Root Square Mean Labeling of Graphs

S. S. Sandhya

Department of Mathematics
 SreeAyyappa College for Women
 Chunkankadai: 629003, India

S. Somasundaram

Department of Mathematics
 ManonmaniamSundaranar University
 Tirunelveli: 627012, India

S. Anusa

Department of Mathematics
 Arunachala College of Engineering for Women
 Vellichanthai-629203, India

Copyright © 2014 S. S. Sandhya, S. Somasundaram and S. Anusa. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \dots, q + 1$ in such a way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor \text{ or } \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil, \text{ then the resulting edge labels are}$$

distinct. In this case f is called a Root Square Mean labeling of G . In this paper we prove that Path P_n , Cycle C_n , Comb, Ladder, Triangular Snake T_n , Quadrilateral Snake Q_n , Star $K_{1,n}$, $n \leq 6$, Complete graph K_n , $n \leq 3$ are Root Square Mean graphs.

Keywords: Graph, Mean labeling, Root Square Mean labeling, Path, Cycle, Comb, Ladder, Triangular snake, Quadrilateral snake, Star $K_{1,n}$, Complete graph K_n

1. Introduction

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary[2]. The concept of mean labeling has been introduced by S.Somasundaram and R.Ponraj in 2004 [3]. Motivated by the above works we introduce a new type of labeling called Root Square Mean labeling. In this paper we investigate the Root Square Mean labeling of Path, Cycle, Comb, Ladder, Triangular Snake, Quadrilateral Snake, Complete graph, Star. We will provide a brief summary of definitions and other information's which are necessary for our present investigation.

Definition1.1:

A walk in which $u_1u_2 \dots u_n$ are distinct is called a path. A path on n vertices is denoted by P_n .

Definition1.2:

A closed path is called a cycle. A cycle on n vertices is denoted by C_n .

Definition1.3:

The graph obtained by joining a single pendent edge to each vertex of a path is called as Comb.

Definition1.4:

The Cartesian product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = (V, E)$ with $V = V_1 \times V_2$ and two vertices $u = (u_1u_2)$ and $v = (v_1v_2)$ are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 is adjacent to v_2) or ($u_2 = v_2$ and u_1 is adjacent to v_1). It is denoted by $G_1 \times G_2$.

Definition1.5:

The product graph $P_2 \times P_n$ is called a ladder and it is denoted by L_n .

Definition1.6:

A Triangular Snake T_n is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n - 1$. That is every edge of a path is replaced by a triangle C_3 .

Definition1.7:

A Quadrilateral Snake Q_n is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i . That is every edge of a path is replaced by a cycle C_4 .

Definition1.8:

A Complete bipartite graph is a bipartite graph with bipartition (V_1, V_2) such that every vertex of V_1 is joined to all the vertices of V_2 . It is denoted by $K_{m,n}$ where $|V_1| = m$ and $|V_2| = n$.

Definition1.9:

A Star graph is a complete bipartite graph $K_{1,n}$.

Definition1.10:

A graph G is said to be complete, if every pair of its distinct vertices are adjacent. A complete graph on n vertices is denoted by K_n .

2. Main Results

Theorem2.1: Any path P_n is a Root Square Mean graph.

Proof: Let P_n be the path $u_1 u_2 \dots u_n$.

Define a function $f: V(P_n) \rightarrow \{1, 2, \dots, q+1\}$ by $f(u_i) = i$, $1 \leq i \leq n$.

Then the edges are labeled with $f(u_i u_{i+1}) = i$, $1 \leq i \leq n-1$.

Hence f is a Root Square Mean labeling.

Example2.2: The Root Square Mean labeling of P_6 is given below.

Figure: 1

Theorem2.3: Any cycle C_n is a Root Square Mean graph.

Proof: Let C_n be the cycle $u_1 u_2 \dots u_n u_1$.

Define a function $f: V(C_n) \rightarrow \{1, 2, \dots, q+1\}$ by $f(u_i) = i$, $1 \leq i \leq n$.

Then the edge labels are distinct. Hence Cycle C_n is a Root Square Mean graph.

Example2.4: Root Square Mean labeling of C_9 is given below.

Figure: 2

Theorem 2.5: Combs are Root Square Mean graphs.

Proof: Let G be a comb with vertex set $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$. Let P_n be the path $u_1 u_2 \dots u_n$ and join a vertex v_i to u_i , $1 \leq i \leq n$.

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_i) = 2i - 1, \quad 1 \leq i \leq n,$$

$$f(v_i) = 2i, \quad 1 \leq i \leq n.$$

Then the edges are labeled with

$$f(u_i u_{i+1}) = 2i, \quad 1 \leq i \leq n-1,$$

$$f(u_i v_i) = 2i - 1, \quad 1 \leq i \leq n.$$

Hence Comb is a Root Square Mean graph.

Example 2.6: Root Square Mean labeling of Comb obtained from P_6 is given below.

Figure: 3

Theorem 2.7: The Ladder $P_n \times P_2$ is a Root Square Mean graph.

Proof: Let G be the Ladder graph. Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$. Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_i) = 3i - 1, \quad 1 \leq i \leq n,$$

$$f(v_i) = 3i - 2, \quad 1 \leq i \leq n.$$

Then the edges are labeled with

$$f(u_i v_i) = 3i - 2, \quad 1 \leq i \leq n,$$

$$f(v_i v_{i+1}) = 3i - 1, \quad 1 \leq i \leq n - 1,$$

$$f(u_i u_{i+1}) = 3i, \quad 1 \leq i \leq n - 1.$$

Then we get distinct edge labels.

Hence Ladder is a Root Square Mean graph.

Example 2.8: The Root Square Mean labeling of L_6 is given below.

Figure: 4

Theorem 2.9: Triangular Snake T_n is a Root Square Mean graph.

Proof: Let T_n be a triangular snake. Define a function $f: V(T_n) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_i) = 3i - 2, \quad 1 \leq i \leq n,$$

$$f(v_i) = 3i - 1, \quad 1 \leq i \leq n - 1.$$

Then the edges are labeled with

$$f(u_i v_i) = 3i - 2, \quad 1 \leq i \leq n - 1,$$

$$f(u_i u_{i+1}) = 3i - 1, \quad 1 \leq i \leq n - 1,$$

$$f(u_{i+1} v_i) = 3i, \quad 1 \leq i \leq n - 1.$$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling.

Example 2.10:

The Root Square Mean labeling of T_6 is given below.

Figure: 5

Theorem 2.11: Any Quadrilateral Snake Q_n is a Root Square Mean graph.

Proof: Let Q_n be the Quadrilateral Snake.

Define a function $f: V(Q_n) \rightarrow \{1, 2, \dots, q + 1\}$ by

$$f(u_1) = 1, f(u_i) = 4i - 5, \quad 2 \leq i \leq n,$$

$$f(v_i) = 4i - 2, \quad 1 \leq i \leq n - 1,$$

$$f(w_i) = 4i, \quad 1 \leq i \leq n - 1.$$

Then the edges are labeled with

$$f(u_i v_i) = 4i - 3, \quad 1 \leq i \leq n - 1,$$

$$f(u_i u_{i+1}) = 4i - 2, \quad 1 \leq i \leq n - 1,$$

$$f(u_{i+1} w_i) = 4i - 1, \quad 1 \leq i \leq n - 1,$$

$$f(v_i w_i) = 4i, \quad 1 \leq i \leq n - 1.$$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling.

Example 2.12 Root Square Mean labeling of Q_6 is given below.

Figure: 6

Theorem 2.13 $K_{1,n}$ is a Root Square Mean graph if and only if $n \leq 6$.

Proof: $K_{1,1}$, $K_{1,2}$ are Root Square Mean graphs by theorem 2.1

Let the central vertex of the star be u . The other vertices are $v_1, v_2, \dots, v_n$ respectively. Here we consider the following cases.

Case (i) $2 \leq n \leq 5$, Assign 1 to u and $i + 1$ to v_i ($1 \leq i \leq n$). The labeling pattern is shown below.

$K_{1,2}$

$K_{1,3}$

$K_{1,4}$

$K_{1,5}$

Figure: 7

Case (ii) For $n = 6$, Assign 2 to u and $v_1 = 1, v_i = i + 1, 2 \leq i \leq n$. Clearly this labeling pattern is a Root Square Mean labeling and is shown below.

$K_{1,6}$

Figure 8

Case (iii) Assume $n > 6$.

Let the label of the vertex u be 2 and $v_1 = 1, v_i = i + 1, 2 \leq i \leq n$.

Figure: 9

The edges uv_6 and uv_7 get the same edgelabels, which is not possible.

Sub case (i) If $f(u) > 2$

Then there is no edge with label 1, which is not possible.

From case(i), case(ii), case(iii), we conclude that $K_{1,n}$ is a Root Square Mean graph if and only if $n \leq 6$.

Theorem 2.14: K_n is a Root Square Mean graph if and only if $n < 4$.

Proof: Here we consider two cases

Case(i) $n = 2, 3, 4$

Clearly K_2 and K_3 are Root Square Mean graphs. The labeling pattern of K_2, K_3 and K_4 are given below.

Figure: 10

Case(ii) $n > 4$

If $n > 4$ we have repetition of edge labels. Which is not possible.

Hence $K_n, n > 4$ is not a Root Square Mean graph.

If $n = 5$ The Root Square Mean labeling of K_5 is given below.

Figure: 11

In the above figure we have the repetition of the edge labels (2,10), (4,8) and (4,10), which is not possible. Hence K_n is a Root Square Mean graph if and only if $n < 4$.

Conclusion

All graphs are not Root Square Mean graphs. It is very interesting to investigate graphs which admit Root Square Mean labeling. In this paper, we proved that Path, Cycle, Comb, Ladder, Triangular Snake, Quadrilateral Snake, Star and Complete graph are Root Square Mean graphs. It is possible to investigate similar results for several other graphs.

Acknowledgements. The authors thank the referees for their comments and valuable suggestions.

References

- [1] J.A.Gallian, 2010, A dynamic Survey of graph labeling. The electronic Journal of Combinatorics17#DS6 .
- [2] F.Harary, 1988, Graph Theory, Narosa Publishing House Reading, New Delhi.
- [3] R.Ponraj and S.Somasundaram2003, Mean labeling of graphs, National Academy of Science Letters vol.26, p210-213.

Received: October 7, 2014; Published: November 19, 2014