Abstract

Let $G$ and $H$ be two digraphs. $G$ is $H$–colored when there exists a function $f : A(G) \rightarrow V(H)$. A walk $C = (v_1, v_2, \ldots, v_n)$ in $G$ is an $H$–walk if the list $(f(v_1, v_2), \ldots, f(v_{n-1}, v_n))$ is a walk in $H$.

In addition, let $N \subseteq V(G)$ $N$ is a kernel by $H$–walks if the following conditions are held:

1) If $x \in V(G) - N$, then there exists an $x, y - H$–walk for some $y \in N$.

2) Let $x, y \in N$, with $x \neq y$, then there is no $x, y - H$–walk in $G$.

When every induced subdigraph of $D$ has a kernel by $H$–walks we say that $D$ is kernel perfect by $H$–walks.

In this work we introduce the following result:

Let $H$ be a digraph such that $|V(H)| \leq 2$, and suppose that $H$ is not isomorphic to the tournament of 2 vertices. For every $H$–colored tournament $T$ such that every $C_3$ (a cycle of length 3) in $T$ contains an $H$–walk of length of at least 2, then $T$ is kernel perfect by $H$–walks.

This result generalizes the one obtained by Sands, Sauer and Woodrow: Every 2–colored tournament has a vertex $x$ such that for every other vertex $u$ in the tournament there exists a $ux$–monochromatic path where all of its arcs are colored alike.

Mathematics Subject Classification: 05C20
Keywords: kernel, $H$-walks, tournaments

1 Introduction

A digraph $D$ is a couple $(V, A)$, where $V \neq \emptyset$ and $A \subseteq V \times V$. The elements of $V$ are called vertices, and the elements of $A$ are called arcs (the elements $(x, x)$ in $A$ are called loops). A list of vertices $(x_1, x_2, \ldots)$ is a walk in $D$ if for every $i$ we have that $(x_i, x_{i+1}) \in A$. Furthermore, if the list is finite and all the vertices are different except the first and the last one we say that the list is a cycle. We denote the cycle of length $n$ (which has $n$ different vertices) by $C_n$. A cycle is Hamiltonian when it contains all the vertices of the digraph. In addition, a walk where any two vertices are different is called a path.

A set $K$ of vertices of the digraph $D$ is a kernel if $K$ satisfies the following conditions:
1) For every vertex $x$ in $V(D) - K$ there exists a vertex $y$ in $K$ such that $(x, y) \in A(D)$.
2) For every pair of vertices of $K$ there exists no arc between them since $K$ is independent.

The definition of kernel was introduced by Von Neumann and Morgenstern in [13] by searching for solutions in the area of Game Theory, because a kernel of a digraph represents a perfect strategy in every game that is represented by a digraph. This is a particular case of domination in graphs, this topic has been widely studied by several authors, a very complete study of this topic is presented in [10, 11]

The concept of kernel was generalized by H. Galeana-Sánchez in [3]. In such generalization it was considered a colored digraph, it means, a digraph where each arc has been assigned one and just one color. That generalization does not demand the independence and absorbency by arcs; instead, it is asking for the independence and absorbency by monochromatic paths. A monochromatic path is a path where all its arcs have been assigned the same color. For this reason these sets are called kernels by monochromatic paths. Since then this generalization has been analyzed in many articles; see for example [4-9,12]. This kind of kernels are used for solving many problems, such as the problems related to optimization and decision making.

Over the years, many studies have again generalized the definition of kernel. In such way, not only monochromatic paths were considered, the initial response was to think in terms of using the vertices of a digraph $H$ to color
another digraph $D$; however, here the generalizations were diversified considering different conditions of absorbency and independence.

In this work we consider the definition of kernel by $H$-walks, which is one of the more extensive generalizations of kernel and kernel by monochromatic paths.

## 2 Kernels by $H$-walks

**Definition 2.1** Let $G$ and $H$ be two digraphs. We say that $G$ is **$H$-colored** when there exists a function $f : A(G) \to V(H)$. A walk $C = (v_1, v_2, \ldots, v_n)$ in $G$ is an **$H$-walk** if the list $(f(v_1, v_2), \ldots, f(v_{n-1}, v_n))$ is a walk in $H$; see Figure 1.

![Figure 1: The walk $(v_2, v_3, v_5, v_1)$ is an $H$-walk, because the list $(1, 2, 1)$ is a walk in $H$, but $(v_4, v_5, v_2, v_1)$ is not an $H$-walk.](image)

**Definition 2.2** Let $H$ be a digraph and $G$ be an $H$-colored digraph, then $S \subseteq V(G)$ is a **kernel by $H$-walks** if it satisfies the following two conditions:

i) For every $x \in V(G) - S$, there exists an $x, y - H$-walk in $G$ for some $y \in S$ (we say that $S$ is absorbent by $H$-walks).

ii) For every $x, y \in S$, there exists no $x, y - H$-walk in $G$; we then say that $S$ is independent by $H$-walks.

Furthermore, when the kernel by $H$-walks is compressed into a single vertex we say that this vertex is an **$H$-sink**, and when every induced subdigraph of a digraph has a kernel by $H$-walks we say that the digraph is **kernel perfect by $H$-walks**.

To facilitate the eventual writing we use the following notation; $x \rightarrow y$ means that there is an arc from $x$ to $y$, and $x \xrightarrow{1} y$ means that there is an arc of color 1 from $x$ to $y$. 
This generalization has not been studied much, although it was briefly analyzed in [1]. Few necessary or sufficient conditions for the existence of kernels by \( H \)-walks in a digraph are known. These conditions can be for the digraph \( G \) or for the digraph \( H \), the one that we use to color [2]. For this reason we are interested in the study of this kind of kernels, especially to establish conditions to make certain the existence of this kernel in a digraph \( G \).

From the study of kernels by monochromatic paths, the following result for tournaments [14] was generated.

**Theorem 2.3** (Sands, Sauer and Woodrow) Every 2-colored tournament has a kernel by monochromatic paths.

The following problem was also introduced in the same article.

**Problem 2.4** Let \( T \) be a tournament which has arcs colored with \( m \) colors and contains no 3-colored \( C_3 \). Does \( T \) have a kernel by monochromatic paths?

According to this problem, in 1988 [12] Shen Minggang proved that if \( T \) is an \( m \)-colored tournament such that every cycle of length 3 and every transitive tournament of order 3 are at most 2-colored, then \( T \) has a kernel by monochromatic paths. In [5] it was proved that the result is the best possible for \( m \geq 4 \); therefore, the problem stated by Sands, Sauer and Woodrow is still open for three colors.

By considering the structure presented in Theorem 2.3 and by using the idea of kernel by \( H \)-walks, we raised a generalized theorem. In this article we give sufficient conditions for the existence of kernels by \( H \)-walks in \( 2 - H \)-colored tournaments.

### 3 Main Results

The following theorem is useful for the proof of the main result.

**Theorem 3.1** Let \( H \) be a digraph such that \( |V(H)| \leq 2 \), and let \( T_m \) be an \( H \)-colored tournament with \( m \) vertices such that every \( C_3 \) has an \( H \)-walk of length of at least 2. Suppose that \( m \) is the smallest number such that \( T_m \) has no \( H \)-sink, then \( T_m \) contains a Hamiltonian cycle \((x_0, x_1, x_2, ..., x_m-1, x_0)\) where every \( x_i \) is an \( H \)-sink of \( T_m - \{x_{i+1}\} \), with the indices taken modulo \( m \).

**Proof.** First, note that \( m > 2 \). Now, since \( T_m \) is the smallest \( H \)-colored tournament that satisfies the conditions of Theorem 3.1, then for every \( x \in V(T_m) \) there exists \( y \in V(T_m - \{x\}) \) which is an \( H \)-sink of \( T_m - \{x\} \).
We then define the function \( f: V(T_m) \to V(T_m) \), as follows: Let \( x \in V(T_m) \), then \( f(x) \) is an \( H \)-sink of \( T_m - \{x\} \).

We have that \( f \) is injective. Suppose that there exists \( \{x, x'\} \subseteq V(T_m) \), such that \( x \neq x' \) and \( f(x) = s = f(x') \), then \( s \) is an \( H \)-sink of \( T_m - \{x\} \) and also \( s \) is an \( H \)-sink of \( T_m - \{x'\} \). In addition, \( s \) absorbs by \( H \)-walks all the vertices of \( T_m \), which is a contradiction.

Since \( |V(T_m)| \) is finite, then \( f \) is bijective. Furthermore, for every vertex \( x \) cannot exist an \( H \)-walk from \( f^{-1}(x) \) to \( x \) in \( T_m \); otherwise \( x \) would be an \( H \)-sink of \( T_m \).

In addition, suppose that there exists \( S \subseteq V(T_m) \) such that \( f(S) = S \). It follows from \( |S| < |V(T_m)| \) and the choice of \( m \) that there exists \( s \in S \) which is an \( H \)-sink of \( (S)_{T_m} \) (the subdigraph of \( T_m \) generated by \( S \)). We obtain that \( s \) absorbs by \( H \)-walks all the elements of \( S \), including \( f^{-1}(s) \in S \); therefore, there exists an \( H \)-walk from \( f^{-1}(s) \) to \( s \), a contradiction. Consequently, \( S \subseteq V(T_m) \) does not exist such that \( f(S) = S \).

Considering the previous results we can form the following Hamiltonian cycle. Let \( x_0 \) be a vertex in \( V(T_m) \); now take \( x_i = f^{-i}(x_0) \) for every \( i \in \{1, 2, 3, ..., m - 1\} \), obtaining the Hamiltonian cycle \( (x_0, x_1, x_2, ..., x_m-1, x_0) \) which satisfies that \( x_i \) is an \( H \)-sink of \( T_m - \{x_{i+1}\} \). □

Now, let us pass to an important detail that we use in the classification of the possible cases analyzed in the proof of our main result: there exist 12 non-isomorphic digraphs with 1 or 2 vertices.

![Figure 2: All the non-isomorphic digraphs with at most 2 vertices.](image-url)
Now, we can pass to the main result. Let $H_1$ be the digraph such that $V(H_1) = \{\text{green}, \text{red}\}$ and $A(H_1) = \{(\text{green}, \text{red})\}$, as seen in Figure 3.

Remember that a digraph $D$ is transitive if for different vertices $u$, $v$ and $w$ such that $\{(u,v),(v,w)\} \subset A(D)$ we have $(u,w) \in A(D)$. Furthermore, a tournament is transitive if and only if it is acyclic.

![Diagram](image)

Figure 3: $H_1$.

**Theorem 3.2** Let $H$ be a digraph such that $|V(H)| \leq 2$ with $H$ not isomorphic to $H_1$, and let $T$ be an $H$–colored finite tournament such that every $C_3$ in $T$ has an $H$–walk of length of at least 2. $T$ then has an $H$–sink.

**Proof.**

Suppose by contradiction, that there exists an $H$–colored tournament that satisfies the hypothesis of Theorem 3.2 and has no $H$–sink. Now, let $T_m$ be a tournament with the minimum number of vertices with these conditions. By Theorem 3.1 there exists a Hamiltonian cycle, say $C = (x_0, x_1, x_2, \ldots, x_{m-1}, x_0)$, such that $x_i$ is an $H$–sink of $T_m - \{x_{i+1}\}$. Note that since $T_m$ has no $H$–sink, then no $x_{i+1}, x_i - H$–walk exists. We may suppose that $A(H) \neq \emptyset$; otherwise, all the $H$–walks are trivial. In addition, $T_m$ cannot contain any $C_3$, so $T_m$ is a transitive tournament which contradicts that it is Hamiltonian. Now, since $T_m$ has no $H$–sink we may assume that $|V(H)| = 2$, say $V(H) = \{\text{green}, \text{red}\}$. Also, note that if $H$ contains a loop $(a, a)$, then $C$ cannot be monochromatic of color $a$.

Now, we will see that all the following cases generate a contradiction with the choice of $T_m$.

**Case 1)** $A(H) = \{(\text{green}, \text{red}), (\text{red}, \text{green})\}$.

Suppose, without loss of generality, that $x_0 \xrightarrow{\text{green}} x_1$. Since $x_0$ is an $H$–sink of $T_m - \{x_1\}$, there then exists an $x_2, x_0 - H$–walk of the form $P_1 = (x_2 = z_0, z_1, z_2, \ldots, z_{k-1}, z_k = x_0)$.

Note that if $z_{k-1} \xrightarrow{\text{red}} x_0$, then $(x_2, z_1, z_2, \ldots, z_{k-1}, x_0, x_1)$ is an $x_2, x_1 - H$–walk, which results in a contradiction; therefore, $z_{k-1} \xrightarrow{\text{green}} x_0$. Now, suppose that there exists $j \in \{1, 2, 3, \ldots, k-1\}$ such that $z_j \rightarrow x_1$. Let $i$ be the smallest index such that $z_i \rightarrow x_1$ with $i \in \{1, 2, 3, \ldots, k-1\}$, then $(x_1, z_{i-1}, z_i, x_1)$ is a $C_3$. Furthermore, it contains an $H$–walk of length of at least 2. It follows that $(x_1, z_{i-1}, z_i)$ is an $H$–walk or $(z_{i-1}, z_i, x_1)$ is an $H$–walk. In the first case we find that $(x_1, z_{i-1}, z_i, z_{i+1}, \ldots, z_{k-1}, z_k = x_0)$ is an $x_1, x_0 - H$–walk, which is a contradiction with the properties of $T_m$. On the other hand, in the
second case \((x_2, z_1, \ldots, z_{i-1}, z_i, x_1)\) is an \(H\)-walk, which also generates a contradiction. Consequently, \(x_1 \rightarrow z_j\) for every \(j \in \{1, 2, 3, 4, \ldots, k - 1\}\), and \((x_1, z_{k-1}, x_0, x_1)\) is a \(C_3\). Since, \(z_{k-1} \xrightarrow{\text{green}} x_0 \xrightarrow{\text{green}} x_1\), then \(x_1 \xrightarrow{\text{red}} z_{k-1}\); thus, \(x_1 \xrightarrow{\text{red}} z_{k-1} \xrightarrow{\text{green}} x_0\) is an \(x_1, x_0 - H\)-walk, which is a contradiction.

**Case 2)** \(A(H) = \{(\text{red}, \text{red})\}\.\)

**Subcase 2.1** Suppose that there exists a change of color in \(C\). Now let \(\{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C)\) be such that \(x_i \xrightarrow{\text{green}} x_{i+1} \xrightarrow{\text{red}} x_{i+2}\). If \(x_{i+2} \rightarrow x_i\), then \((x_i, x_{i+1}, x_{i+2}, x_i)\) is a \(C_3\). The hypothesis thus implies that \(x_{i+2} \xrightarrow{\text{red}} x_i\), then \((x_{i+1}, x_{i+2}, x_i)\) is an \(x_{i+1}, x_i - H\)-walk, which is a contradiction; therefore, \(x_i \rightarrow x_{i+2}\). Hence, there exists an \(x_{i+2}, x_i - H\)-walk of length of at least 2, namely \(P_1\). Since the only \(H\)-walk of length of at least 2 is monochromatic of color \(\text{red}\), then \((x_{i+1}, x_{i+2}) \cup P_1\) is an \(x_{i+1}, x_i - H\)-walk, which is also a contradiction.

**Subcase 2.2** Suppose that \(C\) is monochromatic \(\text{green}\). First, take \(x_0 \xrightarrow{\text{green}} x_1 \xrightarrow{\text{green}} x_2\). Note that if \(x_2 \rightarrow x_0\), then we get a \(C_3\) without any \(H\)-walk of length of at least 2. We then have that \(x_0 \rightarrow x_2\), and there exists an \(H\)-walk from \(x_2\) to \(x_0\), say \(P_1 = (x_2 = z_0, z_1, \ldots, z_{k-1}, z_k = x_0)\) with \(k \geq 2\); \(P_1\) must be monochromatic of color \(\text{red}\). Suppose that there exists \(j \in \{1, 2, 3, \ldots, k - 1\}\) such that \(z_j \rightarrow x_1\). Second, let \(i \in \{1, 2, 3, \ldots, k - 1\}\) be the smallest index such that \(z_i \rightarrow x_1\), then \((x_1, z_{i-1}, z_i, x_1)\) is a \(C_3\), which implies that \((x_1, z_{i-1})\) is red or \((z_i, x_1)\) is red; therefore, \((x_1, z_{i-1}, z_i, z_{i+1}, \ldots, x_0)\) is an \(x_1, x_0 - H\)-walk or \((x_2, z_1, \ldots, z_{i-1}, z_i, x_1)\) is an \(x_2, x_1 - H\)-walk. In both cases we obtain a contradiction.

**Case 3)** \(A(H) = \{(\text{red}, \text{red}), (\text{red}, \text{green})\}\). Note that in this case every \(H\)-walk of length of at least 2 begins with a red arc.

**Subcase 3.1** Suppose that there exists in \(C\) a change of color. Now, let \(\{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C)\) be such that \(x_i \xrightarrow{\text{green}} x_{i+1} \xrightarrow{\text{red}} x_{i+2}\). Note that there must exist an \(x_{i+2}, x_i - H\)-walk of length of at least 2, namely \(P_1\). Since \(P_1\) begins with a \(\text{red}\) arc, and \((\text{red}, \text{red}) \in A(H)\), then \((x_{i+1}, x_{i+2}) \cup P_1\) is an \(H\)-walk, which is a further contradiction.

**Subcase 3.2** Suppose that \(C\) is monochromatic \(\text{green}\). Let \(i \in \{2, 3, 4, \ldots, m - 1\}\) be the smallest index such that \(x_i \rightarrow x_0\); note that this index exists because \(x_{m-1} \rightarrow x_0\). Note that there exists an \(x_{i-1}, x_{m-1} - H\)-walk (since \(x_{i-1} \neq x_0\)), say \(P_1\). As a result, \(P_1\) begins with a \(\text{red}\) arc or is a single green arc. If \(x_0 \xrightarrow{\text{red}} x_{i-1}\), then \((x_0, x_{i-1}) \cup P_1\) is an \(H\)-walk from \(x_0\) to \(x_{m-1}\), which is a contradiction. Now, \(x_0 \xrightarrow{\text{green}} x_{i-1} \xrightarrow{\text{green}} x_{i} \rightarrow x_0\). If \(x_i \xrightarrow{\text{green}} x_0\), then we have a \(C_3\) not admissible with the hypothesis; if \(x_i \xrightarrow{\text{red}} x_0\), then \((x_i, x_0, x_{i-1})\) is an
\[ x_i, x_{i-1} - H \text{-walk, which is a contradiction.} \]

**Case 4)** \( A(H) = \{(\text{red, red}), (\text{green, red})\}. \)

**Subcase 4.1** Suppose that there exists in \( C \) a change of color. Let \( \{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C) \) be such that \( x_i \xrightarrow{\text{red}} x_{i+1} \xrightarrow{\text{green}} x_{i+2} \). Let \( P_1 \) be an \( x_{i+2}, x_i - H \text{-walk} \). Since \( x_{i+2}, x_i - H \text{-walk} \), then \( P_1 \) is a single green arc or ends in a red arc. In any case \( P_1 \cup (x_i, x_{i+1}) \) is an \( x_{i+2}, x_{i+1} - H \text{-walk} \), which is a contradiction with the choice of \( T_m \).

**Subcase 4.2** \( C \) is monochromatic green. Let \( x_i \in V(T_m) \) be such that \( i \) is the smallest index that satisfies \( (x_i, x_0) \in A(T_m) \), then \( (x_0, x_{i-1}) \in A(T_m) \). In this way note that \( (x_0, x_{i-1}, x_i, x_0) \) is a \( C_3 \). If \( x_i \xrightarrow{\text{green}} x_0 \), then \( x_0 \xrightarrow{\text{red}} x_{i-1} \); otherwise we have a \( C_3 \) with no admissible coloration. Hence, \( x_i \xrightarrow{\text{green}} x_0 \xrightarrow{\text{red}} x_{i-1} \), and we obtain an \( x_i, x_{i-1} - H \text{-walk} \), which is a contradiction. Assume that \( x_i \xrightarrow{\text{red}} x_0 \). Since there exists an \( x_i, x_i - H \text{-walk} \), say \( P_1 \) is then a single green arc or ends in a red arc; therefore, \( P_1 \cup (x_i, x_0) \) is an \( x_i, x_0 - H \text{-walk} \), also a contradiction.

**Case 5)** \( A(H) = \{(\text{red, red}), (\text{green, red}), (\text{red, green})\}. \)

**Subcase 5.1** Suppose that there exists a change of color in \( C \). Let \( \{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C) \) be such that \( x_i \xrightarrow{\text{red}} x_{i+1} \xrightarrow{\text{green}} x_{i+2} \). There then exists an \( x_{i+2}, x_i - H \text{-walk} \), say \( P_1 \). Now \( P_1 \cup (x_i, x_{i+1}) \) is an \( x_{i+2}, x_{i+1} - H \text{-walk} \), which is a contradiction with the choice of \( T_m \).

**Subcase 5.2** \( C \) is monochromatic green. Let \( x_i \in V(T_m) \) be such that \( i \) is the smallest index such that \( (x_i, x_0) \in A(T_m) \), then \( (x_0, x_{i-1}) \in A(T_m) \). In this way we may note that \( (x_0, x_{i-1}, x_i, x_0) \) is a \( C_3 \). If \( x_i \xrightarrow{\text{green}} x_0 \), then \( x_0 \xrightarrow{\text{red}} x_{i-1} \); otherwise we have a \( C_3 \) without any \( H \text{-walk} \) of length of at least 2. Hence, \( x_i \xrightarrow{\text{green}} x_0 \xrightarrow{\text{red}} x_{i-1} \), and we obtain an \( x_i, x_{i-1} - H \text{-walk} \), which is a contradiction. If \( x_i \xrightarrow{\text{red}} x_0 \) then \( x_i \xrightarrow{\text{red}} x_0 \xrightarrow{\text{red}} x_{i-1} \) is an \( x_i, x_{i-1} - H \text{-walk} \) no matter the color of the arc \( (x_0, x_{i-1}) \) or a contradiction.

**Case 6)** \( A(H) = \{(\text{red, red}), (\text{green, green})\}. \)
We have that there exists a change of color in \( C \). Let \( \{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C) \) be such that \( x_i \xrightarrow{\text{green}} x_{i+1} \xrightarrow{\text{red}} x_{i+2} \). Let \( P_1 \) be an \( x_{i+2}, x_i - H \text{-walk} \). Since every \( H \text{-walk} \) is monochromatic, then \( P_1 \cup (x_i, x_{i+1}) \) or \( (x_{i+1}, x_{i+2}) \cup P_1 \) is an \( x_{i+2}, x_{i+1} - H \text{-walk} \) or an \( x_{i+1}, x_i - H \text{-walk} \) respectively, which is a contradiction.

**Case 7)** \( A(H) = \{(\text{red, red}), (\text{green, green}), (\text{red, green})\}. \)
We have that there exists a change of color in \( C \). Let \( \{x_i, x_{i+1}, x_{i+2}\} \subseteq V(C) \) be such that \( x_i \xrightarrow{\text{green}} x_{i+1} \xrightarrow{\text{red}} x_{i+2} \). Let \( P_1 \) be an
$x_{i+2}, x_i - H$-walk. Note that no matter which color has the last arc of $P_1$, $P_1 \cup (x_i, x_{i+1})$ is an $x_{i+2}, x_{i+1} - H$-walk which represents a contradiction.

**Case 8**) $A(H) = \{(\text{red, red}), (\text{green, green}), (\text{red, green}), (\text{green, red})\}$. Note that every walk in $T_m$ is an $H$-walk, since each list of vertices of $H$ is a walk. We then obtain a contradiction with the choice of $T_m$.

If $|V(H)| \leq 2$, and $H$ is not isomorphic to $H_1$, then every $H$-colored finite tournament $T$ such that every $C_3$ in $T$ has an $H$-walk of length of at least 2 has a kernel by $H$-walks.

**Corollary 3.3** Let $T$ be a tournament that satisfies the conditions of Theorem 3.2, then $T$ is kernel perfect by $H$-walks.

Further note that Theorem 3.2 generalizes Theorem 2.3. Since this result assures, among other things, the existence of a kernel by monochromatic paths in every 2-colored tournament. Because of this, any cycle of length 3 in a 2-colored tournament contains a monochromatic path of length of at least 2.

**Remark 3.4** The condition that $H$ is not isomorphic to $H_1$ cannot be eliminated from Theorem 3.2, given the following $H_1$-colored tournament.

Note that in the two tournaments every $C_3$ has a change of color; therefore, it satisfies the condition that have an $H$-walk of length of at least 2. Nevertheless, they have no $H$-sink, since each vertex $i$ does not absorb the vertex $i + 1$. 
Figure 4: In both digraphs we use the continuous line to denote the green color, and the dotted line to mark the red color.

References


H-paths in 2-colored tournaments


Received: April 30, 2015; Published: May 18, 2015