Pricing FX Target Redemption Forward under
Regime Switching Model

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Abstract

In this paper, we employ regime switching model in valuing an exotic FX derivative, called ‘target redemption forward’. Market conditions are assumed to be governed by a hidden Markov process and the coupled partial differential equations can be solved numerically.

Keywords: Regime Switching, Hidden Markov Process, Finite Difference Method

1 Introduction

The most simple way to hedge future cash flows of a foreign currency is to enter into an FX forward contract through OTC(Over The Counter) market. FX forward is a contract to buy or sell a foreign currency at the prespecified exchange rate(strike price). But one can sell(buy) at a higher(lower) strike price through exotic FX derivatives. In this paper, we introduce an exotic derivative called FX target redemption forward and use regime switching model in valuing the product numerically.

FX target redemption forward is composed of serial FX forwards, each of which has their own payoff and if the accumulated profit exceeds the prespecified knock out level at the nth fixing time before expiry, the contract terminates. For example, let the contract notional amount and strike price $N, K$ respectively and
there are $n$ fixing times, that is the contract consists of $n$ serial underlying FX forwards. Formally, let $P_i = N \ast (K - S_i)$, $i = 1, \ldots, n$. $S_i$ is the underlying asset price at $i$th fixing time. $C_i = \sum_{k=1}^{i} \max(0, P_k)$ is the accumulated profit for fixing time $t$. Then, $P_i$ is the payoff for the client, that is the client receives $P_i$ if $P_i$ is positive but if $P_i$ is negative, the client pays the absolute value of $P_i$ at each fixing time $i$. If $i$ is the first fixing time such that $C_i \geq M$ for $t \leq n$, $M - C_i$ will be finally received and remaining payoffs $P(t < i)$ are cancelled. This product contains an embedded option that knocks out future cash flows if the accumulated profit reaches the prespecified knock out level. This option cannot be decomposed vanilla options which have analytic pricing formula and should be solved numerically.

Market practice in FX option is to assume the return of a foreign currency follows a log-normal distribution with deterministic volatility. But the volatility is not deterministic but stochastic. A simplified approach to incorporate stochastic behavior of volatility is to adopt a regime switching model. [4] proposed an analytic pricing formula for European option under regime-switching model and [1] developed a lattice method for option valuation using regime-switching model. [2] investigated numerical algorithm for valuing European style exotic option with regime-switching market conditions which are modulated by a hidden Markov process.

In this paper, we use regime-switching model for valuing FX target redemption forward. It is assumed that market conditions are governed by a hidden Markov process and the related coupled partial differential equations will be introduced and are solved numerically. The rest of this paper is organized as follows. Section 2 describes the model and derive the governing partial differentials. A numerical example is presented in Section 3 and Section 4 concludes.

### 2 Model

#### 2.1 Two states regime switching model

Consider a probability triple $(\Omega, F, P)$ with the filtration $\{F_t\}$, which is generated by a standard Brownian motion $B(t)$ and a hidden Markov process $y(t)$. It is assumed that $B(t)$ and $y(t)$ are independent. Under the risk-neutral probability measure $Q$, the price of one unit of foreign currency $S(t)$ is given by

$$dS(t) / S(t) = (r_d(t) - r_f(t))dt + \sigma(t)dB(t),$$

where $r_d(t)$ and $r_f(t)$ are domestic and foreign risk-free interest rate, respectively.
and $\sigma(t)$ is the volatility. We assume that the hidden Markov process $y(t)$ can take two values 0, 1. $r_d(t)$, $r_f(t)$ and $\sigma(t)$ are assumed to take corresponding two state values $(r_{d,0}, r_{d,1}), (r_{f,0}, r_{f,1})$ and $(\sigma_0, \sigma_1)$ respectively. As in [2], $y(t)$ is generated by a transition matrix

$$
\begin{pmatrix}
a_{0,0} & a_{0,1} \\
a_{1,0} & a_{1,1}
\end{pmatrix} = 
\begin{pmatrix}
-\lambda_0 & \lambda_0 \\
\lambda_1 & -\lambda_1
\end{pmatrix}.
$$

2.2 Governing partial differential equations

In this subsection, we construct the partial differential equations for valuing FX target redemption forward which is introduced in the Section 1. The state variables that determine the value of FX target redemption forward are the price of foreign currency $S(t)$ and the accumulated benefit $C_t$. Formally, if we denote $V_t(i)$ the price of FX target redemption forward at time $t$ for regime $i(i=0, 1)$, we can write $V_t(i) \equiv V(t, S(t), C_t, i)$. Let $\Sigma = \{t_1, t_2, \cdots, t_n\}$ be the set of fixing times. $t_n = T$ is the contract maturity. Then, we have the following coupled partial differential equations for the price $V(t, S(t), C_t, i)$.

**Theorem 2.1** $V(t, S(t), C_t, i)$ satisfies the following coupled partial differential equations

$$
\frac{\partial V(t, S(t), C_t, i)}{\partial t} + (r_{d, i} - r_{f, i}) S \frac{\partial V(t, S(t), C_t, i)}{\partial S} + \frac{1}{2} \sigma_s^2 S^2 \frac{\partial^2 V(t, S(t), C_t, i)}{\partial S^2} + d_i \sum_{k=0}^1 a_{k,i} V(t, S, C, k) = 0
$$

between fixing times. The jump conditions at fixing times are given by

$$
\begin{align*}
M < C; \ & V(t, S, C, i) = 0, \\
C \leq M \leq C + \max(P_r, 0); \ & V(t, S, C, i) = M - C,
\end{align*}
$$

with the final conditions

$$
\begin{align*}
M < C; \ & V(T, S, C, i) = 0, \\
C \leq M \leq C + \max(P_r, 0); \ & V(T, S, C, i) = M - C,
\end{align*}
$$

with the final conditions

$$
\begin{align*}
C + \max(P_r, 0) < M; \ & V(T, S, C, i) = P_r.
\end{align*}
$$

**Proof.** The accumulated benefit $C_t$ are constant between fixing times. Therefore, we may consider $C_t$ as a constant between fixing times. Following [2], we can obtain the coupled partial differentials between fixing times. The jump conditions
and final conditions are easily obtained and are similar to those of ‘cliquet’ option, which is introduced in [3].

3 Results

In this section, we apply the numerical pricing procedure explained in the previous section to a USD/KRW target redemption forward.

3.1 Two states regime in the market

In general, when USD/KRW increases, USD/KRW option volatility $\sigma(t)$ also increases and the domestic risk-free interest rate $r_d(t)$ plummets. So, we can assume a two states regime economy, that is regime0 with a lower option volatility with a higher domestic risk-free interest rate and regime1 with a higher option volatility with a lower domestic risk-free interest rate. We assume that the foreign risk-free interest rate is constant, that is $r_{f,0} = r_{f,1}$.

3.2 Numerical results

In this section, we assume that $\lambda_0 = \lambda_1 = \lambda$ and solve the coupled partial differential equations numerically. We use the finite difference method with the iterative algorithm introduced in [2] and the parameters are given as follows: $N=1,000,000$, $K=1,180$, $M=400$, $r_{d,0}=0.025$, $r_{d,1}=0.010$, $r_{f,1}=r_{f,1}=0.005$, $\sigma_0 = 0.10$, $\sigma_1 = 0.25$.

In Table1, we give the prices for different values of transition intensity $\lambda$. First, the effect of regime switching is larger for small FX rates (USD/KRW). Second, as the transition intensity increases, the difference between prices in two regimes decreases and the effect of regime switching decreases.

In Table2, we consider the prices for different values of the contract maturity $T$. We can see that the effect of regime switching is more substantial for large maturities.

<table>
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<tr>
<th>$\lambda$</th>
<th>Regime0 FX=1,160</th>
<th>Regime1 FX=1,160</th>
<th>Regime0 FX=1,180</th>
<th>Regime1 FX=1,180</th>
<th>Regime0 FX=1,200</th>
<th>Regime1 FX=1,200</th>
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<td>-6.72</td>
<td>-6.96</td>
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Pricing FX target redemption forward

Table 2
Prices for different maturity $T$. $\lambda = 1$. (Hundred million won)

<table>
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<th>$T$</th>
<th>Regime 0 $FX=1,160$</th>
<th>Regime 1 $FX=1,160$</th>
<th>Regime 0 $FX=1,180$</th>
<th>Regime 1 $FX=1,180$</th>
<th>Regime 0 $FX=1,200$</th>
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</table>

4 Conclusion

In this paper, we derive the coupled partial differential equation for valuing FX target redemption forward under regime switching market model. We can solve the problem by using the iterative method introduced in [2]. The effect of regime switching is large for small level of FX, small transition intensity and large maturity.

References


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