \subseteq T \) or \( B \subseteq T \).

Proof. (1)\( \Rightarrow \) (2): Assume that \( T \) is left weakly prime. Let \( a \circ H \circ b \subseteq T \) for some \( a \) and \( b \) in \( H \). Then, we have

\[
(H \circ a) \circ (H \circ b) = H \circ (a \circ H \circ b) \subseteq H \circ T \subseteq T.
\]

Since \( H \circ a \) and \( H \circ b \) are left ideals of \( H \) and \( T \) is left weakly prime, we get

\[ H \circ a \subseteq T \text{ or } H \circ b \subseteq T . \]

Let \( H \circ a \subseteq T \). Then, we have

\[
L(a) \circ L(a) = (a \cup H \circ a) \circ (a \cup H \circ a) \\
\subseteq a \circ a \cup a \circ H \circ a \cup H \circ a \circ a \cup H \circ a \circ H \circ a \\
\subseteq H \circ a \subseteq T.
\]

Since \( T \) is left weakly prime and \( L(a) \) is a left hyperideal of \( H \), we have \( a \in L(a) \subseteq L \).

If \( H \circ b \subseteq T \), then, by similar method, we have \( b \in T \).

(2)\( \Rightarrow \) (3): Let \( a, b \in H \), \( a \notin T \) and \( L(a) \circ L(b) \subseteq T \). Then, we have
Thus we have $b \in T$ by (2), since $a \not\in T$.

(3)$\Rightarrow$(4): Assume that $A \circ B \subseteq T, A \not\subseteq T$. Let $a \in A \setminus T$ and $b \in B$. Then we have
\[
L(a) \circ L(b) = (a \cup H \circ a) \circ (b \cup H \circ b)
\subseteq (A \cup H \circ A) \circ (B \cup H \circ B)
\subseteq A \circ B \cup A \circ H \circ B \cup H \circ A \circ B \cup H \circ A \circ H \circ B
\subseteq T \cup H \circ T
\subseteq T.
\]
Hence we have $a \in T$ or $b \in T$ by (3).

(4)$\Rightarrow$(1): It is obvious. $\blacksquare$

**Theorem 3.2** Let $H$ be a semihyper and $T$ a right hyperideal of $H$. Then the following are equivalent:

1. $T$ is right weakly prime.
2. If $a \circ H \circ b \subseteq T$ for some $a$ and $b$ in $H$, then $a \in T$ or $b \in T$.
3. If $R(a) \circ R(a) \subseteq T$ for some $a$ and $b$ in $H$, then $a \in T$ or $b \in T$.
4. If $A$ is a right hyperideal of $H$ and $B$ is any subset of $H$ such that $A \circ B \subseteq T$, then $A \subseteq T$ or $B \subseteq T$.

If $T$ is an hyperideal of $H$, then we get the following corollary:

**Corollary 3.3** Let $H$ be a semihypergroup and $T$ an hyperideal of $H$. The followings are equivalent:

1. $T$ is left weakly prime.
2. $T$ is right weakly prime.
3. $T$ is weakly prime.
4. If $a \circ H \circ b \subseteq T$ for some $a$ and $b$ in $H$, then $a \in T$ or $b \in T$.
5. If $L(a) \circ L(a) \subseteq T$ for some $a$ and $b$ in $H$, then $a \in T$ or $b \in T$.
6. If $R(a) \circ R(a) \subseteq T$ for some $a$ and $b$ in $H$, then $a \in T$ or $b \in T$.
7. If $A$ is a right hyperideal of $H$ and $B$ is any subset of $H$ such that $A \circ B \subseteq T$, then $A \subseteq T$ or $B \subseteq T$. 

(8) If $A$ is any subset of $H$ and $B$ is a left ideal of $H$ such that $A \circ B \subseteq T$, then $A \subseteq T$ or $B \subseteq T$.

**Proof.** We note that a hyperideal is a left and right hyperideal. Hence we can prove that $(1) \Rightarrow (4) \Rightarrow (5) \Rightarrow (8) \Rightarrow (1)$ by Theorem 3.1, $(2) \Rightarrow (4) \Rightarrow (6) \Rightarrow (7) \Rightarrow (2)$ by Theorem 3.2 and $(3) \iff (4)$.

**References**


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